Separation Theorems for Phase-Incoherent Multiple-User Channels

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Abstract

We study the transmission of two correlated and memoryless sources \((U, V)\) over several multiple-user phase asynchronous channels. Namely, we consider a class of phase-incoherent multiple access relay channels (MARC) with both non-causal and causal unidirectional cooperation between encoders, referred to as phase-incoherent unidirectional non-causal cooperative MARC (PI-UNCC-MARC), and phase-incoherent unidirectional causal cooperative MARC (PI-UCC-MARC) respectively. We also consider phase-incoherent interference channels (PI-IC), and interference relay channel (PI-IRC) models in the same context. In all cases, the input signals are assumed to undergo non-ergodic phase shifts due to the channel. The shifts are assumed to be unknown to the transmitters and known to the receivers as a realistic assumption. Both necessary and sufficient conditions in order to reliably send the correlated sources to the destinations over the considered channels are derived. In particular, for all of the channel models, we first derive an outer bound for reliable communication that is defined with respect to the source entropy content (i.e., the triple \(H(U|V), H(V|U), H(U, V)\)). Then, using separate source and channel coding, under specific gain conditions, we establish the same region as the inner bound and therefore obtain tight conditions for reliable communication for the specific channel under study. We thus establish a source-channel separation theorem for each channel and conclude that without the knowledge of the phase shifts at the transmitter sides, separation is optimal. It is further conjectured that separation in general is optimal for all channel coefficients.

Index Terms

Multiple access relay channel, cooperative encoders, interference channel, interference relay channel, phase uncertainty, joint source-channel coding, correlated sources.

I. INTRODUCTION

Incoherence or asynchronism between different nodes of a communication network is an inherent challenge to modern communication systems. In particular, there are major factors in wireless systems, such as feedback delay, the bursty nature of some applications, and reaction delay, which cause time or phase asynchronism between different nodes of a network \[20\]. Furthermore, in multi-user systems, interference from other sources make synchronization much more difficult. Therefore, it is interesting to study multi-user communication problems without assuming synchronism a priori.
In point-to-point wireless systems, achieving receiver synchronization is possible in principle, using training sequences and/or feedback. However, although analytically convenient, full synchronization is rarely a practical or easily justified assumption, and in some cases theoretically infeasible [25]. The first studies of time asynchronism in point-to-point communications goes back to the 60’s ([4], [10]), where the receiver is not accurately aware of the exact time that the encoded symbols were transmitted. The recent work of [20], on the other hand, assumes a stronger form of time asynchronism, that is, the receiver knows neither the time at which transmission starts, nor the timing of the last information symbol. They propose a combined communication and synchronization scheme and discuss information-theoretical limits of the model. Also, in multi-user communication settings, the problem of time asynchronism is addressed for example in [6], [22] for the particular case of multiple access channels.

Besides time asynchronism [20], which is present in most channels, other forms of asynchronism such as phase uncertainty are important in wireless systems. In fading channels, the channel state information (CSI) models amplitude attenuation and phase shifts (phase fading) introduced by the channels between the nodes. In many systems, it is difficult to know phase shifts at the transmitter side due to the delay and resource limits in feedback transmission. In particular, in highly mobile environments, fading in conjunction with feedback delay may result in out of date phase knowledge by the time it reaches the transmitters (see, e.g., [16]).

The issue of phase asynchronism can be analytically seen in the larger framework of channel uncertainty, that is, the communicating parties have to work under situations where the full knowledge of the law governing the channel (or channels in a multi-user setting) is not known to some or all of them [14]. In order to study this general problem from an information-theoretic point of view, the mathematical model of a compound channel (or state-dependent channel) has been introduced by different authors [3], [9], [24]. A compound channel is generally represented by a family of transition probabilities $p_{Y|X}^\theta$, where the index $\theta \in \Theta$ is the state of the channel and $\Theta$ represents the uncertainty of different parties about the exact channel’s transition probability.

In this paper, we consider the problem of joint source-channel coding for a range of compound Gaussian multiple-user channels with phase uncertainty and prove a separation theorem for each. We assume that the phase shifts over channels under consideration are stationary non-ergodic phase fading processes which are chosen randomly and fixed over the block length. Thus, phase asynchronism is formulated in the compound channel framework and the phase information $\theta$ (as the channel parameter) is assumed to be unknown to the transmitters and known to the receiver side(s) as a practical assumption. Consequently, as our main contribution, we find conditions that are both necessary and sufficient for sending a pair of correlated sources over a class of continuous alphabet multiple-user channels under phase uncertainty.

The problem of joint source-channel coding for a network is open in general. Several works, however, have been published on this issue for multiple access channel (MAC). As an example, for lossy source-channel coding, a separation approach is shown in [21] to be optimal or approximately optimal to communicate independent sources. In [5], on the other hand, a sufficient condition based on joint source-channel coding
to send correlated sources over a MAC is given, along with an uncomputable expression for the outer bound. As the sufficient condition in [5] provides a region greater than that ensured to be achieved by separate source and channel coding, it is proved that the separate source-channel coding is not optimal for correlated sources. In [1], [2], however, the authors show that performing separate source and channel coding for the important case of a Gaussian MAC with phase shifts, shown in Fig. I is optimal. Namely, in [1] and [2], F. Abi Abdallah et. al. showed the following separation theorem for a class of phase asynchronous multiple access channels for both non-ergodic, and ergodic i.i.d. phase fading:

**Theorem 1:** Reliable communication over a PI-MAC: A necessary condition for reliable communication of the source pair \((U, V) \sim \prod_i p(u_i, v_i)\) over a class of multiple access channels with unknown phase fading at the transmitters, with power constraints \(P_1, P_2\) on the transmitters, and fading amplitudes \(g_1, g_2 > 0\), is given by

\[
\begin{align*}
H(U|V) &\leq \log(1 + g_1^2 P_1/N), \\
H(V|U) &\leq \log(1 + g_2^2 P_2/N), \\
H(U, V) &\leq \log(1 + (g_1^2 P_1 + g_2^2 P_2)/N),
\end{align*}
\]

where \(N\) is the noise power. Sufficient conditions for the reliable communications are also given by (1)-(3), with \(\leq\) replaced by \(<\).

Also, the recent work [15] addresses the same problem for a phase fading Gaussian multiple access relay channel (MARC) and proves a separation theorem under some channel coefficient conditions. For the achievability part, the authors use the results of [11], [17], and [13] based on a combination of **regular** Markov encoding at the transmitters and **backward** decoding at the receiver [12]. In particular, in order to derive the achievable region for discrete-memoryless MARC, the authors of [17] use codebooks of the same size which is referred to as regular Markov encoding. This is in contrast with block Markov encoding which was introduced by Cover and El Gamal in [8] for the relay channel. There, the encoding is done using codebooks of different sizes and is referred to as **irregular** block Markov encoding.

In this paper, we consider a more general PI-MARC, in which one of the encoders is helped by the other one causally or non-causally. We refer to such networks as phase-incoherent unidirectional cooperative MARCs or PI-UC-MARCs for short. Furthermore, we also prove separation theorems for a phase-incoherent interference channel (PI-IC) under strong interference conditions and phase-incoherent interference relay channel (PI-IRC) under specific strong interference gain conditions.

The networks that we consider and for which we prove our results are listed as follows:

- **PI-UC-MARC with non-causal (NC) cooperation** between transmitters and with strong path gains from transmitters to the relay. We refer to this network as phase incoherent unidirectional non-causal cooperative (PI-UNCC)-MARC. By removing the relay, the results can be specialized to the case of a MAC (PI-UNCC-MAC).
- **PI-UC-MARC with causal (C) cooperation** between transmitters and with strong path gains from transmitters to the relay. This network is called a phase-incoherent unidirectional causal cooperative
(PI-UCC)-MARC. By removing the relay, the results can be specialized to the case of a MAC (PI-UCC-MAC).

- Phase incoherent interference channel (PI-IC) in strong interference regime.
- Phase incoherent interference relay channel (PI-IRC) in a specific strong interference regime with strong path gains from transmitters to the relay.

We show that if the phase shifts are unknown to the transmitters, then the optimal performance is no better than the scenario in which the information sources are first source coded and then channel coded separately, i.e., the correlation between the sources is not helpful to enlarge the achievable region, as opposed to cases where the transmitters have knowledge of the phase shifts and could potentially use beamforming, for example, to joint source-channel code the data and achieve higher rates. Although we assume non-ergodic phase shifts throughout the paper, as in [2], our results are also true for the ergodic case, where the phases change i.i.d. from symbol to symbol. The contributions of this work are stated in the form of four separation theorems that are given in the following sections.

Further, we conjecture that optimality of separation is true not only for the specific gain conditions we state, but also for all possible values of path gains. Hence, we conjecture that separation is optimal for unrestricted forms of the phase incoherent Gaussian phase fading channels discussed in this paper. The approach we used here to prove the separation theorems which is based on computing necessary and sufficient conditions for reliable communication, however, may not be viable to prove the conjecture.

The rest of this paper is organized as follows. We introduce the phase asynchronous multi-user networks considered in this work in Section II along with a key lemma that we use several times in the paper. In Section III, we define the general problem of the joint source-channel coding for a PI-MARC and state a separation theorem for it. In Sections IV and V, we state and prove separation theorems under specific gain conditions for a class of phase asynchronous MARCs in which the encoders cooperate unidirectionally both non-causally and causally respectively. Next, In Sections VI and VII, we consider joint source-channel coding problem for interference channels and interference relay channels under phase uncertainty respectively and likewise state and prove separation theorems for them under strong interference conditions. We finally conclude the results in Section VIII along with a conjecture.

