A mathematical model for precipitation in the Basque Country, Spain

IÑAKI ANTIGÜEDAD
Department of Geodynamics, University of the Basque Country, 48940 Leioa (Bizcaia), Basque Country, Spain
JESÚS GARCÍA MUÑIZ
Provincial Government of Bizcaia, 48009 Bilbao, Basque Country, Spain
JOSÉ LLAMAS
Faculté des Sciences et Génie, Université Laval, Sainte Foy, Quebec, Canada, G1K 7P4

Abstract Most hydrological phenomena in a specific region are, in general, non-homogeneous processes; such is the case of rainfall in the Basque Country of northern Spain. An analysis of precipitation events occurring there permits identification of different regions according to the mean storm occurrence rate and the mean number of storms per unit volume of precipitation. In this paper, a model of regional precipitation (Todorovic, 1967; Llamas, 1986) is applied to the Basque Country using recorded daily precipitation greater than 1 mm. The precipitation parameters mentioned above reveal two distinct pluviometric regions.

Un modèle mathématique des précipitations au Pays Basque, Espagne

Résumé Dans une région donnée la plupart des phénomènes hydrologiques ne sont en général pas homogènes. C'est le cas des précipitations dans le Pays Basque, au nord de l'Espagne. Une analyse des événement pluvieux qui s'y produisent a permis de déterminer différentes zones selon le taux d'occurrence moyen des averses et selon le nombre moyen d'averses par unité de précipitation. Dans le présent article, un modèle régional de précipitations (Todorovic, 1967; Llamas, 1986), s'appuyant sur les enregistrements de pluies journalières supérieures à 1 mm, a été appliqué au Pays Basque. Les paramètres mentionnés ci-dessus ont permis de mettre en évidence deux zones pluviométriquement distinctes.

INTRODUCTION

Broadly speaking, the Basque Country has two clearly distinct climatic regions: the Biscayan region, which drains into the Bay of Biscay; and the Ebro region, which drains, by way of the Ebro River, into the Mediterranean Sea (Fig. 1).

This study focuses on one meteorological variable: daily precipitation. The statistical sample used is composed of data from 38 rain stations, some of which are located in areas bordering the Basque Country.

The two regions mentioned above do not have the same pluviometric behaviour, the Biscayan region having a higher precipitation volume and a

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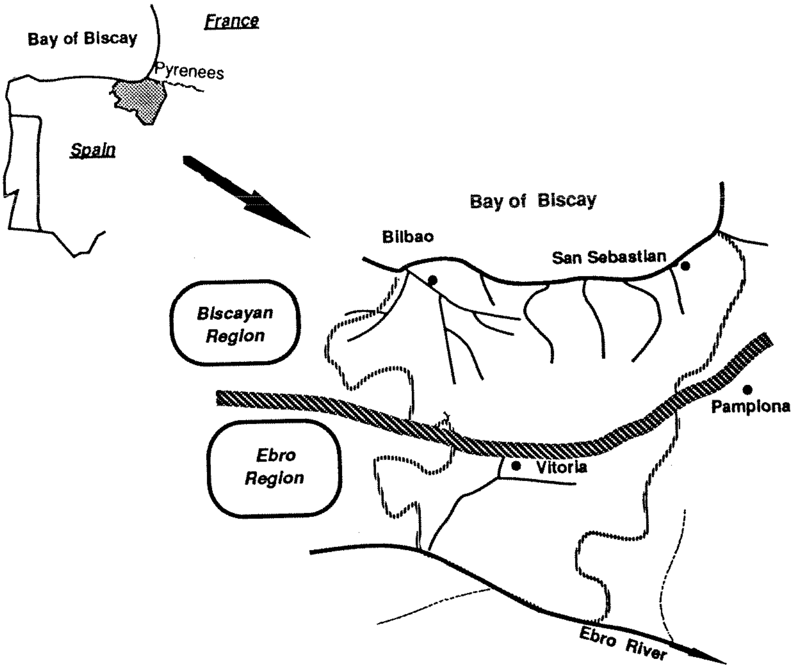

Fig. 1 Location of the Basque Country and its two meteorological regions.

more homogeneous rain pattern than those of the Ebro region. Only in mountainous areas, where orographic factors come into play, do the two regions provide similar pluviometric data. The boundary between the two regions, which are separate catchment areas, is a divide running from the western end of the Pyrenees to the western border of the Basque Country (Fig. 1). The height of the divide decreases progressively toward the west. It is along this divide that precipitation is the greatest.

