LETTER

Electrostatic Solution for Broadside-Coupled Striplines in a Shield

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SUMMARY The electrostatic characteristics of broadside-coupled striplines in a shield are investigated with the mode-matching method. The Fourier series is employed to describe electrostatic potential distributions. Numerical results are shown for coupled transmission line cell applications.

key words: broadside-coupled striplines, coupled transmission line, mode-matching

1. Introduction

Broadside-coupled striplines in a shield are basic guiding structures in coupled transmission line (CTL) cells that are important in EMI/EMC applications [1]–[3]. The CTL cells exhibit good performance in terms of field uniformity and bandwidth. The CTL cells consist of uniform and tapered sections. Higher-order TE and TM wave propagation along the uniform section is governed by Laplace’s equation for the electrostatic potential. The TEM wave in the uniform section was studied in [4]. Higher-order modes for offset striplines in a shield were studied in [5]. However, for the accurate estimation of field distribution in the CTL cells, an understanding of dominant TEM mode in the uniform section is also needed. It is therefore of practical interest to rigorously analyze TEM wave transmission along the uniform section of CTL cells. The TEM wave in the uniform section is governed by Laplace’s equation for the electrostatic potential \( \Phi \). While Helmholtz’s equation was used in [5] for higher-order mode analysis, Laplace’s equation will be used here for the TEM wave analysis. The purpose of this letter is to solve Laplace’s equation for broadside-coupled striplines in a shield using the mode-matching method [4], [5]. The analysis follows next.

2. Field Representations

Figure 1 shows a cross section of broadside-coupled striplines that are surrounded by a perfectly conducting rectangular shield. Assume that the electrostatic potentials at the upper stripline, at the lower stripline, and at the shield are \( V_1 \), \( V_2 \), and 0, respectively. The electrostatic potentials are governed by Laplace’s equation. To solve Laplace’s equation, the interior is divided into 7 regions, as shown in Fig. 1.

In region (I), \((-b < x < b, h + d < y < c)\), the electrostatic potential is given by

\[
\Phi_I = \sum_{m=1}^{+\infty} A_m \sinh(\alpha_m(y - c)) \sin(\alpha_m(x + b))
\]

where

\[
\alpha_m = m\pi/(2b) \quad \text{and} \quad m = 1, 2, 3 \ldots
\]

In region (II), \((-b < x < -a, h < y < h + d)\), the potential, using the superposition principle, is

\[
\Phi_{II} = \sum_{m=1}^{+\infty} (B_m \cosh(\beta_m y) + C_m \sinh(\beta_m y)) \sin(\beta_m(x + b))
\]

\[
+ \frac{4V_1}{\pi} \sum_{k=0}^{+\infty} \frac{\sinh(\gamma_{2k+1}(x + b))}{(2k + 1) \sinh(\gamma_{2k+1}(b - a))} \sin(\gamma_{2k+1}(y - h))
\]