II. NETWORK MODELS AND A KEY LEMMA

Consider two finite alphabet sources \{U_i, V_i\} with correlated outputs that are drawn according to a distribution \(P[U_i = u, V_i = v] = p(u, v)\). The sources are memoryless, i.e., \((U_i, V_i)\)'s are independent and identically distributed (i.i.d). Both of the sources are to be transmitted to the corresponding destinations through continuous alphabet and discrete-time memoryless non-ergodic Gaussian channel models. Channels are parameterized by the phase shifts that are introduced by different paths of the network which are, as a realistic assumption for wireless networks, not known to the transmitters. The vector \(\theta\) denotes the non-ergodic phase fading parameters. For simplicity, throughout the paper, we assume that transmitter node with index \(i \in \{1, 2, r\}\) has power constraint \(P_i\) and the noise power at all corresponding receiving nodes is \(N\).
In the models that we consider, the receiver(s) are fully aware of $\theta$. However, the transmitters do not have access to the channel state information (CSI), $\theta$, but only the knowledge of the family of channels over which the communication is done and the code design must be robust for all $\theta$. Such channels are referred to as *compound* channels \[9], [24]. Nevertheless, in order to avoid ambiguity, we call the particular channel under consideration a *phase-incoherent* (PI) channel with correlated sources. In the sequel, we introduce the channel models to be considered in this paper.

A. Multiple Access Channel (MAC)

A phase incoherent multiple access channel (PI-MAC) $(X_1 \times X_2, Y, p_\theta(y|x_1, x_2))$ with parameter $\theta = (\theta_1, \theta_2) \in [0, 2\pi)^2$ is illustrated in Fig. 1. The MAC is described by the relationship

$$Y_i = h_1 X_{1i} + h_2 X_{2i} + Z_i,$$  \hspace{1cm} (4)

where $X_{1i}, X_{2i}, Y_i \in \mathbb{C}$, $Z_i \sim \mathcal{CN}(0, N)$ is circularly symmetric complex Gaussian noise, $h_1 = g_1 e^{j\theta_1}, h_2 = g_2 e^{j\theta_2}$ are non-ergodic complex channel gains, and parameter $\theta$ represents the phase shifts introduced by the channel to inputs $X_1$ and $X_2$, respectively. The amplitude gains, $g_1$ and $g_2$, are assumed to be known at transmitters and can model e.g., line of sight path gains.

B. MAC with Unidirectional Non-Causal Cooperation Between Transmitters (UNCC-MAC)

A PI-UNCC-MAC $(X_1 \times X_2, Y, p_\theta(y|x_1, x_2))$ is depicted in Fig. 2. The first encoder $X_1$ has non-causal and perfect knowledge of the second source $V$. The channel characteristic is the same as an ordinary PI-MAC given in (4).

C. MAC with Unidirectional Causal Cooperation Between Transmitters (UCC-MAC)

Another multi-user model that is considered in this paper is a cooperative variation of the multiple access channel $(X_1 \times X_2, Y_1 \times Y, p_\theta(y_1, y|x_1, x_2))$ where one of the transmitters can play the role of a relay for the other. This channel model is shown in Fig. 3 where the first transmitter (node indicated by $X_1$) can...
help the second transmitter (node indicated by $X_2$) to transmit its information to the destination. However, node 2 cannot help node 1 and thus we refer to such a channel as a unidirectional cooperative MAC. The received signal of the PI-UCC-MAC at the destination is also given by (4). At the transmitter/relay node, node 1, we have

$$Y_{1i} = g_{21} e^{j\theta_{21}} X_{2i} + Z_{1i},$$

(5)

where $g_{21}$ and $\theta_{21}$ are the path gain and the phase shift of the channel from node 2 to node 1 respectively. The vector $\theta$ for the PI-MAC has three elements and is defined as $\theta = (\theta_1, \theta_2, \theta_{21})$.

**D. Multiple Access Relay Channel (MARC)**

A multiple access relay channel is a network with four nodes, two transmitters, a relay and a destination. As depicted in Fig. 4 in a MARC with phase fading $(\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_r, \mathcal{Y}, p_\theta(y, y_r|x_1, x_2, x_r))$, two transmitters wish to reliably send their information to a common destination, with the help of a relay. There are five paths in the network. The phase parameters are not known to the transmitters and hence we refer to the MARC as PI-MARC. The received signal at the destination is given by

$$Y_i = h_1 X_{1i} + h_2 X_{2i} + h_r X_r + Z_i,$$  

(6)
where $X_{1i}, X_{2i}, Y_i \in \mathbb{C}$, $Z_i \sim \mathcal{C}\mathcal{N}(0, N)$ is circularly symmetric complex Gaussian noise, $h_1 = g_1 e^{j\theta_1}$, $h_2 = g_2 e^{j\theta_2}$, $h_r = g_r e^{j\theta_r}$ are non-ergodic complex channel gains, and $\theta_1, \theta_2, \theta_r$ represent the phase shifts introduced by the channel to inputs $X_1, X_2$ and $X_r$, respectively.

Moreover, the signal received at the relay can be written as

$$Y_{ri} = h_{1r}X_{1i} + h_{2r}X_{2i} + Z_{ir}$$

(7)

where $Z_{ir} \sim \mathcal{C}\mathcal{N}(0, N)$ and $h_{1r} = g_{1r} e^{j\theta_{1r}}, h_{2r} = g_{2r} e^{j\theta_{2r}}$ are the complex path gains with unknown phases $\theta_{1r}, \theta_{2r}$ at transmitters. The parameter $\theta = (\theta_1, \theta_2, \theta_r, \theta_{1r}, \theta_{2r}) \in [0, 2\pi)^5$ of the PI-MARC includes all of the fading phases in different paths.

E. MARC with Unidirectional Non-Causal Cooperation Between Transmitters (UNCC-MARC)

An important multi-user network that we consider is a unidirectional cooperative MARC, in which the first encoder has non-causal access to the second source $V$. Indeed, UNCC-MARC is a UNCC-MAC with a relay. The channel model is similar to an ordinary MARC, but the setup of the sources and encoders are different. Fig. 5 depicts a PI-UNCC-MARC.

Like PI-MARC, the input/output relationships of the channel for the receiver and the relay are given by (6) and (7). The parameter $\theta$ is the same as that of the ordinary MARC.

F. MARC with Unidirectional Causal Cooperation Between Transmitters (UCC-MARC)

We also consider sending sources $U, V$ over a PI-MARC with causal unidirectional cooperation between the encoders denoted by $(X_1 \times X_2 \times X_r, Y_1 \times Y_r, p_\theta(y_1, y_r, y|x_1, x_2, x_r))$. As it can be seen from Figure 6 the encoder $X_1$ does not have non-causal knowledge about $V$, but it receives a noisy phase faded version of $X_2$ through the link from node 2 to node 1. Again, (6) and (7) describe the input/output relationships of the channel for the receiver and the relay. Additionally, the relationship

$$Y_{1i} = g_{21} e^{j\theta_{21}} X_{2i} + Z_{1i}$$

(8)

describes the cooperative link from node 2 to node 1 which completes the definition of a PI-UCC-MARC. The parameter $\theta$ for the PI-UCC-MARC is the vector $\theta = (\theta_1, \theta_2, \theta_r, \theta_{1r}, \theta_{2r}, \theta_{12})$. 

Fig. 4. Correlated sources and phase incoherent multiple access relay channel.
Fig. 5. Correlated sources and phase incoherent multiple access relay channel with unidirectional non-causal cooperation between the encoders.

Fig. 6. Correlated sources and phase incoherent multiple access relay channel with unidirectional causal cooperation between the encoders.

G. Interference Channel (IC)

Another network model we consider in this paper is the two-user interference channel with strong interference. A continuous alphabet, discrete-time memoryless interference channel (IC) with phase fading is denoted by \((X_1 \times X_2, Y_1 \times Y_2, p_{\theta_1, \theta_2}(y_1, y_2|x_1, x_2))\) and its probabilistic characterization is described by the relationship

\[
Y_1 = g_{11} e^{j\theta_{11}} X_{1i} + g_{21} e^{j\theta_{21}} X_{2i} + Z_{1i}, \quad (9)
\]

\[
Y_2 = g_{12} e^{j\theta_{12}} X_{1i} + g_{22} e^{j\theta_{22}} X_{2i} + Z_{2i}, \quad (10)
\]

where \(X_{1i}, X_{2i}, Y_i \in \mathbb{C}, Z_i \sim \mathcal{CN}(0, N)\) is circularly symmetric complex Gaussian noise, \(g_{11}, g_{12}, g_{21}, g_{22}\) are non-ergodic complex channel gains, and parameters \(\theta_1 = (\theta_{11}, \theta_{21}) \in [0, 2\pi)^2, \theta_2 = (\theta_{12}, \theta_{22}) \in [0, 2\pi)^2\) represents the phase shifts introduced by the channel to inputs \(X_1\) and \(X_2\), respectively. Figure 7 depicts such a channel. We refer to the IC defined by (9) and (10) as PI-IC if we assume the phase shift parameters \(\theta_1, \theta_2\) are not known to the transmitters.
H. Interference Relay Channel (IRC)

