Adaptive Resource Allocation for a Two-Way OFDM Relay Network with Fairness Constraints*

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SUMMARY In this letter, we propose a new resource allocation scheme for a two-way OFDM relay network with fairness constraints. To maximize sum capacity, subcarriers and their power are successively allocated to the relays based on channel conditions. Also, the power constraint is imposed on each relay to achieve fairness for the relays. Simulation results show that the proposed scheme improves sum capacity and fairness significantly.

key words: two-way, OFDM, resource allocation, fairness

1. Introduction

In cooperative relaying, multiple terminals with a single antenna share their resources and assist each other in data transmission to obtain the benefits of multiple input multiple output (MIMO) systems [1]. However, under half-duplex constraints, most cooperative relaying techniques suffer from an inherent spectral loss since resource allocation for each transmission is done either in the time domain or the frequency domain. In two-way cooperative relaying, two terminals exchange information with each other via a single or multiple relays [2]–[4]. By using simultaneous transmission at two terminals and the bidirectional relaying at the relays, two-way cooperative relaying provides improved spectral efficiency compared with one-way cooperative relaying. However, the resource allocation, which can effectively use the resources such as bandwidth and power, for a two-way relay network with multiple relays has not been extensively investigated, yet.

In this letter, we propose a suboptimal but efficient resource allocation scheme for a two-way OFDM relay network with multiple relays. This letter is organized as follows. In Sect. 2, a system model and an optimization problem is described. A new resource allocation scheme is proposed in Sect. 3. In Sect. 4, simulation results are shown. Finally, conclusions are drawn in Sect. 5.

2. System Model and Problem Formulation

2.1 System Model

Consider a two-way OFDM relay network which consists of two terminals, A and B, and M potential relays, each with a single antenna. Assume that there is no direct path between the two terminals which can not transmit and receive simultaneously. Suppose that K relays out of M potential relays are selected to support communication between the two terminals. Assume that all relays are connected with a central unit [5]. The central unit has the functions to calculate and executes the resource allocation and sends the signaling messages to the related relays and the terminals.

Suppose that an OFDM symbol consists of N subcarriers. Let \( f_k^{(n)} \) and \( g_k^{(n)} \) denote the instantaneous channel coefficients of the \( n \)-th subcarrier from the terminal A to the relay \( k \) and from the terminal B to the relay \( k \), respectively. We consider the following assumptions: (i) the channel of each subcarrier is frequency-flat; (ii) the instantaneous channel coefficient of each subcarrier is reciprocal and constant over an OFDM frame which consists of several OFDM symbols; (iii) perfect channel state information is available at all terminals and potential relays.

Assuming an amplify-and-forward protocol, a symbol is transmitted in two phases: The multiple-access and broadcast phases. In the multiple-access phase, both the terminals A and B transmit symbols to relays simultaneously. Assuming all terminals and relays are symbol synchronous, the received symbol at the relay \( k \) on the \( n \)-th subcarrier is given by

\[
y_k^{(n)} = \sqrt{P_A^{(n)} f_k^{(n)}} x_A^{(n)} + \sqrt{P_B^{(n)} g_k^{(n)}} x_B^{(n)} + n_k^{(n)}
\]

where \( P_A^{(n)} \) and \( P_B^{(n)} \) denote the transmit power of the terminals A and B on the \( n \)-th subcarrier, respectively, \( x_A^{(n)} \) and \( x_B^{(n)} \) denote the transmit symbols of the terminals A and B on the \( n \)-th subcarrier, respectively, and \( n_k^{(n)} \) is the zero-mean complex Gaussian noise with variance \( N_0 \) on the \( n \)-th subcarrier at the relay \( k \).

In the broadcast phase, the relay \( k \) amplifies the received signal \( y_k^{(n)} \) and forwards it to both terminals A and B on the same \( n \)-th subcarrier. The amplification factor of the relay \( k \) on the \( n \)-th subcarrier is given by

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where $p^{(n)}_k$ is the transmit power of the relay $k$ on the $n$-th subcarrier. The received symbols at the terminals $A$ and $B$ from the relay $k$ are given by

$$y^{(n)}_A = p^{(n)}_k f^{(n)}_k g^{(n)}_k + n^{(n)}_A,$$

$$y^{(n)}_B = p^{(n)}_k g^{(n)}_k y^{(n)}_k + n^{(n)}_B,$$

respectively, where $n^{(n)}_A$ and $n^{(n)}_B$ are zero-mean complex Gaussian noises with variance $N_0$ on the $n$-th subcarrier at the terminals $A$ and $B$, respectively.