In this study a mathematical model of regional precipitation (Todorovic, 1967; Llamas, 1986) is applied to each of the two regions, analysing the distribution function of the number of storms and the distribution of precipitation volume in a given period of time.

METHODOLOGY

For this analysis, precipitation samples were collected at each of the 38 stations from 1981 to 1988, taking the months of the calendar year (i.e. January-December) as sampling periods. With a sound hypothesis as a basis, one can obtain the distribution function of a hydrological phenomenon, using statistical methods for parameter estimation. This is the approach used in the analysis of precipitation in the Basque Country.

The precipitation phenomenon in a region may be defined in terms of events called independent storms. Throughout this study a storm is defined as a sequence of daily precipitation bounded at both ends of the wet period by two days of no precipitation or precipitation lower than a pre-established threshold. Owing to the lack of instrumental precision this threshold was taken as 1 mm day^{-1}. Lower precipitation amounts are barely detectable by existing raingauges.

The major part of the mathematical structure of the model used in this paper was first derived by Todorovic (1967) who established, under particular assumptions, that the distribution functions of storm events are of the Poisson type, and that the total precipitation depth, conditioned on a fixed number of storms, has a Gamma-type distribution.

Let $(0,t)$ be a time period in which a sample of ν complete storms has been recorded, and let λ_1 be the mean storm occurrence rate, i.e. the mean number of storms per unit of time. Under the assumption of a small time period, and if the numbers of storms in two non-overlapping periods are independent, the distribution function of ν is of the Poisson type (Dubreuil, 1974; Llamas, 1993):

$$P\left(E_{\nu}^{0,t}\right) = e^{-\lambda_1 t}\left(\lambda_1 t\right)^{\nu}/\nu! \tag{1}$$

The period $(0,t)$ must be meteorologically homogeneous, meaning that the probability of a storm occurring is the same at any moment in the period. The parameter λ_1 can be estimated by the relationship:

$$\lambda_1 = \bar{\nu}/t$$

where $\bar{\nu}$ is the mean number of storms observed during several homogeneous periods of common length t. If only one homogeneous period is available, $\bar{\nu}$ represents the total number of storms during that period.

The distribution function of total precipitation, x, produced by ν storms is (Todorovic,1967):

$$F(x \mid \nu) = 1 - e^{-\lambda_2 x} \sum_{j=0}^{\nu-1} \frac{(\lambda_2 x)^j}{j!} \tag{2}$$

The physical meaning of λ_2 is the inverse of the mean precipitation depth produced by a single storm. It can be estimated by:

$$\lambda_2 = \frac{\lambda_1 t}{x - x_0}$$

where x_0 is the total amount of precipitation at the beginning of the process.

This conditional distribution is of Gamma type; its mathematical justification arises from the assumption that λ_2 is invariable throughout the homogeneous period.

Combining this latter relationship with equation (1), according to the theorem of total probabilities the unconditional distribution of the total precipitation volume during a homogeneous period of length t is (Koulekey, 1977):

$$F(x,t) = P(x=0) + \sum_{\nu=1}^{M} \left[e^{-\lambda_1 x} \frac{(\lambda_1 x)^{\nu}}{\nu!} \left[1 - e^{-\lambda_2 x} \sum_{j=0}^{\nu-1} \frac{(\lambda_2 x)^j}{j!} \right] \right] \quad (3)$$

where M = maximum number of storms that can occur during the discrete period (0,t); and since the unit interval is one day, $M = t/2$ if t is even and $M = (t + 1)/2$ if t is odd.