The last network model we consider is an interference channel with two transmitters and a relay referred to as interference relay channel (IRC), depicted in Figure 8. Again, we consider phase fading at all paths, unknown to the transmitters and thus call the channel a phase-incoherent IRC (PI-IRC). The PI-IRC \((X_1 \times X_2 \times Y_1 \times Y_2 \times Y_r, p_\theta(y_1, y_2, y_r| x_1, x_2, x_r))\) is described by relationships

\[
\begin{align*}
Y_{1i} &= g_{11}e^{j\theta_{11}}X_{1i} + g_{21}e^{j\theta_{21}}X_{2i} + g_{r1}e^{j\theta_{r1}}X_{ri} + Z_{1i}, \\
Y_{2i} &= g_{12}e^{j\theta_{12}}X_{1i} + g_{22}e^{j\theta_{22}}X_{2i} + g_{r2}e^{j\theta_{r2}}X_{ri} + Z_{2i}, \\
Y_{ri} &= g_{1r}e^{j\theta_{1r}}X_{1i} + g_{2r}e^{j\theta_{2r}}X_{2i} + Z_{ri},
\end{align*}
\]

where \(X_{1i}, X_{2i}, X_{ri}, Y_{1i}, Y_{2i}, Y_{ri} \in \mathbb{C}, Z_{1i}, Z_{2i}, Z_{ri} \sim CN(0, N)\) are circularly symmetric complex Gaussian noises, \(g_{11}, g_{21}, g_{r1}, g_{12}, g_{22}, g_{r2}\) are non-ergodic complex channel gains, and parameter \(\theta = (\theta_{11}, \theta_{21}, \theta_{r1}, \theta_{12}, \theta_{22}, \theta_{r2}, \theta_{1r}, \theta_{2r}) \in [0, 2\pi)^8\) represents the phase shifts introduced by the channel to inputs \(X_1, X_2\) and \(X_r\), respectively.
1. Key Lemma

Definition 1: Let $X = (X_1, X_2, \ldots, X_m)$, be a vector of random variables with joint distribution $p_X$ and $\max_i \mathbb{E}\|X_i\|^2 \leq \infty$. Also let the scalar RV $V \triangleq \sum_{i=1}^{m} g_i e^{j\theta_i} X_i + Z$, where $g_i e^{j\theta}$ are arbitrary complex coefficients and $Z \sim \mathcal{CN}(0, N)$.

We now state the following lemma which asserts that the minimum over $\theta = (\theta_1, \theta_2, \ldots, \theta_m)$ of the mutual information between $X$ and $V$, is maximized when $X$ is a zero-mean Gaussian vector with independent elements, i.e., RVs $X_1, X_2, \ldots, X_m$ are independent Gaussians with zero mean.

Notation: For convenience, we denote the mutual information between $X$ and $V$ by $B_{\theta}(p_X) \triangleq I(X; \sum_{i=1}^{m} g_i e^{j\theta_i} X_i + Z)$.

Lemma 1: Let $\mathcal{P} = \{p_X : \mathbb{E}\|X_i\|^2 \leq P_i, \forall i\}$ and $p_X^* \in \mathcal{P}$ be a zero-mean Gaussian distribution with independent elements and $\mathbb{E}\|X_i\|^2 = P_i, \forall i$. Then,

$$\max_{p_X \in \mathcal{P}} \min_{\theta} B_{\theta}(p_X) = \log \left(1 + \sum_{i=1}^{m} g_i^2 P_i / N\right) = B_{\theta}(p_X^*),$$

i.e., when $\theta$ is chosen adversarially, the best $X$ is a zero-mean Gaussian vector with independent elements and $\text{Var}(X_i) = P_i, \forall i$.

Proof:

By definition, we have

$$B_{\theta}(p_X) = h(g_1 e^{j\theta_1} X_1 + g_2 e^{j\theta_2} X_2 + \cdots + g_N e^{j\theta_m} X_m + Z) - h(Z).$$

By letting $\mathbb{E}(X_i X_j) = \rho_{ij} \sqrt{P_i P_j}$, it can be easily seen that the RV $V$ has a fixed variance $\sigma_V^2$, which is equal to

$$\sigma_V^2 = \left(\sum_{i=1}^{m} g_i^2 P_i\right) + N + 2 \sum_{i < j} g_i g_j \sqrt{P_i P_j} \text{Re}\left\{\rho_{ij} e^{j(\theta_i - \theta_j)}\right\}. \quad (11)$$

Using the fact that for a given variance $\sigma_V^2$, the Gaussian distribution maximizes the differential entropy $h(V)$ [7], we can bound $B_{\theta}(p_X)$ as

$$B_{\theta}(p_X) \leq \frac{1}{2} \log(2\pi e \sigma_V^2) - h(Z). \quad (12)$$

Next, note that $\min_{\theta} \sigma_V^2$ is maximized when $\rho_{ij} = 0, \forall i, j$. It can be seen from (11) that if $\rho_{ij} \neq 0$, the parameters $\theta_1, \theta_2, \ldots, \theta_m$ can be chosen such that the term $2 \sum_{i < j} g_i g_j \sqrt{P_i P_j} \text{Re}\left\{\rho_{ij} e^{j(\theta_i - \theta_j)}\right\}$ is strictly negative. Therefore, independent Gaussians ($\rho_{ij} = 0, \forall i, j$) maximize the right hand side of (12) and the lemma is proved.

Remark 1: For the ergodic setting, where $\theta$ is i.i.d. from channel use to channel use, uniformly distributed over $[0, 2\pi)^m$, and the averaged mutual information over $\theta$ is to be maximized, a similar result is given in
Specifically,

\[
\max_{p_X} \mathbb{E}_{\theta} B_\theta(p_X) = \log \left( 1 + \sum_{i=1}^{m} g_i^2 P_i / N \right).
\]

III. PHASE INCOHERENT MULTIPLE ACCESS RELAY CHANNEL

In this section, we formulate the problem of source-channel coding for the PI-MARC introduced in Section II-D and state a separation theorem for it [15]. The definitions and problem formulation given in this section will be of use for the other networks in the paper.

A. Preliminaries

Definition 2: Joint source-channel code: A joint source-channel code of length \( n \) for the PI-MARC introduced in Section II-D with correlated sources is defined by

1) Two encoding functions

\[
(x_{11}, x_{12}, \cdots, x_{1n}) = x_{1}^n : \mathcal{U}^n \rightarrow \mathcal{X}_1^n
\]

\[
(x_{21}, x_{22}, \cdots, x_{2n}) = x_{2}^n : \mathcal{V}^n \rightarrow \mathcal{X}_2^n,
\]

that map the source outputs to the codewords. Furthermore, we define relay encoding functions by

\[
x_{ri} = f_i(y_{r1}, y_{r2}, \cdots, y_{r(i-1)}), \quad i = 1, 2, \cdots, n.
\]

The sets of codewords are denoted by the codebook \( \mathcal{C} = \{ (x_1(u), x_2(v)) : u \in \mathcal{U}^n, v \in \mathcal{V}^n \} \).

2) Power constraint \( P_1, P_2 \) and \( P_r \) at the transmitters, i.e.,

\[
\mathbb{E} \left[ \frac{1}{n} \sum_{i=1}^{n} \| X_{ji} \|^2 \right] \leq P_j, \quad j = 1, 2, r,
\]

where \( \mathbb{E} \) is the expectation operation over the distribution induced by \( \mathcal{U}^n, \mathcal{V}^n \).

3) A decoding function

\[
g_\theta^n : \mathcal{Y}^n \rightarrow \mathcal{U}^n \times \mathcal{V}^n.
\]

Upon reception of the received vector \( Y^n \), the receiver decodes \( (\hat{U}^n, \hat{V}^n) = g_\theta(Y^n) \) as the transmitted source outputs. The probability of an erroneous decoding depends on \( \theta \) and is given by

\[
P_e^n(\theta) = P\{(U^n, V^n) \neq (\hat{U}^n, \hat{V}^n) | \theta \}
\]

\[
= \sum_{(u^n, v^n) \in \mathcal{U}^n \times \mathcal{V}^n} p(u^n, v^n) \times P\{(\hat{U}^n, \hat{V}^n) \neq (u^n, v^n) | (u^n, v^n), \theta \}.
\]

Definition 3: We say the source \( \{ U_i, V_i \}_{i=1}^{n} \) of i.i.d. discrete random variables with joint probability mass function \( p(u, v) \) can be reliably sent over the PI-MARC, if there exists a sequence of encoding functions \( \mathcal{E}_n \triangleq \{ x_1^n(U^n), x_2^n(V^n), f_1, f_2, \cdots, f_n \} \) and decoders \( g_\theta^n \) such that the output sequences \( U^n \) and \( V^n \) of the
source can be estimated with asymptotically small probability of error (uniformly over all parameters $\theta$) at the receiver side from the received sequence $Y^n$, i.e.,

$$\left[ \sup_{\theta} P_e^n(\theta) \right] \rightarrow 0, \text{ as } n \rightarrow \infty. \quad (14)$$

**Theorem 2:** Reliable communication over a PI-MARC: Consider a PI-MARC with power constraints $P_1, P_2, P_r$ on the transmitters, fading amplitudes $g_1, g_2, g_r > 0$ between the nodes and the receiver and $g_{1r}, g_{2r} > 0$ between the transmitter and the relay, and the gain conditions

$$g_{1r}^2 P_1 \geq g_1^2 P_1 + g_r^2 P_r, \quad (15)$$

$$g_{2r}^2 P_1 \geq g_2^2 P_1 + g_r^2 P_r. \quad (16)$$

A necessary condition for reliably sending the source pair $(U, V) \sim \prod_i p(u_i, v_i)$, over a PI-MARC, is given by

$$H(U|V) \leq \log(1 + (g_{1r}^2 P_1 + g_r^2 P_r)/N), \quad (17)$$

$$H(V|U) \leq \log(1 + (g_{2r}^2 P_2 + g_r^2 P_r)/N), \quad (18)$$

$$H(U, V) \leq \log(1 + (g_{1r}^2 P_1 + g_{2r}^2 P_2 + g_r^2 P_r)/N). \quad (19)$$

Moreover, (17)-(19) also describes sufficient conditions for reliable communications with $\leq$ replaced by $<$.

