Application of the Huygens Absorbing Boundary Condition to Wave-Structure Interaction Problems

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Introduction

- The Huygens ABC was introduced in three papers in 2003-2007.

- Can be viewed as just another implementation of analytical ABCs. (Mur, Higdon). However, provides us with more possibilities.

- In this presentation the HABC is combined with a real stretch of the mesh to realize an effective ABC for wave-structure interaction problems. This combination is:
  - more effective than any previous analytical ABC. It can challenge the PML ABC in terms of computational cost and accuracy, at least from 2D tests.
  - simpler to implement than the PML ABC.
The Principle of the Huygens ABC

Introduced independently in special cases: **Multiple absorbing surfaces** (Sudiarta, 2003) and **Teleportation** (Diaz and Scherbatko, 2004). Later generalized as **Huygens ABC** (Berenger, 2007).

The principle is simple: radiating a wave opposite to the outgoing field.
The Huygens ABC: need of an operator

A problem to implement this simple idea: the outgoing field is not known where the Huygens surface must be enforced (if it were known this would be a perfect ABC!).

An operator (Higdon, others) is needed to evaluate the field to be radiated by the Huygens surface.

Theory shows that the overall reflection is the same as the one of the operator. => The new ABC is nothing but another implementation of operator ABC’s.

Advantage of the new implementation: Easy combination with other ABC’s (PML or Analytical ABC).
Wave-Structure Interaction problems

With Operator ABC the object-ABC separation must be large, typically ~ object size

Object = 300-cell thin plate

The evanescent waves are not absorbed by Higdon operator (nor by Mur ABC)

The incident wave is a Unit-Step wave
Wave-Structure Interaction problems

With a HABC (Higdon) and a PEC behind it (separation = 2 FDTD cells)

As expected, the results are like with the Higdon Operator ABC

⇒ Used “as is”, the HABC is just another implementation of Higdon ABC, with same drawback (no absorption of evanescent waves).
Combination of Huygens ABC with a stretched mesh

The proposed idea:

- A Huygens ABC to absorb traveling waves (highest frequencies).
- Outside the HABC only low frequency evanescent waves are present. For a scatterer of size $W$ they decrease in function of distance as:

$$E(d) = E_0 \ e^{-\frac{d}{W}}$$

$\Rightarrow$ A very coarse mesh region (real stretch of coordinates) can be used to “absorb” evanescent waves (i.e. to permit natural decrease). We can hope the needed coarse mesh be < 10-20 cells in thickness.
To render the HABC equivalent, rigorously, to an operator ABC, extensions of HABC lines (surfaces in 3D) must be added in corner regions.

- The results presented in the following have been computed with the extensions.
- The need of extensions was not discovered in 2003-2007 papers. It is the subject of a submitted paper (F. Costen, JP Berenger, Journ. Comp. Phys.)
Can the HABC be placed close to the object?

The FDTD cell is uniform in the FDTD domain

The HABC does absorb very well the traveling waves even if it is quite close to the object (here, 3-cell separation with a 300-cell object).

The evanescent waves are absorbed (natural decrease) in the large surrounding mesh.

=> YES
Use of a stretched mesh outside the HABC

- HSG 3 cells from object
- 4 cells of constant size $\Delta$
- $ng$ cells that grow geometrically from $\Delta$ to $\Delta_{\text{max}}$

Characteristic length of decrease of surrounding evanescent waves = size $w$ of object

$\Rightarrow$ Maximum step $\Delta_{\text{max}}$ probably of the order of size $w$. 
An experiment with a stretched mesh

In this experiment the maximum step $\Delta_{\text{max}}$ is constant ($= W / 5$) and the transition region varies from $ng = 0$ to $ng = 8$ cells.

$\Rightarrow 26$ stretched cells can replace $900$ non-stretched cells
An experiment with a stretched mesh

In this experiment the transition region is constant \((ng = 8\text{ cells})\) and the maximum step \(\Delta_{\text{max}}\) varies from \(w / 10\) to \(W / 1\).

\[\text{dPEC} < 40\text{ cells} \leftrightarrow 900\text{ non-stretched cells}\]

\[\text{dHABC} = 3\text{ cells} \quad ng = 8\text{ cells}\]

\[\Rightarrow 17\text{ stretched cells can replace 900 non-stretched cells}\]
Comparison with the CFS-PML

The Physics behind Optimized CFS-PML and HABC+stretched mesh is the same. Both methods rely on separation of evanescent waves with traveling waves: evanescent waves at low frequency, traveling waves at high frequency.

\[ f_0 = \frac{2\omega}{c} \]
\[ \lambda = 2w \]

-> From this we can expect similar performance in terms of computational cost

-> HABC is more general: CFS-PML requires \( \omega \sinh \chi = \text{constant} \)
Comparison with the PML

- 15 cells thick normal PML
- 5 cells thick CFS-PML
- HABC + Stretched grid

Domain sizes:

With PML: [Diagram]

With HABC: [Diagram]

- HABC: big FDTD domain, but filled with big FDTD cells.
- HABC is intermediate between normal PML and Optimized CFS-PML in terms of CPU time (one cell of PML is 1.5-2 times more costly than one cell of vacuum).
- HABC is simpler to implement.
- Further works to improve HABC (higher order operator, optimization of the mesh stretch).
Conclusion

- Combination of a HABC with a strongly stretched mesh realizes an effective ABC for the solution of wave-structure interaction problems.
- This ABC can challenge the PML ABC in terms of computational cost reduction. Only the optimized CFS-PML remains slightly better, but HABC has not yet been optimized.
- HABC implementation is simpler than PML implementation.
- The same principle could be used with other problems where evanescent waves are only present at low frequency (waveguides).
- Extension to 3D is currently under investigation.
- References -

The Huygens ABC