where

\[
\beta_m = \frac{m\pi}{b - a}, \quad \gamma_{2k+1} = \frac{(2k + 1)\pi}{d}, \quad \text{and} \quad k = 0, 1, 2, \ldots
\]
Similarly, in region (III) \((a < x < b, h < y < h + d)\),
\[
\Phi^{III} = \sum_{m=1}^{\infty} (D_m \cosh (\beta_m y) + E_m \sinh (\beta_m y)) \sin (\beta_m (x - b)) - 4V_1 \sum_{k=0}^{\infty} \frac{\sinh (\gamma_{2k+1} (x - b))}{\pi} \sum_{n=1}^{\infty} (2k+1) \sinh (\gamma_{2k+1} (b - a)) \sin (\gamma_{2k+1} (y - h)) \quad (3)
\]
In region (IV) \((-b < x < b, -h < y < h)\)
\[
\Phi^{IV} = \sum_{m=1}^{\infty} (F_m \cosh (\alpha_m y) + G_m \sinh (\alpha_m y)) \sin (\alpha_m (x + b)) \quad (4)
\]
In region (V) \((-b < x < -a, -h - d < y < -h)\)
\[
\Phi^{V} = \sum_{m=1}^{\infty} (H_m \cosh (\beta_m y) + I_m \sinh (\beta_m y)) \sin (\beta_m (x + b)) - 4V_1 \sum_{k=0}^{\infty} \frac{\sin (\gamma_{2k+1} (x + b))}{\pi} \sum_{n=1}^{\infty} (2k+1) \sinh (\gamma_{2k+1} (b - a)) \sin (\gamma_{2k+1} (y + h)) \quad (5)
\]
In region (VI) \((a < x < b, -h - d < y < -h)\)
\[
\Phi^{VI} = \sum_{m=1}^{\infty} (J_m \cosh (\beta_m y) + K_m \sinh (\beta_m y)) \sin (\beta_m (x + b)) + 4V_1 \sum_{k=0}^{\infty} \frac{\sin (\gamma_{2k+1} (x + b))}{\pi} \sum_{n=1}^{\infty} (2k+1) \sinh (\gamma_{2k+1} (b - a)) \sin (\gamma_{2k+1} (y + h)) \quad (6)
\]
In region (VII) \((-b < x < b, -c < y < -h - d)\)
\[
\Phi^{VII} = \sum_{m=1}^{\infty} L_m \sinh (\alpha_m (y + c)) \sin (\alpha_m (x + b)) \quad (7)
\]
3. Boundary Conditions and Computations

In the previous section, 12 unknown modal coefficients \((A_m, B_m, \ldots, K_m, L_m)\) were introduced. To determine these modal coefficients, 12 boundary conditions are needed. For instance, the Dirichlet condition at \(y = h + d\) is
\[
\Phi^I = \begin{cases} 
\Phi^{II}, & \text{for } -b < x < -a \\
V_1, & \text{for } -a < x < a \\
\Phi^{III}, & \text{for } a < x < b
\end{cases} \quad (8)
\]
Similarly three additional Dirichlet conditions can be obtained from \(y = -h\), \(y = -h - d\), and \(y = h\). The Neumann boundary conditions at \(y = h + d\) are
\[
\frac{\partial \Phi^I}{\partial y} \bigg|_{y=h+d} = \frac{\partial \Phi^{II}}{\partial y} \bigg|_{y=h+d}, \text{ for } -b < x < -a \\
\frac{\partial \Phi^I}{\partial y} \bigg|_{y=h+d} = \frac{\partial \Phi^{III}}{\partial y} \bigg|_{y=h+d}, \text{ for } a < x < b \quad (9)
\]
Similarly additional Neumann conditions can be obtained from \(y = -h\), \(y = -h - d\), and \(y = h\). The mode-matching method utilizes the orthogonality of sinusoidal functions. For instance, we multiply the both sides of (8) by \(\sin (\alpha_n (x + b))\) and integrate it with respect to \(x\) from \(-b\)
to \(b\). Then (8) becomes
\[
b \sum_{m=1}^{\infty} A_m \sinh (\alpha_m (h + d - c)) \delta_{mn}
\]
\[
= \sum_{m=1}^{\infty} (B_m \cosh (\beta_m (h + d)) + C_m \sinh (\beta_m (h + d))) M_{1mn}
\]
\[+ \frac{2V_1}{\alpha_n} \sin (a \alpha_n) \sin (b \alpha_n)
\]
\[+ \sum_{m=1}^{\infty} (D_m \cosh (\beta_m (h + d)) + E_m \sinh (\beta_m (h + d))) M_{2mn} \quad (11)
\]
Here, \(\delta_{mn}\) is the Kronecker delta, and
\[
M_{1mn} = \int_{-b}^{a} \sin (\beta_m (x + b)) \sin (\alpha_n (x + b)) \, dx \quad (12)
\]
\[
M_{2mn} = \int_{-d}^{a} \sin (\beta_m (x - b)) \sin (\alpha_n (x + b)) \, dx \quad (13)
\]
Applying the mode-matching method to the remaining boundary conditions, it is possible to obtain a set of simultaneous equations for the modal coefficients. The simultaneous equations can be rewritten in compact matrix form (see Appendix). Computations were performed to check the accuracy of the mode-matching model. Figure 2 shows the equipotential lines when \(V_1 = 1, V_2 = -1\). The numbers of modes used are \(M = 5\) (in regions (I), (IV) and (VII)) and \(P = 1\) (in regions (II), (III), (V) and (VI)). Table 1 shows the values of series coefficient \(G_m\) when \(M = 5\) and \(P = 1\). We note that the series converge fast. However, care must be exercised in choosing \(M\) and \(P\) to ensure solution accuracy. The value \(\Phi\) near the edges of striplines is sensitive to the choice of \(M\) and \(P\). Thus an appreciable error may exist near the edges of striplines. Our solution, nonetheless,