**Proof:** The theorem is the same as [15, Theorem 4].

In Sections IV and V, we study a more general version of the PI-MARC in which the transmitters cooperate in a specific way. Indeed, we consider a pair of correlated sources to be communicated over a phase incoherent (PI) multiple access relay channel where one of the transmitters has causal or non-causal side information about the message of the other. We refer to such channels as UC-MARC. In the non-causal case (see Fig. 2), there is no path between the transmitters and the first encoder knows both sources outputs $U, V$, whereas in the causal case (see Fig. 6), the first transmitter works as a relay for the other while communicating its own information. For the situations where the channel gains between the relay and the transmitters are large enough, we prove that the separation approach is optimal. This may correspond to the physical proximity of the relay and the transmitters to each other. For the causal case, we have an additional condition on the gain between the encoders. The phase fading information is not known to the transmitters while it is known at the receivers.

**IV. UC-MARC with non-causal side information**

In this section, we study the PI-UNCC-MARC introduced in Section II-F with the pair of arbitrarily correlated sources $(U, V)$. The definition of a joint source-channel code for the PI-UNCC-MARC is identical to the one defined for a PI-MARC in section II-A except for the definition of the encoding function $x_1$ which is replaced by
\[(x_{11}, x_{12}, \cdots, x_{1n}) = x_1^n(U, V).\]

**Theorem 3:** Reliable Communication over a PI-UNCC-MARC: Consider a PI-UNCC-MARC with non-causal cooperation and with power constraints \(P_1, P_2, P_r\) on transmitters and relay, fading amplitudes \(g_1, g_2, g_r > 0\) between the nodes and the receiver and \(g_{1r}, g_{2r} > 0\) between the transmitter and the relay. Moreover, assume the gain conditions

\[
g_1^2 P_1 \geq g_1^2 P_1 + g_r^2 P_r, \tag{20}
\]

\[
g_1^2 P_1 + g_2^2 P_2 \geq g_1^2 P_1 + g_2^2 P_2 + g_r^2 P_r. \tag{21}
\]

A necessary condition for sending a source pair \((U, V)\) over such PI-UC-MARC is given by

\[
H(U|V) \leq \log(1 + (g_1^2 P_1 + g_r^2 P_r)/N), \tag{22}
\]

\[
H(U, V) \leq \log(1 + (g_1^2 P_1 + g_2^2 P_2 + g_r^2 P_r)/N). \tag{23}
\]

Furthermore, eqs. \(22\)-\(23\) also give the sufficient conditions for reliable communications over such PI-UD-MARC with \(\leq\) replaced by <.

The proof of the theorem is divided into two parts: achievability and converse. The achievability part is obtained by a separate source and channel coding approach. The source coding part involves Slepian-Wolf coding followed by a channel coding technique which is based on the block Markov coding. The converse and achievability parts of Theorem 3 are discussed and proved in the sequel.

**A. Converse**

We derive an outer bound on the capacity region of the PI-UC-MARC (both causal and non-causal) under gain conditions \(20\)-\(21\) and prove the converse part of Theorem 3.

**Lemma 2:** Converse: Let \(E_n\), and \(g^n_\theta\) be a sequence in \(n\) of encoders and decoders for the PI-UC-MARC for which \(\sup_\theta P_e^n(\theta) \to 0\), as \(n \to \infty\). Then

\[
H(U|V) \leq \min_\theta I(X_1, X_r; g_1 e^{j\theta_1} X_1 + g_r e^{j\theta_r} X_r + Z),
\]

\[
H(U, V) \leq \min_\theta I(X_1, X_2, X_r; g_1 e^{j\theta_1} X_1 + g_2 e^{j\theta_2} X_2 + g_r e^{j\theta_r} X_r + Z),
\]

for some joint distribution \(p_{X_1, X_2, X_r}\) such that \(\mathbb{E}|X_1|^2 \leq P_1, \mathbb{E}|X_2|^2 \leq P_2, \mathbb{E}|X_r|^2 \leq P_r\), with \(Z \sim \mathcal{CN}(0, N)\).

**Proof:**

First, fix a PI-UC-MARC with given parameter \(\theta\), a codebook \(C\), and induced empirical distribution \(p_{\theta}(u, v, x_1, x_2, x_r, y)\) by the codebook. Since for this fixed choice of \(\theta\), \(P_e^n(\theta) \to 0\), from Fano’s inequality, we have

\[
\frac{1}{n} H(U, V|Y, \theta) \leq \frac{1}{n} P_e^n(\theta) \log \|U^n \times V^n\| + \frac{1}{n} \delta_n(\theta), \tag{24}
\]

\[
\frac{1}{n} \delta_n(\theta) \to 0.
\]
and \( \epsilon_n(\theta) \to 0 \), where convergence is uniform in \( \theta \) by [14]. Defining \( \sup_{\theta} \epsilon_n(\theta) = \epsilon_n \) and following the similar steps as in [5] Section 4, we have

\[
H(U|V) = \frac{1}{n} H(U|V)
\]

\[
\leq \frac{1}{n} I(U; Y|V, X_2, \theta)
\]

\[
= \frac{1}{n} I(U; Y|V, X_2, \theta) + \frac{1}{n} H(U|V, Y, X_2, \theta)
\]

\[
\leq \frac{1}{n} I(U; Y|V, X_2, \theta) + \epsilon_n
\]

\[
\leq \frac{1}{n} I(X_1; Y|V, X_2, \theta) + \epsilon_n,
\]

where (a) follows from the fact that \( X_2 \) is only a function of \( V \), (b) follows from (24), and (c) follows from data processing inequality. Similarly, it can be shown that

\[
H(U, V) = \frac{1}{n} I(U, V; Y|\theta) + \frac{1}{n} H(U, V|Y, \theta)
\]

\[
\leq \frac{1}{n} I(X_1, X_2, X_r; Y|\theta) + \epsilon_n.
\]

We now define the region \( C_n(\theta) \) as

\[
C_n(\theta) = \left\{ (R_1, R_2) : R_1 < \frac{1}{n} I(X_1^n; Y^n|V^n, X_2^n, \theta) + \epsilon_n, \right. \\
R_2 < \frac{1}{n} I(X_1^n, X_2^n; Y^n|\theta) + \epsilon_n \right\},
\]

for the empirical distribution induced by the \( n \)th codebook

\[
\prod_{i=1}^n p(u_i, v_i)p(x_1^n|u)p(x_2^n|v) \prod_{i=1}^n p(\theta, y_{ri}|x_{1i}, x_{2i}, x_{ri}) \times p(\theta, y_{ri}p(x_{ri}|y_{ri}, y_{r2}, \ldots, y_{r(i-1)}).
\]

Hence, the outer bounds (25) and (26) can be equivalently described by \( C_n(\theta) \):

\[
(H(U|V), H(U, V)) \in C_n(\theta).
\]

We then note that the outer bound is true for all \( \theta \) and thus can be tightened by taking intersection over \( \theta \) and letting \( n \to \infty \). We now further upper bound \( C_n(\theta) \) and then take the limit and intersection.

First, we expand \( Y \) in the right hand side of (25) to upper bound \( H(U|V) \) as follows:

\[
H(U|V) \leq \frac{1}{n} I(X_1, X_r; Y|V, X_2, \theta) + \epsilon_n
\]

\[
= \frac{1}{n} I(X_1, X_r; g_1 e^{j\theta}; X_1 + g_2 e^{j\theta}; X_2 + g_r e^{j\theta}; X_r + Z|V, X_2) + \epsilon_n
\]

\[
= \frac{1}{n} I(X_1, X_r; g_1 e^{j\theta}; X_1 + g_r e^{j\theta}; X_r + Z|V, X_2) + \epsilon_n
\]

\[
= \frac{1}{n} \left[ h(g_1 e^{j\theta}; X_1 + g_r e^{j\theta}; X_r + Z|V, X_2) - h(Z) \right] + \epsilon_n
\]

\[
\leq \frac{1}{n} \left[ h(g_1 e^{j\theta}; X_1 + g_r e^{j\theta}; X_r + Z) - h(Z) \right] + \epsilon_n
\]

\[
= \frac{1}{n} I(X_1, X_r; g_1 e^{j\theta}; X_1 + g_r e^{j\theta}; X_r + Z) + \epsilon_n
\]
\[
\leq \frac{1}{n} \sum_{i=1}^{n} I(X_{1i}, X_{ri}; g_1 e^{j\theta_1} X_{1i} + g_r e^{j\theta_r} X_{ri} + Z_i) + \epsilon_n
\]

\overset{(a)}{=} I(X_1, X_r; g_1 e^{j\theta_1} X_1 + g_r e^{j\theta_r} X_r + Z|W) + \epsilon_n

\[
= \left[ h(g_1 e^{j\theta_1} X_1 + g_r e^{j\theta_r} X_r + Z|W) - h(Z) \right] + \epsilon_n
\]

\[
\leq \left[ h(g_1 e^{j\theta_1} X_1 + g_r e^{j\theta_r} X_r + Z) - h(Z) \right] + \epsilon_n
\]

\[
= I(X_1, X_r; g_1 e^{j\theta_1} X_1 + g_r e^{j\theta_r} X_r + Z) + \epsilon_n.
\]  

(27)

where (a) follows by defining new random variables

\[
X_j = X_{jW}, \ j \in \{1, 2, r\},
\]

(28)

\[
Z = Z_W,
\]

(29)

\[
W \sim \text{Uniform}\{1, 2, \ldots, n\}.
\]

(30)

From (13), the input signals \(X_1, X_r\) satisfy the power constraints

\[
E|X_j|^2 = E \left[ \frac{1}{n} \sum_{i=1}^{n} \|X_{ji}\|^2 \right] \leq P_j, \ j = 1, r,
\]

(31)

and \(Z \sim \mathcal{CN}(0, N)\).