Assuming perfect self-interference cancelation, the capacity of the two-way relay network on the $n$-th subcarrier is given by [2]

$$c^{(n)}_k = \frac{1}{2} \log_2 \left( 1 + \frac{|p^{(n)}_k f^{(n)}_k g^{(n)}_k|^2 P^{(n)}_B}{|p^{(n)}_k f^{(n)}_k g^{(n)}_k|^2 + 1 N_0} \right)$$

$$+ \frac{1}{2} \log_2 \left( 1 + \frac{|p^{(n)}_k g^{(n)}_k y^{(n)}_k|^2 P^{(n)}_A}{|p^{(n)}_k g^{(n)}_k y^{(n)}_k|^2 + 1 N_0} \right)$$

Then, the sum capacity over all allocated subcarriers is given by

$$C = \sum_{k=1}^{K} \sum_{n=1}^{N} c^{(n)}_k$$

where $c^{(n)}_k$ denotes the subcarrier assignment indicator variable for the relay $k$ on the $n$-th subcarrier. If the $n$-th subcarrier is assigned to the relay $k$, $c^{(n)}_k = 1$; otherwise, $c^{(n)}_k = 0$. Assume that each subcarrier is allocated to only one relay so that there is no interference between relays.

### 2.2 Problem Formulation

Assume that $K$ relays out of $M$ potential relays are selected prior to resource allocation. The relay selection metric is given by [6]

$$W_m = \min \left\{ \sum_{m=1}^{N} \left| f^{(n)}_m \right|^2, \sum_{m=1}^{N} \left| g^{(n)}_m \right|^2 \right\}, m = 1, \ldots, M.$$  

By using the above metric, $K$ relays which have the $K$ largest metric values are selected to participate in relaying.

To enhance the fairness of relay utilization, power constraints are imposed on the selected relays. The optimization problem with fairness constraints is formulated as

$$C^* = \max \sum_{k=1}^{K} \sum_{n=1}^{N} p^{(n)}_k \xi^{(n)}_k$$

subject to $p^{(n)}_k \in [0, 1], \forall k, n$, $\sum_{k=1}^{K} p^{(n)}_k = 1, \forall n$, $\sum_{n=1}^{N} p^{(n)}_A \leq P_A$, $\sum_{n=1}^{N} p^{(n)}_B \leq P_B$, and $\sum_{n=1}^{N} p^{(n)}_k \leq P_k, \forall k$, where $P_A$, $P_B$, and $P_k$ are the transmit power of the terminal $A$, the terminal $B$, and the relay $k$, respectively.

### 3. Proposed Resource Allocation Scheme

The optimization problem in (8) jointly considers subcarrier allocation and power distribution. To reduce computational complexity, we propose a suboptimal resource allocation scheme which separates the optimization problem into two steps, i.e., subcarrier allocation and power distribution.

#### 3.1 Proposed Subcarrier Allocation Criterion and Subcarrier Allocation Algorithm

#### 3.1.1 Proposed Subcarrier Allocation Criterion

The problem in (8) is a combinatorial optimization problem having both discrete and continuous variables, which is too complex to obtain an optimal solution. To make the problem tractable, let the constraint on $p^{(n)}_k$ be relaxed so that it could take a real value in $[0, 1]$ [7]. Then, the Lagrangian for the optimization problem with the relaxed constraint is given by

$$L = -\sum_{k=1}^{K} \sum_{n=1}^{N} \lambda_{n}^{(n)} \left[ c^{(n)}_k - 1 \right] - \sum_{n=1}^{N} \eta \left[ P^{(n)}_A - P_A \right] - \gamma \left[ P^{(n)}_B - P_B \right] - \sum_{n=1}^{N} \mu_{k} p^{(n)}_k - P_k - \sum_{n=1}^{N} \alpha_{k}^{(n)} p^{(n)}_A - \sum_{n=1}^{N} \beta_{k} p^{(n)}_B - \sum_{n=1}^{N} \gamma_{k,n} p^{(n)}_k - \sum_{n=1}^{N} \zeta_{k,n} p^{(n)}_k$$