Owing to further restrictions imposed on M, equation (3) is only an approximation to the true probability distribution. For $M = 15$ or 16 ($t = 30$ or 31) the asymptotical behaviour of $F(x,t)$ is quite satisfactory. $F(0,t)$ is the probability of having a completely dry period. When applied to the Basque Country, this model differentiates between the two aforementioned regions according to their pluviometric behaviour.

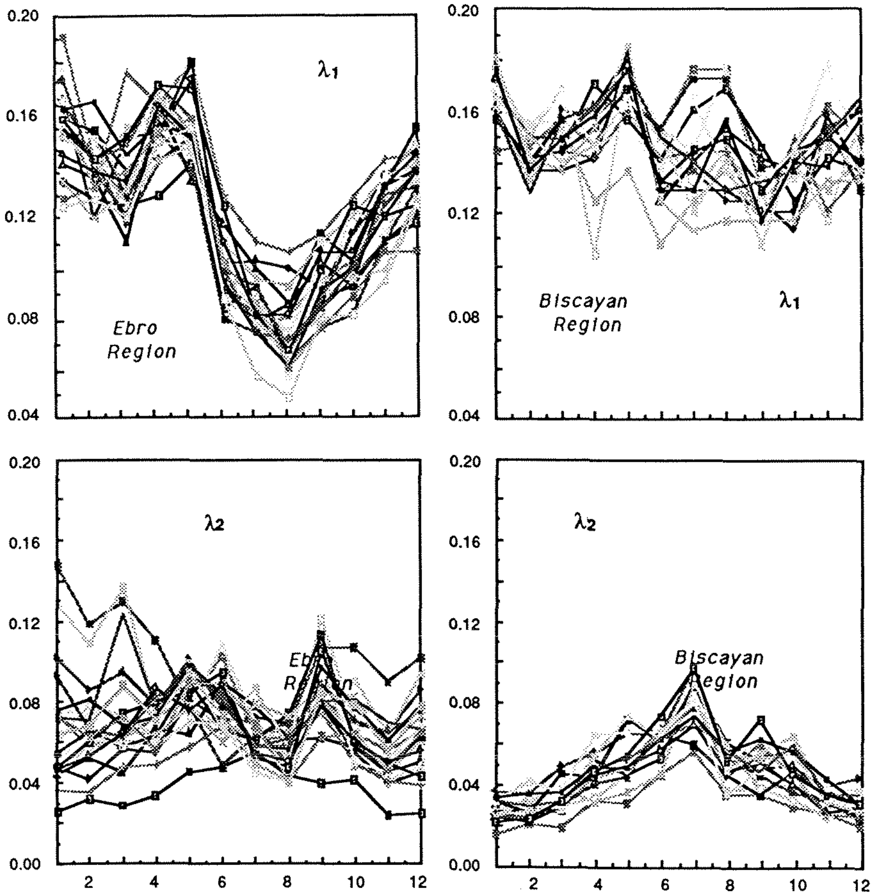

Fig. 2 Monthly fluctuations of λ_1 and λ_2 in the two regions of the Basque Country.

STATISTICAL ANALYSIS

The sample analysed consists of the daily precipitation series recorded at the 38 rain stations over an eight year period (January 1981-December 1988). A mathematical program Delta, developed by the authors, was used for data analysis.

The parameters λ_1 and λ_2, computed for each station and for each month, are plotted over a one year period in Fig. 2. The fluctuations of the parameters reveal that the Basque Country has two different meteorological regions: to the north, the Biscayan area, drained by rivers flowing into the Bay of Biscay; and to the south, the Ebro River basin, which ultimately drains into the Mediterranean Sea. The parameters λ_1 and λ_2 are plotted at monthly intervals because their fluctuations are such that time periods longer than one month are non-homogeneous.

With the mean monthly values of λ_1 and λ_2 evaluated for the two regions, it was convenient to model those parameters by statistical regression using a polynomial adjustment of the computed values. The objective of this transformation was only to facilitate computer programming using a mathematical expression instead of a table of 12 different values. Obviously, the polynomial transformation would be acceptable only if the main objective, i.e. the generation of extreme values of precipitation, were reached with similar accuracy using computed or transformed values of the parameters. This condition has been proven conveniently by regression.