![Fig. 2](image-url) Equipotential lines for \(V_1 = 1\) and \(V_2 = -1\) \((a = 0.22, b = 0.4, c = 0.325, d = 0.005, h = 0.23\) Unit: [m]).

<table>
<thead>
<tr>
<th>Table 1</th>
<th>(G_m) coefficients.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m = 1)</td>
<td>(m = 3)</td>
</tr>
<tr>
<td>(G_m)</td>
<td>0.993</td>
</tr>
</tbody>
</table>
yields stable and accurate results for the central region that is far away from the edges of striplines. The error near the edges of striplines poses no problem for CTL cell application since the CTL cells mostly utilize the central region as a test zone. Figure 3 shows the equipotential lines when \( V_1 = V_2 = 1 \). The rate of series convergence is almost the same as that in Fig. 2. The electric field line of TEM mode can be straightforwardly obtained from \(-\nabla \Phi\). A discussion between the electric field and the electrostatic potential for TEM mode is given in [6]. Note that the proposed mode-matching model provides an accurate estimate of the electric field uniformity, which is a very important parameter in assessing cell performance in EMI/EMC applications.

4. Conclusion

The electrostatic boundary-value problem of broadside-coupled striplines in a shield was solved. The mode-matching method was utilized to obtain the analytic and rigorous solution. The mode-matching method provides the exact solution, thus providing a simple estimate for field distribution in the CTL cells.

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References


Appendix: Matrix Elements

The matrix equation is:

\[
[\psi]P = Q
\]

where

\[
P = [A, B, C, D, \ldots, K, L]^T, Q = [\Omega_1, \Omega_2, \ldots, \Omega_{12}]^T
\]

Here \( A, B, C, \ldots \) are column vectors of the modal coefficients, and the superscript \( T \) denotes a transpose. The non-zero elements of \( \psi \) are \( \psi_{ij} \) where \( i, j = 1, \ldots, 12 \).