Moreover, following similar steps, we have

\[
H(U, V) = \frac{1}{n} H(U, V)
\]

\[
= \frac{1}{n} I(U, V; Y|\theta) + \frac{1}{n} H(U, V|Y, \theta)
\]

\[
\leq \frac{1}{n} I(U, V; Y|\theta) + \epsilon_n
\]

\[
\leq \frac{1}{n} I(X_1, X_2; Y|\theta) + \epsilon_n
\]

\[
\leq \frac{1}{n} I(X_1, X_2, X_r; Y|\theta) + \epsilon_n
\]

\[
= \frac{1}{n} I(X_1, X_2, X_r; g_1 e^{j\theta_1} X_1 + g_2 e^{j\theta_2} X_2 + g_r e^{j\theta_r} X_r + Z) + \epsilon_n
\]

\[
\leq \frac{1}{n} \sum_{i=1}^{n} I(X_{1i}, X_{2i}, X_{ri}; g_1 e^{j\theta_1} X_{1i} + g_2 e^{j\theta_2} X_{2i} + g_r e^{j\theta_r} X_{ri} + Z_i) + \epsilon_n
\]

\[
\leq I(X_1, X_2, X_r; g_1 e^{j\theta_1} X_1 + g_2 e^{j\theta_2} X_2 + g_r e^{j\theta_r} X_r + Z) + \epsilon_n.
\]

(32)

where the last step follows with the same RVs as in (28)-(30).

The constraints defined by (27) and (32) is an outer bound on \(C_n(\theta)\). But since it applies for a fixed \(\theta\), it is also true for all choices of \(\theta\). By taking intersection over all values of \(\theta\) and letting \(n \to \infty\), the lemma is proved.

To prove the converse part of Theorem 2, we note by Lemma 2 that each of the bounds of Lemma 2 are simultaneously maximized by independent Gaussians. The proof of the converse is complete.

Remark 2: Note that to prove the converse part of the Theorem 3 we do not need the receiver to know the CSI \(\theta\). This is indeed true for other separation theorems of the paper as well.
<table>
<thead>
<tr>
<th>Encoder</th>
<th>Block 1</th>
<th>Block 2</th>
<th>Block $B$</th>
<th>Block $B+1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x_1(1, W_{11}, W_{21}, 1)$</td>
<td>$x_1(W_{11}, W_{12}, W_{22}, W_{21})$</td>
<td>$x_1(W_{1(B-1)}, W_{1B}, W_{2B}, W_{2(B-1)})$</td>
<td>$x_1(W_{1B}, 1, 1, W_{2B})$</td>
</tr>
<tr>
<td>2</td>
<td>$x_2(1, W_{21})$</td>
<td>$x_2(W_{21}, W_{22})$</td>
<td>$x_2(W_{2(B-1)}, W_{2B})$</td>
<td>$x_2(W_{2B}, 1)$</td>
</tr>
<tr>
<td>$r$</td>
<td>$x_r(1, 1)$</td>
<td>$x_r(W_{11}, W_{21})$</td>
<td>$x_r(W_{1(B-1)}, W_{2(B-1)})$</td>
<td>$x_r(W_{1B}, W_{2B})$</td>
</tr>
</tbody>
</table>

**TABLE I**

**Block Markov encoding scheme for UNCC-MARC.**

**B. Achievability**

We now establish the same region as achievable for the PI-UNCC-MARC with non-causal cooperation between the encoders. To derive the achievable region, we perform separate source-channel coding. The source coding is performed by Slepian-Wolf coding and the channel coding argument is based on regular block Markov encoding in conjunction with backward decoding [17]. Both source coding and channel coding schemes are explained as follows.

**Source Coding:** Recall that the first encoder has non-causal access to the second source $V$. From Slepian-Wolf coding [19], for asymptotically lossless representation of the source $((U, V), V)$, we should have the rates $(R_1, R_2)$ satisfying

$$R_1 > H(U|V),$$

$$R_1 + R_2 > H(U, V).$$

The source codes are represented by indices $W_1, W_2$ which are then channel coded before being transmitted.

**Channel Coding:** An achievable region for the discrete memoryless UC-MARC with 2 users is given based on the block Markov coding scheme shown in Table I combined with backward decoding.

First fix a distribution $p(x_1)p(x_2)p(x_r)$ and construct random codewords $x_1, x_2, x_r$ based on the corresponding distributions. The message $W_i$ of each encoder is divided to $B$ blocks $W_{i1}, W_{i2}, \cdots, W_{iB}$ of $2^{nR_i}$ bits each, $i = 1, 2$. The codewords are transmitted in $B + 1$ blocks based on the block Markov encoding scheme depicted in Table I. Using its non-causal knowledge of the second source, transmitter 1 sends the information using the codeword $x_1(W_{1(t-1)}, W_{1t}, W_{2t}, W_{2(t-1)})$, while transmitter 2 uses codeword $x_2(W_{2(t-1)}, W_{2t})$ and the relay sends the codeword $x_r(W_{1(t-1)}, W_{2(t-1)})$. We let $B \to \infty$ to approach the original rates $R_1, R_2$.

At the end of each block $b$, the relay decodes $W_{1b}, W_{2b}$, referred to as forward decoding [8]. Indeed, at the end of the first block, the relay decodes $W_{11}, W_{21}$ from the received signal $Y_r(W_{1b}, W_{2b})$. In the second block, nodes 1 and 2 transmit $x_1(W_{11}, W_{12}, W_{22}, W_{21})$ and $x_2(W_{21}, W_{22})$, respectively. The relay decodes $W_{12}, W_{22}$, using the knowledge of $W_{11}, W_{21}$, and this is continued until the last block. Using random coding arguments and forward decoding from the first block, for reliable decoding of messages $W_{1(b-1)}, W_{2(b-1)}$ at the relay after the $b$th block, when $n \to \infty$, it is sufficient to have

$$R_1 < I(X_1; Y_r|X_2, X_r, \theta),$$

(33)
\[ R_1 + R_2 < I(X_1, X_2; Y_r | X_r, \theta). \] (34)

The decoding at the destination, however, is performed based on backward decoding \[13], \[23], i.e., starting from the last block back to the former ones. As depicted in Table I at the end of block \( B + 1 \), the receiver can decode \( W_{1B}, W_{2B} \). Afterwards, by using the knowledge of \( W_{1B}, W_{2B} \), the receiver goes one block backwards and decodes \( W_{1(B-1)}, W_{2(B-1)} \). This process is continued until the receiver decodes all of the messages. Thus, by applying regular block Markov encoding and backward decoding as shown in Table I one finds that the destination can decode the messages reliably if \( n \to \infty \) and

\[ R_1 < I(X_1, X_r; Y_r | X_2, \theta), \] (35)

\[ R_1 + R_2 < I(X_1, X_2, X_r; Y_r | \theta). \] (36)

The achievability part is complete by first choosing \( X_1, X_2, \) and \( X_r \) as independent Gaussians and observing that under conditions (20) and (21), (35) and (36) are tighter bounds than (33) and (34).

As a result of Theorem 3 in the following corollary, we state a separation theorem for the PI-UNCC-MAC introduced in Section II-C with non-causal cooperation between encoders.

**Corollary 1:** Reliable Communication over a PI-UNCC-MAC: Necessary conditions for reliable communication of the source \((U, V)\) over a PI-UNCC-MAC with power constraints \(P_1, P_2\) on transmitters, fading amplitudes \(g_1, g_2 > 0\), and source pair \((U, V) \sim \prod_i p(u_i, v_i)\), are given by

\[ H(U | V) \leq \log(1 + g_1^2 P_1 / N), \] (37)

\[ H(U, V) \leq \log(1 + (g_1^2 P_1 + g_2^2 P_2) / N). \] (38)

Sufficient conditions for reliable communication are also given by (56)-(57), with \( \leq \) replaced by \(<\). \(\Box\)

**Proof:**

The PI-UNCC-MAC is equivalent to a PI-UNCC-MARC where the relay has power constraint \( P_r = 0 \). As the relay is thus silent, we may assume without loss that \( g_{1r}, g_{2r} \) are arbitrarily large, and the conditions (20) and (21) are trivially satisfied.

\section{V. UC-MARC with Causal Side Information}

In this section, we state and prove a separation Theorem for another class of UC-MARC in which the encoders cooperate causally and by means of a wireless phase fading link between transmitters 1 and 2. Unlike the noncausal case discussed in Section II-F, \( X_{1i} \) is a function of the source signal \( U \) and its past received signals \( Y_{1}^{(i-1)} \). In the sequel, we state and prove a separation theorem for the causal PI-UD-MARC under specific gain conditions.