Table 1 shows the mean monthly values of λ_1 and λ_2 computed from the historical data and adjusted by the polynomial transformation (computed (E) and transformed (P) values). Figure 3 shows the polynomial regression of the computed values.

Biscayan region:

$$\begin{aligned}\lambda_1 = {} & 0.24 - 0.12x + 5.13\ 10^{-2}\,x^2 - 9.35\ 10^{-3}\,x^3 + 7.51\ 10^{-4}\,x^4 \\ & - 2.19\ 10^{-5}\,x^5 \qquad r = 0.818\end{aligned}$$

$$\begin{aligned}\lambda_2 = {} & 5.07\ 10^{-2} - 3.54\ 10^{-2}\,x + 1.52\ 10^{-2}\,x^2 - 2.04\ 10^{-3}\,x^3 \\ & + 1.00\ 10^{-4}\,x^4 - 1.34\ 10^{-6}\,x^5 \qquad r = 0.923\end{aligned}$$

Ebro region:

$$\begin{aligned}\lambda_1 = {} & 0.27 - 0.17x + 8.64\ 10^{-2}\,x^2 - 1.82\ 10^{-2}\,x^3 + 1.61\ 10^{-3}\,x^4 \\ & - 5.06\ 10^{-5}\,x^5 \qquad r = 0.944\end{aligned}$$

$$\begin{aligned}\lambda_2 = {} & 5.74\ 10^{-2} + 7.73\ 10^{-3}\,x - 1.60\ 10^{-3}\,x^2 + 1.42\ 10^{-4}\,x^3 \\ & - 6.59\ 10^{-6}\,x^4 + 1.16\ 10^{-7}\,x^5 \qquad r = 0.442\end{aligned}$$

where $x = 1$ (January) ... 12 (December).

The small correlation coefficient of λ_2 (0.422) in the Ebro region is due exclusively to the unusually weak intensity of precipitation in the month of September in this region. This fact in no way invalidates the application of the

Table 1 Mean monthly values of λ_1 and λ_2 for the two regions

Month	Computed values (*E*)				Values derived by polynomial regression (*P*)			
	Biscayan		Ebro		Biscayan		Ebro	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
1	0.167	0.028	0.169	0.065	0.166	0.029	0.168	0.063
2	0.141	0.029	0.151	0.065	0.144	0.026	0.150	0.067
3	0.151	0.033	0.147	0.072	0.147	0.034	0.159	0.069
4	0.148	0.046	0.171	0.066	0.154	0.046	0.163	0.070
5	0.168	0.053	0.169	0.077	0.155	0.057	0.150	0.070
6	0.137	0.062	0.106	0.073	0.150	0.064	0.123	0.069
7	0.139	0.081	0.088	0.060	0.141	0.065	0.095	0.067
8	0.145	0.050	0.075	0.054	0.133	0.061	0.078	0.066
9	0.125	0.051	0.099	0.084	0.130	0.053	0.083	0.065
10	0.136	0.047	0.110	0.064	0.135	0.042	0.109	0.064
11	0.145	0.033	0.130	0.051	0.144	0.033	0.142	0.063
12	0.145	0.029	0.146	0.061	0.145	0.030	0.142	0.060