\[
\psi_{11}^{mn} = b \sinh (\alpha_m (h + d - c)) \delta_{mn}
\]
\[
\psi_{12}^{mn} = -b \cosh (\alpha_m (h + d)) M_{mn}^{nn}
\]
\[
\psi_{13}^{mn} = -b \sinh (\alpha_m (h + d)) M_{mn}^{nn}
\]
\[
\psi_{14}^{mn} = -b \cosh (\alpha_m (h + d)) M_{mn}^{nn}
\]
\[
\psi_{15}^{mn} = -b \sinh (\alpha_m (h + d)) M_{mn}^{nn}
\]
\[
\psi_{16}^{mn} = -b \sinh (\alpha_M (h + d)) \delta_{mn}
\]
\[
\psi_{17}^{mn} = -b \cosh (\alpha_M (h + d)) \delta_{mn}
\]
\[
\psi_{21}^{mn} = b \cosh (\alpha_m (h + d)) \delta_{mn}
\]
\[
\psi_{22}^{mn} = b \sinh (\alpha_m (h + d)) M_{mn}^{nn}
\]
\[
\psi_{23}^{mn} = b \cosh (\alpha_m (h + d)) M_{mn}^{nn}
\]
\[
\psi_{24}^{mn} = b \sinh (\alpha_m (h + d)) M_{mn}^{nn}
\]
\[
\psi_{25}^{mn} = b \cosh (\alpha_M (h + d)) \delta_{mn}
\]
\[
\psi_{26}^{mn} = b \sinh (\alpha_M (h + d)) \delta_{mn}
\]
\[
\psi_{31}^{mn} = -b \sinh (\alpha_m (h + d)) \delta_{mn}
\]
\[
\psi_{32}^{mn} = -b \cosh (\alpha_m (h + d)) \delta_{mn}
\]
\[
\psi_{33}^{mn} = -b \sinh (\alpha_m (h + d)) \delta_{mn}
\]
\[
\psi_{34}^{mn} = -b \cosh (\alpha_m (h + d)) \delta_{mn}
\]
\[
\psi_{35}^{mn} = -b \sinh (\alpha_M (h + d)) \delta_{mn}
\]
\[
\psi_{36}^{mn} = -b \cosh (\alpha_M (h + d)) \delta_{mn}
\]
\[
\psi_{41}^{mn} = b \sinh (\alpha_M (h + d)) \delta_{mn}
\]
\[
\psi_{42}^{mn} = b \cosh (\alpha_M (h + d)) \delta_{mn}
\]
\[
\psi_{43}^{mn} = b \sinh (\alpha_M (h + d)) \delta_{mn}
\]
\[
\psi_{44}^{mn} = b \cosh (\alpha_M (h + d)) \delta_{mn}
\]
\[
\psi_{45}^{mn} = b \sinh (\alpha_M (h + d)) \delta_{mn}
\]
\[
\psi_{46}^{mn} = b \cosh (\alpha_M (h + d)) \delta_{mn}
\]
\[
\psi_{51}^{mn} = -b \sinh (\alpha_m (h + d) - c) \delta_{mn}
\]
\[
\psi_{52}^{mn} = -b \cosh (\alpha_m (h + d) - c) M_{mn}^{nn}
\]
\[
\psi_{53}^{mn} = -b \sinh (\alpha_M (h + d) - c) \delta_{mn}
\]
\[
\psi_{54}^{mn} = -b \cosh (\alpha_M (h + d) - c) M_{mn}^{nn}
\]
\[
\psi_{55}^{mn} = \frac{b}{2} \beta_m \sinh (\alpha_m (h + d) - c) \delta_{mn}
\]
\[
\psi_{56}^{mn} = \frac{b}{2} \beta_m \cosh (\alpha_m (h + d) - c) \delta_{mn}
\]
\[ \psi_{6,5}^m = -\frac{b - a}{2} \beta_m \cosh (\beta_m (h + d)) \delta_{mn} \]
\[ \psi_{7,2}^m = -\frac{b - a}{2} \beta_m \sinh (\beta_m h) \delta_{mn} \]
\[ \psi_{7,3}^m = -\frac{b - a}{2} \beta_m \cosh (\beta_m h) \delta_{mn} \]
\[ \psi_{7,6}^m = \alpha_m \sinh (\alpha_m h) M_1^m \]
\[ \psi_{7,7}^m = \alpha_m \cosh (\alpha_m h) M_1^m \]