**Theorem 4:** Reliable communication over a PI-UCC-MARC: Consider a PI-UCC-MARC with power constraints \( P_1, P_2, P_r \) on transmitters and the relay, fading amplitudes \( g_1, g_2, g_r, g_{1r}, g_{2r}, g_{21} > 0 \) as shown in Figure 3 and source pair \((U, V) \sim \prod_i p(u_i, v_i)\). Furthermore, assume the gain conditions

\[ g_{1r}^2 P_1 \geq g_{2r}^2 P_1 + g_r^2 P_r, \] (39)
Encoder | Block 1 | Block 2 | Block $B$ | Block $B + 1$
---|---|---|---|---
1 | $x_1(1, W_{11}, 1)$ | $x_1(W_{11}, W_{12}, W_{21})$ | $x_1(W_{1(B-1)}, W_{1B}, W_{2(B-1)})$ | $x_1(W_{1B}, 1, W_{2B})$
2 | $x_2(1, W_{21})$ | $x_2(W_{21}, W_{22})$ | $x_2(W_{2(B-1)}, W_{2B})$ | $x_2(W_{2B}, 1)$
$r$ | $x_r(1, 1)$ | $x_r(W_{11}, W_{21})$ | $x_r(W_{1(B-1)}, W_{2(B-1)})$ | $x_r(W_{1B}, W_{2B})$

TABLE II
Block Markov encoding scheme for UCC-MARC.

\[
g_2^2 P_1 \geq g_1^2 P_1 + g_2^2 P_r, \tag{40}
\]
\[
1 + \frac{g_2^2 P_2}{N} \geq 2^{-H(U|V)} \left( 1 + \frac{g_1^2 P_1 + g_2^2 P_2 + g_r^2 P_r}{N} \right). \tag{41}
\]

Then, a necessary condition of reliable communication of the correlated sources $(U, V)$ over such channel with or without knowledge of $\theta$ at the receiver, is given by
\[
H(U|V) \leq \log(1 + g_1^2 P_1 + g_2^2 P_r / N), \tag{42}
\]
\[
H(U, V) \leq \log(1 + (g_1^2 P_1 + g_2^2 P_2 + g_r^2 P_r) / N). \tag{43}
\]
Conversely, (42) and (43) also describe sufficient conditions for the causal PI-UCC-MARC with $\leq$ replaced by $<$.

\begin{proof}

Converse: The proof of the converse part of Theorem 4 is exactly the same as that of Theorem 3 as all of the steps remain unchanged in the causal setting.

Achievability: For the achievability part, similar to Section [IV-B], we use separate source and channel coding. We need to show that given (42) and (43), we can first losslessly source code the sources to indices $W_1 \in [1, 2^{nR_1}], W_2 \in [1, 2^{nR_2}]$ and then send $W_1, W_2$ over the channel with arbitrarily small error probability.

Source Coding: Using Slepian-Wolf coding [19], for asymptotically lossless representation of the source $(U, V)$, we should have the rates $(R_1, R_2)$ satisfying
\[
R_1 > H(U|V), \tag{44}
\]
\[
R_2 > H(V|U), \tag{45}
\]
\[
R_1 + R_2 > H(U, V). \tag{46}
\]

Channel Coding: Similar to that given in Section [IV-B] for the noncausal PI-UC-MARC, the channel coding argument is again based on block Markov coding with backward decoding as shown in Table II.

Since $V$ is not perfectly and non-causally known to the first encoder, node 1 needs to first decode $W_{2t}$ after block $t$ from its received signal over the link between the encoders. In order to guarantee correct decoding at the relay and correct backward decoding at the destination, using standard random coding arguments, the following conditions should be satisfied:
\[
R_1 < I(X_1; Y_r|X_2, X_r, \theta), \tag{47}
\]
\[ R_2 < I(X_2; Y_r | X_1, X_r, \theta), \]  
\[ R_1 + R_2 < I(X_1, X_2; Y_r | X_r, \theta), \]  
for decoding at the relay and

\[ R_1 < I(X_1, X_r; Y | X_2, \theta), \]  
\[ R_1 + R_2 < I(X_1, X_2, X_r; Y | \theta), \]

for decoding at the destination respectively.

Additionally, to reliably decode the second encoder’s message at the first encoder (which plays the role of a relay), we need to satisfy the condition

\[ R_2 < I(X_2; Y_1 | X_1, X_r), \]  
\[ R_1 + R_2 < I(X_1, X_2; Y_1 | X_r, \theta). \]

Computing these conditions for independent Gaussian inputs and using conditions (39) and (40), we find the following achievable region for channel coding:

\[ R_1 < \log(1 + \frac{g_1^2 P_1 + g_2^2 P_r}{N}), \]  
\[ R_2 < \log(1 + \frac{g_2^2 P_2}{N}), \]  
\[ R_1 + R_2 < \log(1 + \frac{g_1^2 P_1 + g_2^2 P_2 + g_2^2 P_r}{N}). \]

In order to make the inner bounds of (53)-(55) coincide with the outer bounds (42), (43), we need to have

\[ \log(1 + \frac{g_1^2 P_1 + g_2^2 P_2 + g_2^2 P_r}{N}) - R_1 < \log(1 + \frac{g_2^2 P_2}{N}), \]

so that we can drop (54) from the achievability constraints. But by choosing \( R_1 = H(U|V) + \epsilon, \) with \( \epsilon > 0 \) arbitrary, condition (41) makes (54) dominated by (55) for the Gaussian input distributions. Therefore, since \( \epsilon > 0 \) is arbitrary, one can easily verify that given (42) and (43) with \( \leq \) replaced by \( < \), along with the conditions (39)-(41), source and channel codes of rates \( R_1, R_2 \) can be found such that (44)-(46), and (47)-(52) simultaneously hold.

**Corollary 2:** Reliable communication over a PI-UCC-MAC: Necessary conditions for reliable communication of the sources \((U, V)\) over the causal PI-UCC-MAC with power constraints \(P_1, P_2\) on transmitters, fading amplitudes \(g_1, g_2 > 0\), and source pair \((U, V) \sim \prod_i p(u_i, v_i)\), is given by

\[ H(U|V) \leq \log(1 + g_1^2 P_1 / N), \]  
\[ H(U, V) \leq \log(1 + (g_1^2 P_1 + g_2^2 P_2) / N), \]

provided

\[ 1 + \frac{g_2^2 P_2}{N} \geq 2^{-H(U|V)} \left( 1 + \frac{g_1^2 P_1 + g_2^2 P_2}{N} \right). \]

Given (58), sufficient conditions for reliable communications are also given by (56) and (57), with \( \leq \) replaced by \( < \).
Proof:
The argument is similar to the proof of the Corollary [1]. The PI-UCC-MAC is equivalent to a PI-UCC-MARC where the relay has power constraint \( P_r = 0 \). As the relay is thus silent, we may assume without loss that \( g_{1r}, g_{2r} \) are arbitrarily large. The conditions (39)-(40) of Theorem 4 with (41) being changed to (58) are then trivially satisfied.

VI. INTERFERENCE CHANNEL

We now study the communication of the arbitrarily correlated sources \((U, V)\) over a phase-asynchronous interference channel introduced in Section II-G. The definition of the joint source-channel code and power constraints are similar to the ones given in Section III-A. However, since there are two decoders in this setup, we define two indexed decoding functions \( g^n_{\theta_1} \) and \( g^n_{\theta_2} \) and two error probability functions

\[
P_{e_1}^n(\theta_1) = P\{U^n \neq \hat{U}^n | \theta_1\} = \sum_{u^n \in U^n} p(u^n) \times P\{\hat{U}^n \neq u^n | u^n, \theta_1\},
\]

\[
P_{e_2}^n(\theta_2) = P\{V^n \neq \hat{V}^n | \theta_2\} = \sum_{(v^n) \in V^n} p(v^n) \times P\{\hat{V}^n \neq v^n | v^n, \theta_2\}.
\]

for each of the corresponding receivers.

Consequently, reliable communications for the PI-IC is defined as:

**Definition 4:** We say the source \( \{U_i, V_i\}_{i=1}^n \) of i.i.d. discrete random variables with joint probability mass function \( p(u, v) \) can be reliably sent over the PI-IC, if there exists a sequence of encoding functions \( E_n \equiv \{x_1^n(U^n), x_2^n(V^n)\} \) and decoders \( g^n_{\theta_1}, g^n_{\theta_2} \) such that the output sequence \( U^n \) can be reliably estimated at the first receiver and \( V^n \) can be reliably estimated at the second receiver over all parameters \( \theta_1, \theta_2 \) respectively. That is,

\[
\left[ \sup_{\theta} P_{e_1}^n(\theta_1) \right] \longrightarrow 0, \text{ as } n \to \infty, \quad (59)
\]

\[
\left[ \sup_{\theta} P_{e_2}^n(\theta_2) \right] \longrightarrow 0, \text{ as } n \to \infty. \quad (60)
\]

**Theorem 5:** Reliable Communications over a PI-IC: A necessary condition of reliably sending arbitrarily correlated sources \((U, V)\) over a PI-IC with power constraints \( P_1, P_2 \) on transmitters, fading amplitudes \( g_{11}, g_{12}, g_{21}, g_{22} > 0 \), and source pair \((U, V) \sim \prod_i p(u_i, v_i)\), with the strong interference condition

\[
g_{11} \geq g_{12}
\]

\[
g_{22} \geq g_{21}
\]

with or without knowledge of \( \theta \) at the receiver, is given by

\[
H(U|V) \leq \log(1 + g_{11}^2 P_1 / N), \quad (63)
\]

\[
H(V|U) \leq \log(1 + g_{22}^2 P_2 / N), \quad (64)
\]
\[ H(U, V) \leq \min \left\{ \log(1 + (g_{11} P_1 + g_{21} P_2)/N), \log(1 + (g_{12} P_1 + g_{22} P_2)/N) \right\}. \] (65)

The same conditions (63)-(65) with \( \leq \) replaced by \(<\) describe the achievability region.