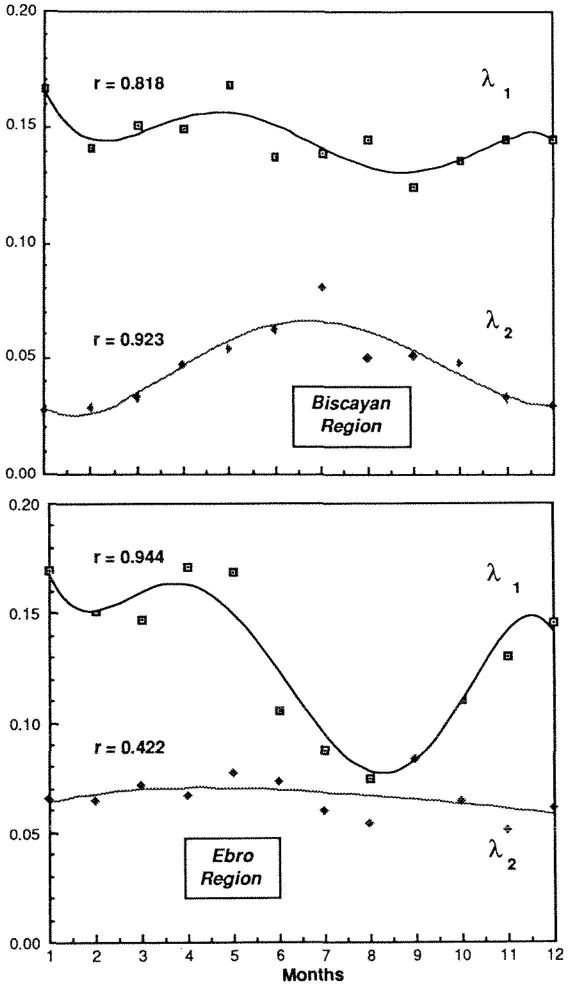

Fig. 3 Mean monthly values of λ_1 and λ_2 for the two regions and the corresponding polynomial transformation.

model to the Basque Country, because precipitation values, with several return periods, computed by the computed values of λ_2 have a significantly high correlation with the same values computed by the adjusted values of λ_2 estimated by polynomial regression. This correlation is a guarantee of model applicability.

For illustrative purposes, Table 2 gives precipitation values of 50 years return period obtained with the computed values of the parameters.

In order to complete the adequacy of the polynomial transformation of λ_1 and λ_2 a linear regression analysis was made with the values of precipitation corresponding to several return periods. Precipitation values were computed from equation (3) in two different ways: using computed (*E*) values of λ_1 and λ_2 and then by polynomial transformation (*P*).

Tables 3 and 4 show two different sets of total monthly precipitation corresponding to several return periods. The high correlations confirm, as shown on Fig. 4, that polynomial transformations of λ_1 and λ_2 can be

Table 2 Monthly precipitation values for a 50-year return period at the 38 stations