\[ \psi_{8,4}^m = -\frac{b - a}{2} \beta_m \sinh (\beta_m h) \delta_{mn} \]
\[ \psi_{8,5}^m = -\frac{b - a}{2} \beta_m \cosh (\beta_m h) \delta_{mn} \]
\[ \psi_{8,6}^m = -\alpha_m \sinh (\alpha_m h) M_2^m \]
\[ \psi_{8,7}^m = -\alpha_m \cosh (\alpha_m h) M_2^m \]
\[ \psi_{9,6}^m = \alpha_m \sinh (\alpha_m h) M_1^m \]
\[ \psi_{9,7}^m = -\alpha_m \cosh (\alpha_m h) M_1^m \]
\[ \psi_{9,8}^m = -\frac{b - a}{2} \beta_m \sinh (\beta_m h) \delta_{mn} \]
\[ \psi_{9,9}^m = -\frac{b - a}{2} \beta_m \cosh (\beta_m h) \delta_{mn} \]
\[ \psi_{10,6}^m = \alpha_m \sinh (\alpha_m h) M_2^m \]
\[ \psi_{10,7}^m = -\alpha_m \cosh (\alpha_m h) M_2^m \]
\[ \psi_{10,10}^m = -\frac{b - a}{2} \beta_m \sinh (\beta_m h) \delta_{mn} \]
\[ \psi_{10,11}^m = -\frac{b - a}{2} \beta_m \cosh (\beta_m h) \delta_{mn} \]
\[ \psi_{11,8}^m = -\frac{b - a}{2} \beta_m \sinh (\beta_m (h + d)) \delta_{mn} \]
\[ \psi_{11,9}^m = -\frac{b - a}{2} \beta_m \cosh (\beta_m (h + d)) \delta_{mn} \]
\[ \psi_{11,12}^m = \alpha_m \cosh (\alpha_m (-h - d + c)) M_1^m \]
\[ \psi_{12,10}^m = -\frac{b - a}{2} \beta_m \sinh (\beta_m (h + d)) \delta_{mn} \]
\[ \psi_{12,11}^m = \alpha_m \cosh (\alpha_m (h - d + c)) M_2^m \]
\[ \Omega_1 = \frac{2V_1}{\alpha_n} \sin (\alpha q_n) \sin (b a_n) \]
\[ \Omega_2 = \frac{2V_1}{\alpha_n} \sin (\alpha q_n) \sin (b a_n) \]
\[ \Omega_3 = \frac{2V_2}{\alpha_n} \sin (\alpha q_n) \sin (b a_n) \]
\[ \Omega_4 = \frac{2V_2}{\alpha_n} \sin (\alpha q_n) \sin (b a_n) \]
\[ \Omega_5 = -\frac{4V_1}{d} \sum_{k=0}^{+\infty} \frac{\beta_n (-1)^n}{\gamma_{2k+1}^2 + \beta_n^2} \]
\[ \Omega_6 = -\frac{4V_1}{d} \sum_{k=0}^{+\infty} \frac{\beta_n (-1)^n}{\gamma_{2k+1}^2 + \beta_n^2} \]
\[ \Omega_7 = -\frac{4V_1}{d} \sum_{k=0}^{+\infty} \frac{\beta_n (-1)^n}{\gamma_{2k+1}^2 + \beta_n^2} \]
\[ \Omega_8 = -\frac{4V_2}{d} \sum_{k=0}^{+\infty} \frac{\beta_n (-1)^n}{\gamma_{2k+1}^2 + \beta_n^2} \]
\[ \Omega_9 = -\frac{4V_2}{d} \sum_{k=0}^{+\infty} \frac{\beta_n (-1)^n}{\gamma_{2k+1}^2 + \beta_n^2} \]
\[ \Omega_{10} = \frac{4V_2}{d} \sum_{k=0}^{+\infty} \frac{\beta_n (-1)^n}{\gamma_{2k+1}^2 + \beta_n^2} \]
\[ \Omega_{11} = \frac{4V_2}{d} \sum_{k=0}^{+\infty} \frac{\beta_n (-1)^n}{\gamma_{2k+1}^2 + \beta_n^2} \]
\[ \Omega_{12} = \frac{4V_2}{d} \sum_{k=0}^{+\infty} \frac{\beta_n (-1)^n}{\gamma_{2k+1}^2 + \beta_n^2} \]