A. Converse

In this section, we derive an outer bound on the capacity region and prove the converse part of Theorem 5 for the interference channel.

Lemma 3: Converse: Let \(\{x_1^n(u^n), x_2^n(v^n)\}\), and \(g^n_{\theta_1}, g^n_{\theta_2}\) be sequences in \(n\) of codebooks and decoders for the PI-IC for which (59) and (60) hold. Then we have

\[ H(U|V) \leq \min_{\theta_1} I(X_1; e^{j\theta_1} X_1 + Z), \]

\[ H(V|U) \leq \min_{\theta_2} I(X_2; e^{j\theta_2} X_2 + Z), \]

\[ H(U, V) \leq \min \left\{ \min_{\theta_1} I(X_1, X_2; e^{j\theta_1} X_1 + g_{21} e^{j\theta_2} X_2 + Z), \min_{\theta_2} I(X_1, X_2; g_{12} e^{j\theta_1} X_1 + e^{j\theta_2} X_2 + Z) \right\} \]

(68)

for some joint distribution \(p_{X_1, X_2}\) such that \(\mathbb{E}|X_1|^2 \leq P_1, \mathbb{E}|X_2|^2 \leq P_2\).

Proof:

First, fix a PI-IC with given parameters \((\theta_1, \theta_2)\), a codebook \(C\), and induced empirical distribution \(p_\theta(u, v, x_1, x_2, y_1, y_2)\). Then, we note that by using the strong interference conditions of (61) and (62), one can argue that both of the receivers can decode both of the sequences \(U, V\) provided there are encoders and decoders such that each receiver can reliably decode its own source sequence (see [18] for details). Thus, \(U, V\) can both be decoded from both \(Y_1, Y_2\). Thus, we have the intersection of two PI-MACs and the result follows from Theorem 1.

B. Achievability

The achievability part of Theorem 5 can be obtained by noting that if we make joint source-channel codes such that both receivers are able to decode both messages, then we will have an achievable region. Thus, the interference channel will be divided to two PI-MACs and the achievable region will be again the intersection of the achievable regions of the two PI-MACs as given in Theorem 1.

VII. INTERFERENCE RELAY CHANNEL (IRC)

In this section, we prove a separation theorem for the PI-IRC introduced in Section II-H under some non-trivial constraints on the channel gains which can be considered as a strong interference situation for the IRC. The definitions of reliable communication and joint source-channel codes for the PI-IRC are similar to those for the PI-IC. We first state the separation theorem and consequently give the proofs of the converse and achievability parts.

Theorem 6: Reliable communication over a PI-IRC: Consider a PI-IRC with power constraints \(P_1, P_2, P_r\) on transmitters, fading amplitudes \(g_{11}, g_{21}, g_{12}, g_{22} \geq 0\) between the transmitters and the receivers, \(g_{r1}, g_{r2} \geq 0\) on receivers.
0 between the relay and the receivers, and \( g_{1r}, g_{2r} > 0 \) between the transmitters and the relay. Assume also that the network operates under the gain conditions

\[
\frac{g_{11}}{g_{12}} = \frac{g_{r1}}{g_{r2}} = \alpha < 1, \quad (69)
\]

\[
g_{11}^2 P_1 + g_{r1}^2 P_r \leq g_{1r}^2 P_1, \quad (70)
\]

\[
g_{22}^2 P_2 + g_{r2}^2 P_r \leq g_{2r}^2 P_2, \quad (71)
\]

\[
(1 - \alpha^2) \frac{g_{12}^2 P_1}{P_2} + \frac{(1 - \alpha^2) g_{r2}^2 P_r}{P_2} + g_{22}^2 \leq g_{21}^2. \quad (73)
\]

Then, a necessary condition for reliably sending a source pair \((U, V) \sim \prod_i p(u_i, v_i)\), over such PI-IRC is given by

\[
H(U|V) \leq \log (1 + (g_{11}^2 P_1 + g_{r1}^2 P_r)/N), \quad (74)
\]

\[
H(V|U) \leq \log (1 + (g_{22}^2 P_2 + g_{r2}^2 P_r)/N), \quad (75)
\]

\[
H(U, V) \leq \log (1 + (g_{12}^2 P_1 + g_{22}^2 P_2 + g_{r2}^2 P_r)/N). \quad (76)
\]

Moreover, a sufficient condition for reliable communication is also given by (74)-(76), with \( \leq \) replaced by \(<\), when \( \theta \) is known at the receivers.

The proof of Theorem 6 is discussed in the two following subsections. First, the converse is proved and afterwards, we prove the achievability part of Theorem 6.

A. Converse

**Lemma 4: PI-IRC Converse:** Let \( \mathcal{E}_n \) be a sequence in \( n \) of encoders, and \( g_{1\theta_i}^n, g_{2\theta_i}^n \) be sequences in \( n \) of decoders for the PI-IRC for which \( \sup_{\theta} P_{e1}^n(\theta), P_{e2}^n(\theta) \to 0 \), as \( n \to \infty \), then we have

\[
H(U|V) \leq \min_{\theta \in \Phi_c} I(X_1, X_r; g_{11} e^{j\theta_1} X_1 + g_{r1} e^{j\theta_r} X_r + Z), \quad (77)
\]

\[
H(V|U) \leq \min_{\theta \in \Phi_c} I(X_2, X_r; g_{22} e^{j\theta_2} X_2 + g_{r2} e^{j\theta_r} X_r + Z), \quad (78)
\]

\[
H(U, V) \leq \min_{\theta \in \Phi_c} I(X_1, X_2, X_r; g_{12} e^{j\theta_{12}} X_1 + g_{22} e^{j\theta_{22}} X_2 + g_{r2} e^{j\theta_r} X_r + Z), \quad (79)
\]

for some joint distribution \( p_{X_1, X_2, X_r} \) such that \( \mathbb{E}|X_1|^2 \leq P_1, \mathbb{E}|X_2|^2 \leq P_2, \mathbb{E}|X_r|^2 \leq P_r \), where \( \Phi_c \triangleq \{ \theta : \theta_{11} = \theta_{12}, \theta_{r1} = \theta_{r2} \} \).

**Proof:**

First, fix a PI-IRC with given parameter \( \theta \in \Phi_c \), a codebook \( C \), and induced empirical distribution \( p_{\theta}(u, v, x_1, x_2, x_r, y_1, y_2) \). Since for this fixed choice of \( \theta \), \( P_{e1}^n(\theta), P_{e2}^n(\theta) \to 0 \), from Fano’s inequality, we have

\[
\frac{1}{n} H(U|Y_1, \theta) \leq \frac{1}{n} P_{e1}^n(\theta) \log \|U^n\| + \frac{1}{n} \triangleq \epsilon_{1n}(\theta), \quad (79)
\]

\[
\frac{1}{n} H(V|Y_2, \theta) \leq \frac{1}{n} P_{e2}^n(\theta) \log \|V^n\| + \frac{1}{n} \triangleq \epsilon_{2n}(\theta), \quad (80)
\]
and $\epsilon_{1n}(\theta), \epsilon_{2n}(\theta) \to 0$, where convergence is uniform in $\theta$. Defining $\sup_{\theta} \epsilon_{in}(\theta) = \epsilon_{in}, i = 1, 2$ and following similar steps as those resulting in (25), we have

$$H(U|V) \leq \frac{1}{n} I(X_1, X_r; Y_1|V, X_2, \theta) + \epsilon_{1n}, \quad (80)$$
$$H(V|U) \leq \frac{1}{n} I(X_2, X_r; Y_2|U, X_1, \theta) + \epsilon_{2n}. \quad (81)$$

As in Section IV-A we can upper bound (80), (81) and derive (77) and (78). Next, to derive (79), we define a random vector $\tilde{Z}_1 \sim CN(0, (1-\alpha)NI)$ with $I$ the $n \times n$ identity matrix, and bound $H(U, V)$ as follows:

$$H(U, V) = \frac{1}{n} H(U, V)$$
$$= \frac{1}{n} H(V) + \frac{1}{n} H(U|V)$$
$$= \frac{1}{n} H(V) + \frac{1}{n} H(U|V, X_2)$$
$$= \frac{1}{n} I(V; Y_2|\theta) + \frac{1}{n} I(U; Y_1|V, X_2, \theta) + \frac{1}{n} H(V|Y_2, \theta) + \frac{1}{n} H(U|V, X_2, Y_1, \theta)$$
$$\leq \frac{1}{n} I(X_2; Y_2|\theta) + \frac{1}{n} I(X_1; Y_1|V, X_2, \theta) + \epsilon_{1n} + \epsilon_{2n}$$
$$\leq \frac{1}{n} I(X_2; Y_2|\theta) + \frac{1}{n} I(X_1, X_r; Y_1|V, X_2, \theta) + \epsilon_{1n} + \epsilon_{2n}$$
$$\leq \frac{1}{n} I(X_2; Y_2|\theta) + \frac{1}{n} [h(Y_1|X_2, \theta) - h(Z_1)] + \epsilon_{1n} + \epsilon_{2n}$$
$$= \frac{1}{n} I(X_2; Y_2|\theta) + \frac{1}{n} I(X_1, X_r; Y_1|X_2, \theta) + \epsilon_{1n} + \epsilon_{2n}$$
$$\leq \frac{1}{n} I(X_2; Y_2|\theta) + \frac{1}{n} I(X_1, X_r; g_{11} e^{j\theta_1} X_1 + g_{1r} e^{j\theta_r} X_r + Z_1|X_2) + \epsilon_{1n} + \epsilon_{2n} \quad (82)$$