Station	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Bermeo	568.8	341.7	453.0	285.3	303.2	189.4	144.1	323.8	275.5	365.0	432.7	439.0
Durango	513.0	451.0	375.1	256.3	193.2	184.5	125.3	211.4	216.1	217.6	331.6	379.4
Urkulu	392.6	334.5	360.3	143.0	253.9	183.8	179.2	207.3	179.6	212.3	320.2	393.6
Valdegovia	160.3	178.3	171.0	231.2	153.6	111.4	129.8	75.8	132.3	143.6	214.6	160.8
Salinas	188.2	168.1	105.0	185.2	137.4	121.2	163.6	131.0	112.8	131.9	164.9	179.0
Haro	89.2	105.2	90.2	116.9	159.3	175.8	130.9	110.3	81.9	90.6	116.0	111.0
Balmaseda	513.3	451.0	375.5	256.3	193.1	184.5	125.3	211.4	216.0	217.5	331.6	379.5
Igorre	445.0	424.1	389.5	275.0	264.3	171.2	150.7	245.8	173.7	268.0	407.0	478.6
Armiñon	137.1	137.7	116.0	157.9	129.9	119.7	128.3	152.7	102.0	126.4	156.5	138.1
Zambrana	180.7	160.5	127.5	181.8	104.5	121.5	140.7	156.1	99.8	108.0	172.9	148.3
Vitoria	273.0	220.0	187.9	157.0	173.0	118.7	158.5	172.6	128.5	172.4	231.7	228.8
Lagran	248.0	189.7	214.1	202.9	165.9	137.7	140.2	120.5	93.6	171.8	251.1	212.0
Treviño	152.2	173.5	179.3	161.2	142.1	115.6	143.9	128.8	99.1	139.4	172.8	162.7
Miranda	102.2	107.0	84.0	164.3	127.5	136.5	97.7	127.8	112.0	107.4	151.9	124.1
Gamiz	190.8	160.5	146.4	193.3	170.0	93.1	129.3	144.1	97.2	183.9	207.7	191.9
Etxebarria	559.4	492.9	412.5	312.1	299.8	185.0	173.3	250.5	252.8	326.8	389.7	499.6
Amurrio	374.7	317.5	263.5	224.2	202.8	167.3	119.6	243.6	196.4	185.8	307.4	299.5
Ordunte	422.3	372.7	400.4	222.9	193.3	171.0	112.3	217.7	185.7	167.3	327.3	380.0
Legazpia	518.4	412.9	411.7	254.5	272.3	174.5	147.5	219.7	210.4	217.8	374.3	438.0
Peñacerrada	183.0	159.4	188.3	180.3	134.4	113.2	139.6	126.6	95.4	129.1	181.4	158.3
Opakua	269.3	196.0	217.6	217.8	184.4	137.0	200.9	175.1	100.0	207.5	256.7	252.1
Campezo	233.7	156.1	163.7	206.6	173.6	135.7	157.6	133.9	70.7	168.3	259.3	248.2
Bernedo	294.0	195.8	221.2	232.1	192.8	137.0	145.2	139.6	94.3	204.3	265.6	199.1
Iturrieta	375.3	316.5	307.1	277.8	255.7	166.1	174.1	163.8	142.2	232.3	300.6	325.3
Otxandio	601.1	459.6	399.7	272.5	251.6	163.4	122.1	215.9	157.6	256.3	347.8	433.0
Undurraga	428.4	366.2	381.3	255.7	259.0	176.9	158.5	218.7	222.0	197.3	324.2	369.8
Gazeta	285.6	219.3	236.3	202.3	190.9	120.9	127.1	146.5	159.1	161.3	256.1	240.7
Eibar	560.2	507.0	429.1	277.4	278.9	184.4	170.5	260.9	229.6	285.3	457.7	444.0
Añarbe	829.0	539.1	678.0	377.1	433.7	276.9	244.3	393.7	217.4	408.7	494.1	619.0
Bajauri	290.8	254.2	236.6	236.2	175.2	140.7	140.4	143.2	117.8	196.2	296.9	271.8
Arrieta	216.8	202.6	191.5	193.5	164.5	95.1	128.4	133.9	116.2	157.8	215.1	196.0
Sondika	416.6	278.6	322.8	183.1	196.9	180.9	134.1	335.7	207.2	206.8	302.5	306.8
Gordexola	362.4	373.4	252.0	257.5	175.3	159.2	142.2	214.1	257.8	244.0	308.0	380.9
Salvatierra	286.8	198.4	176.5	176.0	169.4	118.9	173.9	161.9	93.2	188.9	222.3	267.7
Laguardia	265.3	174.7	181.6	216.0	167.1	173.0	122.4	130.7	116.5	139.6	209.4	164.2
Abadiño	293.8	418.7	447.2	453.9	279.4	220.5	146.5	220.5	231.2	278.7	378.8	469.2
Bera Bida	546.7	338.1	328.3	341.5	316.8	215.5	123.5	194.1	269.1	284.8	367.2	417.9
Arluzea	460.0	348.9	399.4	340.1	275.1	180.8	173.0	175.6	222.8	227.5	428.3	439.5

Table 3 Precipitation values corresponding to several return periods in the Biscayan region

Month	*E* 5	*P* 5	*E* 10	*P* 10	*E* 50	*P* 50	*E* 100	*P* 100
1	290.6	266.8	367.0	331.5	490.9	461.8	570.7	513.0
2	214.3	239.3	280.9	304.4	404.5	436.7	480.0	489.0
3	216.3	202.2	273.3	253.8	392.9	358.6	436.9	400.4
4	149.4	151.0	187.8	189.5	273.4	267.6	304.7	298.3
5	148.7	126.5	184.8	158.3	256.9	222.1	286.0	247.2
6	103.2	106.6	131.1	134.1	187.6	189.7	210.4	211.7
7	83.0	101.6	104.2	128.1	148.2	182.0	165.6	203.2
8	138.3	103.0	174.0	130.8	246.1	187.5	275.3	209.4
9	119.8	114.4	154.0	145.9	217.2	210.0	256.5	235.6
10	141.2	151.2	178.8	191.5	255.3	273.5	285.4	305.2
11	203.1	197.1	256.5	248.9	264.9	354.2	401.5	395.5
12	242.0	227.7	295.9	286.5	419.2	405.4	468.0	452.9