where $\lambda_{n}, \eta, \nu, \mu_{k}, \alpha_{n}, \beta_{n}, \gamma_{k,n},$ and $\zeta_{k,n}$ are nonnegative Lagrange multipliers. Any optimal solution to the relaxed problem has to satisfy the following Karush-Kuhn-Tucker (KKT) conditions [8]:

$$\frac{\partial L}{\partial p^{(n)}_k} = -c^{(n)}_k + \lambda_{n}^{(n)} + \mu_{k} p^{(n)}_k - \zeta_{k,n} = 0, \forall k, n$$

$$\frac{\partial L}{\partial p^{(n)}_A} = \frac{\partial L}{\partial p^{(n)}_B} = \frac{\partial L}{\partial p^{(n)}_k} = 0, \forall k, n$$

$$\eta \left[ P^{(n)}_A - P_A \right] = \nu \left[ P^{(n)}_B - P_B \right] = 0$$

$$\mu_{k} \left[ \sum_{n=1}^{N} p^{(n)}_k - P_k \right] = 0, \forall k$$

$$\alpha_{n}^{(n)} p^{(n)}_A = \beta_{n} p^{(n)}_B = 0, \forall n$$

$$\gamma_{k,n} p^{(n)}_k = 0, \forall k, n$$

$$\zeta_{k,n} p^{(n)}_k = 0, \forall k, n.$$
\[ \lambda_n \geq c_k^{(n)} - \mu_k p_k^{(n)}. \]  

(11)

From (10a) and (10g), we obtain \( p_k^{(n)}(\lambda_n - c_k^{(n)} + \mu_k p_k^{(n)}) = 0. \) If the \( n \)-th subcarrier is allocated to the relay \( k \), the equality in (11) holds. Then, to maximize the sum capacity and meet the power constraint of each relay, the \( n \)-th subcarrier is allocated to the relay \( k' \) such that

\[ k' = \arg \max_k \left( c_k^{(n)} - \mu_k p_k^{(n)} \right). \]

(12)

### 3.1.2 Proposed Subcarrier Allocation Algorithm

Assume that each relay has the same maximum transmit power constraint and allocates equal power to the subcarriers allocated to it. Then, based on the above subcarrier allocation criterion, we propose the new subcarrier allocation algorithm, as shown in Algorithm 1.

**Algorithm 1: Proposed subcarrier allocation algorithm**

**Step 1:** Initialization

Set \( N = [1, 2, \ldots, N] \), \( K = [1, 2, \ldots, K] \), and \( p_k^{(n)} = 0 \), \( \forall k, n \);

while \( K \neq 0 \) do

\[ (k^*, n^*) = \arg \max_{k,n} c_k^{(n)} \text{, } k \in K, n \in N; \]

\[ \rho_k^{(n)} = 1, N = N - \{n^*\}; \]

update \( p_k = p_k - \sum_{n=1}^{N} \rho_k^{(n)} p_k^{(n)} \);

if \( (p_k - \sum_{n=1}^{N} \rho_k^{(n)} p_k^{(n)}) < P_k \) then

\[ K = K - \{k^*\}; \]

**Step 2:** Subcarrier allocation under fairness constraints

**Step 3:** Remaining subcarrier allocation

Set \( K = [1, 2, \ldots, K] \)

while \( N \neq 0 \) do

\[ (k^*, n^*) = \arg \max_{k,n} c_k^{(n)} \text{, } k \in K, n \in N; \]

\[ \rho_k^{(n)} = 1; \]

\[ N = N - \{n^*\}, K = K - \{k^*\}; \]