(a) $\frac{1}{n} I(X_2; Y_2) + \frac{1}{n} I(X_1, X_r; g_{11} e^{j\theta_1} X_1 + g_{1r} e^{j\theta_r} X_r + \alpha Z_1 + \tilde{Z}_1|X_2) + \epsilon_{1n} + \epsilon_{2n}$

(b) $\frac{1}{n} I(X_2; Y_2) + \frac{1}{n} I(X_1, X_r; g_{11} e^{j\theta_1} X_1 + g_{1r} e^{j\theta_r} X_r + \alpha Z_2 + \tilde{Z}_1|X_2) + \epsilon_{1n} + \epsilon_{2n} \quad (83)$

(c) $\frac{1}{n} I(X_2; Y_2) + \frac{1}{n} I(X_1, X_r; \alpha g_{12} e^{j\theta_2} X_1 + \alpha g_{2r} e^{j\theta_r} X_r + \alpha Z_2 + \tilde{Z}_1|X_2) + \epsilon_{1n} + \epsilon_{2n}$

(d) $\frac{1}{n} I(X_2; Y_2) + \frac{1}{n} I(X_1, X_r; \alpha g_{12} e^{j\theta_1} X_1 + \alpha g_{2r} e^{j\theta_r} X_r + \alpha Z_2|X_2) + \epsilon_{1n} + \epsilon_{2n}$

$$= \frac{1}{n} I(X_2; Y_2) + \frac{1}{n} I(X_1, X_r; \alpha Y_2|X_2) + \epsilon_{1n} + \epsilon_{2n}$$

(e) $\frac{1}{n} I(X_2; Y_2) + \frac{1}{n} I(X_1, X_r; Y_2|X_2) + \epsilon_{1n} + \epsilon_{2n}$

$$= \frac{1}{n} I(X_1, X_2, X_r; Y_2) + \epsilon_{1n} + \epsilon_{2n}, \quad (84)$$

where (a), (b) follows from the fact that by preserving the noise marginal distribution, the mutual information does not change. The noise term $Z_1$ in (82) is thus divided into two independent terms $\alpha Z_1 + \tilde{Z}_1$, and then $Z_1$ is replaced by $Z_2$ to obtain (83). Also, (c) follows from (69) and the fact that in $\Phi_r, \theta_{11} = \theta_{12}$ and $\theta_{r1} = \theta_{r2}$. (d) follows since reducing the noise may only increase the mutual information, and (e) follows from the fact that linear transformation does not change mutual information: $I(X; Y) = I(X; \alpha Y)$. 
for some input distribution $p$. By letting

where $\theta \in \Phi_c$, we derive the following necessary conditions to find reliable channel codes for a compound IRC with decoding at the receivers (note: both receivers decode all messages) and forward decoding at the relay, to satisfy (44)-(46).

**Source Coding:** Using Slepian-Wolf coding, the source $(U, V)$ is source coded, requiring the rates $(R_1, R_2)$ to satisfy (44) with the power constraints similar to (31). By letting $n \to \infty$, the proof of the lemma is complete.

Using the key lemma, we maximize the upper bounds of Lemma 3 with the independent Gaussians and the proof of the converse part is complete.

**B. Achievability**

The achievability part is again proved by separate source-channel coding:

**Source Coding:** Using Slepian-Wolf coding, the source $(U, V)$ is source coded, requiring the rates $(R_1, R_2)$ to satisfy (44) with the power constraints similar to (31). By letting $n \to \infty$, the proof of the lemma is complete.

**Channel Coding:** Using the block Markov coding shown in Table III in conjunction with backward decoding at the receivers (note: both receivers decode all messages) and forward decoding at the relay, we derive the following necessary conditions to find reliable channel codes for a compound IRC with 2 transmitters and a relay $r$:

$$R_1 < \min \{I(X_1; Y_r | X_2, X_r, \theta), I(X_1, X_r | Y_1, X_2, \theta), I(X_1, X_r | Y_2, X_2, \theta)\}, \quad (85)$$

$$R_2 < \min \{I(X_2; Y_r | X_1, X_r, \theta), I(X_2, X_r | Y_1, X_1, \theta), I(X_2, X_r | Y_2, X_1, \theta)\}, \quad (86)$$

$$R_1 + R_2 < \min \{I(X_1, X_2; Y_r | X_2, \theta), I(X_1, X_2, X_r; Y_1 | \theta), I(X_1, X_2, X_r; Y_2 | \theta)\}, \quad (87)$$

for some input distribution $p(x_1)p(x_2)p(x_r)$.

<table>
<thead>
<tr>
<th>Encoder</th>
<th>Block 1</th>
<th>Block 2</th>
<th>Block B</th>
<th>Block B + 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x_1(1, W_{11})$</td>
<td>$x_1(W_{11}, W_{12})$</td>
<td>$x_1(W_{B-1}, W_{1B})$</td>
<td>$x_1(W_{1B}, 1)$</td>
</tr>
<tr>
<td>2</td>
<td>$x_2(1, W_{21})$</td>
<td>$x_2(W_{21}, W_{22})$</td>
<td>$x_2(W_{B-1}, W_{2B})$</td>
<td>$x_2(W_{2B}, 1)$</td>
</tr>
<tr>
<td>$r$</td>
<td>$x_r(1, 1)$</td>
<td>$x_r(W_{11}, W_{21})$</td>
<td>$x_r(W_{B-1}, W_{2B})$</td>
<td>$x_r(W_{1B}, W_{2B})$</td>
</tr>
</tbody>
</table>

**TABLE III**

**Block Markov encoding scheme for IRC.**
Computing the mutual informations in (85)-(87) for independent Gaussians $X_1 \sim \mathcal{CN}(0, P_1)$, $X_2 \sim \mathcal{CN}(0, P_2)$, $X_r \sim \mathcal{CN}(0, P_r)$, we find by (70) and (73) that

$$I(X_1; Y_r | X_2, X_r, \theta) \geq I(X_1, X_r; Y_1 | X_2, \theta),$$
$$I(X_1, X_r; Y_1 | X_2, \theta) \geq I(X_1, X_r; Y_1 | X_2, \theta),$$

respectively, and by (71) and (73) that

$$I(X_2; Y_r | X_1, X_r, \theta) \geq I(X_2, X_r; Y_2 | X_1, \theta),$$
$$I(X_2, X_r; Y_1 | X_1, \theta) \geq I(X_2, X_r; Y_2 | X_1, \theta),$$

respectively. Also, the conditions (70)-(72) together result in

$$I(X_1, X_2; Y_r | X_r, \theta) \geq I(X_1, X_2, X_r; Y_2 | \theta),$$

while the condition (73) makes

$$I(X_1, X_2, X_r; Y_1 | \theta) \geq I(X_1, X_2, X_r; Y_2 | \theta).$$

Hence, due to (70)-(73), the larger terms will drop off from the constraints (85)-(87) and we may rewrite the sufficient conditions as

$$R_1 \leq \log(1 + (g_{11}^2 P_1 + g_{r1}^2 P_r)/N),$$
$$R_2 \leq \log(1 + (g_{22}^2 P_2 + g_{r2}^2 P_r)/N),$$
$$R_1 + R_2 \leq \log(1 + (g_{12}^2 P_1 + g_{22}^2 P_2 + g_{r2}^2 P_r)/N).$$

Thus, combining the source coding and channel coding, the achievable region is the same as the outer bound and the proof of Theorem 6 is complete.

VIII. CONCLUSION

The problem of sending arbitrarily correlated sources over a class of phase asynchronous multiple-user channels with non-ergodic phase fadings is considered. Necessary and sufficient conditions for reliable communication are presented and several source-channel separation theorems are proved by observing the coincidence of both sets of conditions. Namely, outer bounds on the source entropy content ($H(U|V), H(V|U), H(U, V)$) are first derived using phase uncertainty at the encoders, and then are shown to match the achievable regions required by separate source-channel coding under some restrictions on the channel gains. Although, our results are for fixed $\theta$, they are also true for the ergodic case:

**Remark 3:** In all of the above theorems, we assumed that the vector $\theta$ is fixed over the block length. It can be shown that the theorems also hold for the ergodic phase fading, i.e., the phase shifts change from symbol to symbol in an i.i.d. manner, forming a matrix of phase shifts $\Theta$. The achievability parts of the theorems remain unchanged by the assumption of perfect CSI at the receiver(s), while in the proofs of the converses, the expectation operation over $\Theta$ is used instead of taking the minimum (as in [3]). One can
then use the results of Remark 4 as a key lemma to prove the optimality of independent Gaussians for the converse parts.

As a result, joint source-channel coding is not necessary under phase incoherence for the networks studied in this work. We also conjecture that source-channel separation is in fact optimal for all channel coefficients and not only for the constraints presented in this paper.

REFERENCES