Table 4 Precipitations corresponding to several return periods in the Ebro region

Month	*E* 5	*P* 5	*E* 10	*P* 10	*E* 50	*P* 50	*E* 100	*P* 100
1	134.0	121.0	167.0	150.2	232.5	208.9	258.3	231.8
2	105.8	80.1	134.1	101.4	191.5	144.8	211.3	161.9
3	105.6	106.0	132.7	132.2	187.7	184.9	208.2	205.7
4	117.1	104.0	144.6	129.8	201.5	181.8	224.7	202.3
5	99.4	100.0	123.2	125.3	168.9	176.6	184.3	196.8
6	67.9	83.3	90.4	106.9	131.8	155.1	149.5	174.2
7	71.9	69.8	95.6	91.8	145.0	137.5	164.5	155.8
8	68.7	60.6	93.4	81.6	140.5	125.8	166.4	143.5
9	58.7	63.7	76.0	85.6	113.9	131.4	128.6	149.8
10	85.1	85.7	110.0	111.0	161.3	163.0	181.8	183.7
11	124.9	108.0	156.4	136.6	225.3	194.8	247.7	217.7
12	118.4	116.4	148.9	146.8	210.5	208.4	230.3	232.7

performed, without significant risk, in the general precipitation model, thus facilitating programming. This stability is demonstrated specially well by the correlation coefficients obtained for the Biscayan region, which are consistently high and virtually identical for all four return periods.

ANALYSIS OF RESULTS

The regional precipitation analysis reveals the different pluviometric behaviour of the two regions that make up the Basque Country (Fig. 1). Monthly fluctuations of λ_1 and λ_2 throughout the year clearly demonstrate this difference. The general behaviour of λ_1 is quite stable on the Biscayan region, with values ranging between 0.11 and 0.19, whereas in the Ebro region the oscillation is wider, from 0.05 to 0.21, the lowest values corresponding to the summer months (Fig. 2). This means that, throughout the year, in the Biscayan region the number of storms is greater, and the variability is smaller, than in the Ebro region.

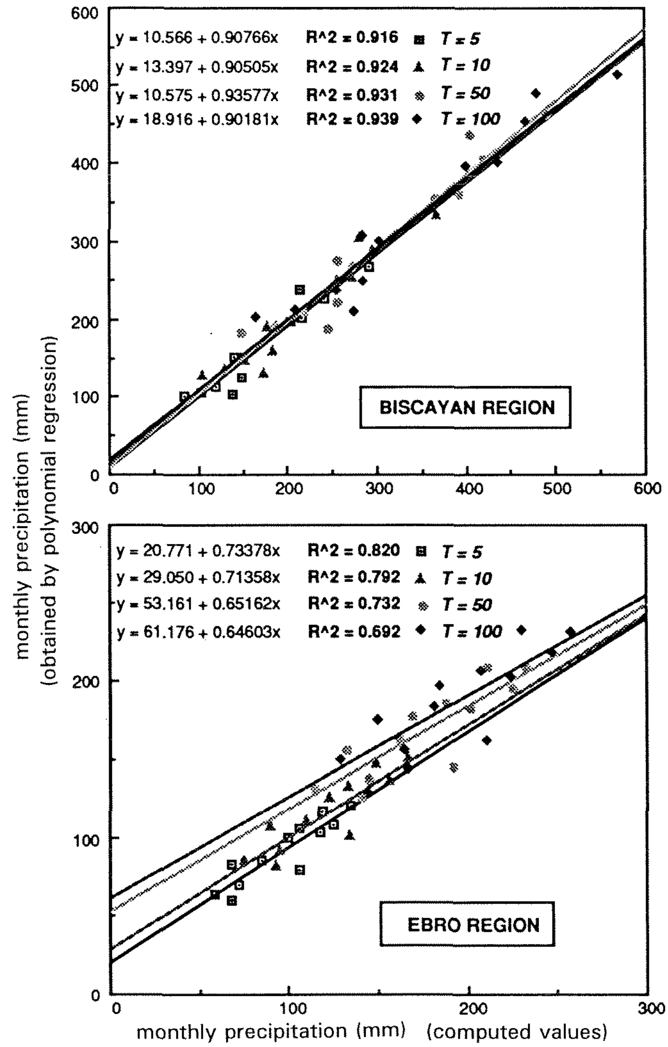

Fig. 4 Linear regression between computed and derived values for different return periods.