**end while**

end while

end while

3.2 Power Allocation for a Determined Subcarrier Allocation

After subcarrier allocation is determined, based on the dual decomposition technique [4], [9], the optimization problem for power allocation is formulated as

\[ C^* = \max_{\{p_k^T \geq P_k \forall k\}} \sum_{k=1}^{K} \sum_{n=1}^{N} \rho_k^{(n)} c_k^{(n)} \]

subject to \( 1^T p_A \leq \rho_A \), \( 1^T p_B \leq \rho_B \), and \( \rho_k p_k \leq P_k \), \( \forall k \), where \( \rho_A = [\rho_A^{(1)}, \rho_A^{(2)}, \ldots, \rho_A^{(N)}]^T \), \( \rho_B = [\rho_B^{(1)}, \rho_B^{(2)}, \ldots, \rho_B^{(N)}]^T \), \( p_k = [p_k^{(1)}, p_k^{(2)}, \ldots, p_k^{(N)}]^T \), \( \forall k \), and \( \rho_k = [\rho_k^{(1)}, \rho_k^{(2)}, \ldots, \rho_k^{(N)}]^T \), \( \forall k \).

The Lagrangian for the problem in (13) is given by

\[ L_p(p_A, p_B, p_1, \ldots, p_K; \eta, \nu, \mu) = \sum_{k=1}^{K} \sum_{n=1}^{N} \rho_k^{(n)} c_k^{(n)} \]

\[ - \eta (1^T p_A - \rho_A) - \nu (1^T p_B - \rho_B) - \sum_{k=1}^{K} \mu_k (p_k^T p_k - P_k) \]

(14)

where \( \eta \), \( \nu \), and \( \mu \) are nonnegative Lagrange multipliers, and \( \mu = [\mu_1, \mu_2, \ldots, \mu_K]^T \).

Then, the Lagrange dual function is given by

\[ f(\eta, \nu, \mu) = \max \left\{ \sum_{k=1}^{K} \rho_k^{(n)} c_k^{(n)} - \eta \rho_A^{(n)} - \nu \rho_B^{(n)} - \sum_{k=1}^{K} \mu_k \rho_k^{(n)} \right\} \]

(15)

subject to \( p_A, p_B, p_1, \ldots, p_K \geq 0 \).

Finally, the Lagrange dual problem for the problem in (13) is formulated as

\[ f^* = \min f(\eta, \nu, \mu) \]

(16)

subject to \( \eta, \nu, \mu \geq 0 \).

The difference between two optimal values \( C^* \) and \( f^* \), which is called duality gap, reduces to zero as the number of subcarriers \( N \) increases [10]. Hence, the solution of the optimization problem in (13) can be obtained by solving the dual problem in (16). To simplify the Lagrange dual problem in (16), the dual decomposition technique is used [9]. By modifying the Lagrangian in (14), the Lagrange dual function \( f(\eta, \nu, \mu) \) is decomposed into the set of \( N \) independent subproblems which is given by

\[ f(n) = \min \left\{ \sum_{k=1}^{K} \rho_k^{(n)} c_k^{(n)} - \eta \rho_A^{(n)} - \nu \rho_B^{(n)} - \sum_{k=1}^{K} \mu_k \rho_k^{(n)} \right\} \]

(17)

for \( n = 1, \ldots, N \). Then, \( f(\eta, \nu, \mu) \) is rewritten as

\[ f(\eta, \nu, \mu) = \sum_{n=1}^{N} f(n) \]

(18)

Thus, \( f(\eta, \nu, \mu) \) is decomposed as \( N \) maximization subproblems which can be independently solved. By substituting (18) into (16), we obtain the simplified Lagrange dual problem.

To solve the simplified Lagrange dual problem, we use a subgradient method [11] which is based on iterative searching. Given the initial values of \( \eta \), \( \nu \), and \( \mu \), we solve the \( N \) subproblems independently to obtain a solution for power allocation. Based on the obtained power allocation, the values of \( \eta \), \( \nu \), and \( \mu \) are updated as

\[ \eta^{l+1} = [\eta^l + \epsilon^l (1^T p_A - \rho_A)]^+ \]

(19a)

\[ \nu^{l+1} = [\nu^l + \epsilon^l (1^T p_B - \rho_B)]^+ \]

(19b)

\[ \mu_k^{l+1} = [\mu_k^l + \epsilon^l (p_k^T p_k - P_k)]^+, \forall k \]

(19c)

where \( l \) is the iteration number and \( \epsilon^l \) is the step size for the \( l \)-th iteration. It is known that if \( \epsilon^l \) is sufficiently small, each of \( \eta \), \( \nu \), and \( \mu \) is guaranteed to converge to its optimal value after a number of iterations [11]. Based on the optimal values of \( \eta \), \( \nu \) and \( \mu \), we can obtain the solution for optimal
power allocation. Then, we normalize $p_A$, $p_B$, and $p_k$ so that the power constraints are satisfied.