The precipitation volume per storm, as measured by the parameter λ_2, is also different in the two regions. The values of λ_2 are higher in the Ebro region, where 0.025 to 0.15 storms are required to produce 1 mm of precipitation (Fig. 2). Maxima of λ_2 occur in May, June, and September; minima in January, August and November.

On the other hand, the Biscayan region needs between 0.015 and 0.10 storms to produce 1 mm of precipitation with minima in December and January and a maximum in July. Hence the storm volume throughout the year is greater in the Biscayan region, with the one exception of July, when it is greater in the Ebro region.

The mean values of λ_1 and λ_2 (Table 1) reveal the same patterns. From January to May the number of precipitation events (λ_1) is highest in both regions though more markedly so in the Ebro region, whereas this number is lowest in August in this region and in September in the Biscayan region.

The heaviest storms, indicated by low values of λ_2, occur from November to March in the Biscayan region and from July to December in the Ebro region, with the exception of September, when storms are very light. The storm volume during the latter period in the Ebro region is similar to that found from May to October in the Biscayan region, with the exception of July in the latter region, when storms are very light.

The difference between the precipitation patterns of the two regions can be clearly seen on Fig. 2, as well as the degree of homogeneity of those patterns throughout the year. As indicated in the same Figure, neither region has perfect homogeneity from station to station. However, some stations present similar values of λ_1 and λ_2 in both regions because of the effect of orographic factors.

CONCLUSION

The model of regional precipitation has permitted identification of two different meteorological regions within the Basque Country according to the mean storm occurrence rate (λ_1) and the mean number of storms per unit volume of precipitation (λ_2) in each region. In both the Biscayan and Ebro regions, the storm occurrence rate during the wet months (November-May) is quite similar, oscillating within the same range of values, but the number of storms needed to yield 1 mm of precipitation is significantly higher in the Ebro region. During the dry period, the occurrence rate remains steady in the Biscayan region but decreases significantly in the Ebro region, whereas the storm volume decreases in the Biscayan region but increases (except in September) in the Ebro region. The two meteorological regions are separated by a water divide and some stations on its opposite sides show similar pluviometric behaviour because of their specific orographic environment.

The computer program, extended for extreme data generation, was significantly simplified by a polynomial transformation, the validity of which was demonstrated by the high correlation (0.83-0.97) between the precipitation values (for 5, 10, 50 and 100 year return periods) evaluated with computed values of λ_1 and λ_2 and those found with adjusted values of λ_1 and λ_2 using polynomial regression (Figs 3 and 4). This operation is, of course, optional and it can be performed only if precipitation values computed with rough and smooth parameters are similar, with respect to a statistical test. In the case here, correlation coefficients and Kolmogorov's statistic verify the validity of the polynomial adjustment (Llamas, 1986). This transformation simplifies computer processing without jeopardizing model accuracy.

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REFERENCES

Dubreuil, P. (1974) *Initiation à l'analyse hydrologique*. ORSTOM. Masson, Paris, France.

Koulekey, C. K. (1977) *Modèles mathématiques de précipitation*. Application à l'étude régionale de la pluviométrie au Togo. Thesis de Maîtrise. Université Laval, Québec, Canada.

Llamas, J. (1986) *Análisis matemático en hidrología*. Universidad Autónoma del Estado de Queretano. México.

Llamas, J. (1993) *Hydrologie générale: Principes et Applications* (2^e edn), ed. Gaètan Morin. Québec, Canada.

Todorovic, P. (1967) Applied Probability Course Notes. Fort Collins, Colorado, USA.

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