4. Simulation Results

Assume that two terminals and all relays are located in a circle centered at the origin of the two-dimensional plane with radius 1. Suppose that the two terminal are located at $(-0.5, 0)$ and $(0.5, 0)$, respectively, and $M$ relays are uniformly distributed in the circle. Suppose that the number of subcarriers is 64, the path loss exponent is 4, and $P_A = P_B = KP_k$. Similar to [12], the received SNR is defined as the average received SNR at the relay which is located on the middle of two terminals. We adopt the ITU pedestrian B model for the frequency-selective channel [13]. The performance of the proposed scheme is compared with those of the static allocation, opportunistic relaying [6], and greedy allocation schemes [7]. In the static allocation scheme, each relay is allocated with predetermined subcarriers regardless of the instantaneous channel condition. In the opportunistic relaying scheme, a single best relay is selected and allocated with all subcarriers. In the greedy allocation scheme, a subcarrier is allocated to the relay which maximizes the capacity, while fairness is not considered. Also, to compare the fairness of the proposed scheme and other schemes, we consider the fairness index which is defined as $F = \left( \sum_{k=1}^{K} \left( \sum_{n=1}^{N} \rho_k(n) p_k(n) \right)^2 / (K \sum_{k=1}^{K} \left( \sum_{n=1}^{N} \rho_k(n) p_k(n) \right)^2 \right)$. In [14].

Figure 1 shows the average capacity per subcarrier vs. signal to noise ratio (SNR) for $K = 4$ and $M = 100$. Unlike two-way relaying, in the one-way relaying the two terminals do not simultaneously transmit and the relays do not perform the bidirectional relaying. As a result, the two-way relaying provides much larger average capacity per subcarrier than the one-way relaying. It is shown that the proposed scheme achieves larger average capacity per subcarrier than either the static allocation scheme or the opportunistic relaying scheme. Also, the average capacity per subcarrier of the proposed scheme is slightly smaller than that of the greedy allocation scheme.

Figure 2 shows the average capacity per subcarrier vs. number of relays for $K = 4$ and SNR = 10 dB. Similar to Fig. 1, it is shown that the proposed scheme achieves much larger average capacity per subcarrier than either the static allocation scheme or the opportunistic relaying scheme, while the average capacity per subcarrier of the proposed scheme is slightly smaller than that of the greedy allocation scheme. Also, comparing the proposed scheme with the opportunistic relaying and static allocation, the performance gap between the proposed scheme and other two schemes increases as the number of relays increases.

Figure 3 shows the fairness index vs. the number of selected relays for $M = 100$ and SNR = 15 dB. It is shown that the fairness index of the proposed scheme is 1, which im-

![Fig. 1 Average capacity per subcarrier versus SNR. $K = 4$ and $M = 100$.](image1)

![Fig. 2 Average capacity per subcarrier versus number of relays. $K = 4$ and SNR = 10 dB.](image2)

![Fig. 3 Fairness index versus number of selected relays. $M = 100$ and SNR = 15 dB.](image3)
plies that fairness for the relay transmit power is maximized [14]. Also, the difference between the fairness indexes of the proposed scheme and the greedy allocation scheme increases as the number of selected relays increases. The proposed scheme achieves the same fairness as the static allocation scheme.

5. Conclusions

In this letter, we proposed a new resource allocation scheme for a two-way OFDM relay network with fairness constraints. In the proposed scheme, subcarriers are heuristically allocated to selected relays and then transmit power is allocated to the subcarriers under fairness constraints. Simulation results show that the proposed scheme improves sum capacity and fairness significantly.

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References