

# Measuring Organization and Asymmetry in Bihemispheric Topographic Maps

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## Abstract

We address the problem of measuring the degree of hemispheric organization and asymmetry of organization in a computational model of a bihemispheric cerebral cortex. A theoretical framework for such measures is developed and used to produce algorithms for measuring the degree of organization, symmetry, and lateralization in topographic map formation. The performance of the resulting measures is tested for several topographic maps obtained by self-organization of an initially random network, and the results are compared with subjective assessments made by humans. It is found that the closest agreement with the human assessments is obtained by using organization measures based on sigmoid-type error averaging. Measures are developed which correct for large constant displacements as well as curving of the hemispheric topographic maps.

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# 1 Introduction

Qualitative factors leading to topographic and computational map formation in neural networks have been studied previously [5]. Work has also been done concerning the quantitative measurement of the degree of topographic map formation [2]. We are interested in the special issues that arise in measuring hemispheric organization and lateral asymmetry of organization in computational models of topographic map development in a bihemispheric system. These issues are of concern in any study on cerebral lateralization. While there has been extensive work regarding lateralization in humans [3], the computational modelling of the development and measurement of lateralization has to our knowledge not yet been undertaken in any systematic fashion.

In this report we develop a theoretical framework for the construction of such performance measures. We show that heuristic suggestions made by others (e.g., [4]) for the measurement of lateral asymmetry arise as special cases of our general framework, thus providing these past measures with a rigorous theoretical foundation. This introductory section contains some basic background information concerning our bihemispheric computational cortex models as well as the notion of topographic map used in our studies. Subsequent sections describe the development of our theoretical framework, culminating in the definition of our measures, and examples of the performance of the measures as judged by human subjects provided with pictures of bihemispheric topographic maps. We conclude the paper with a discussion of the results and a description of work in progress.

## 1.1 Bihemispheric Cortex Model

We have interconnected two computational models of sensory cortex via a simulated *corpus callosum*, and provided sensory connections from a simulated *sensory surface* as shown in Figure 1. Each hemisphere of this system models a small patch of sensory cortex. The model cortices are two-dimensional, with individual elements of the model representing cortical columns in the actual sensory cortex. These elements tessellate the cortex in a regular hexagonal fashion, with each element connected to its nearest neighbors. Connections in the model represent multiple synaptic interactions in the actual cortex. Connections to the nearest neighbors are excitatory, while potentially inhibitory connections are allowed to the next-nearest elements. The intrahemispheric interconnection pattern is hexagonal, as shown in Figure 2.

Connections are provided between the hemispheres. Each element connects to those lying within a certain radius of the element homotopic to it in the opposite hemisphere. This radius of divergence may be varied, leading to different behaviors during self-organization.

The sensory surface is two-dimensional as well. Each sensory element connects symmetrically to the two elements homotopic to it, one in each of the two cortical hemispheres. Sensory elements are indexed by a pair of integers denoting the elements' spatial location within the regular two-dimensional lattice modelling the sensory surface. Cortical elements inherit the index of their homotopically located sensory elements.

To avoid edge effects, the top and bottom edges of the sensory surface and of each hemisphere are identified, as are the left and right edges. Thus, the sensory surface and the

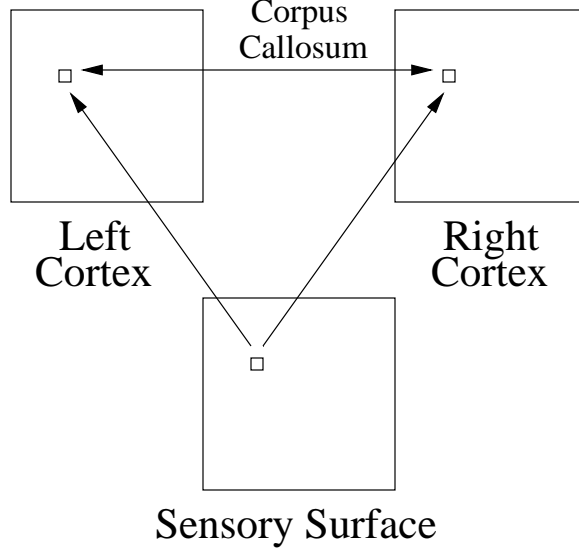


Figure 1: *The bihemispheric cortex model*

hemispheres are actually two-dimensional tori rather than planar rectangles or parallelograms.

A real-valued activation value  $a_i(t)$  is associated with the  $i$ -th element in continuous time  $t$ . The activation dynamics are governed by:

$$\frac{da_i}{dt} = in_i^+(M - a_i) + c_s a_i - in_i^- \quad (1)$$

$$in_i^+ = \sum c_{ij} a_j + e_i \quad (2)$$

$$in_i^- = \sum c_{ij} a_j \quad (3)$$

The sum for  $in_i^+$  ranges over elements in the same cortical hemisphere as the  $i$ -th element, as well as over those in the sensory surface. The sum for  $in_i^-$  ranges over elements in the opposite cortical hemisphere, representing inhibitory signals acting through the corpus callosum. In both cases, only elements lying within a certain radius of the (element homotopic to) the  $i$ -th element yield non-zero contributions to the corresponding sum.

It is known that lateral inhibition is necessary to allow development of structured topographic maps [5]. In our model, all cortical afferent and intrahemispheric connections are excitatory. Virtual inhibition is achieved by having each sensory element competitively distribute its output among the receiving cortical elements in proportion to their activation levels:

$$c_{ki} = c_p \frac{w_{ki}(a_k^v + q)}{\sum_j w_{ji}(a_j^v + q)} \quad (4)$$

Cortical elements competitively distribute their activation among their neighbors also. Such competitive activity distribution has been shown in the past to produce peristimulus inhibition similar to that produced by actual inhibitory connections, and topographic map formation in systems with competitive activity distribution has been demonstrated.

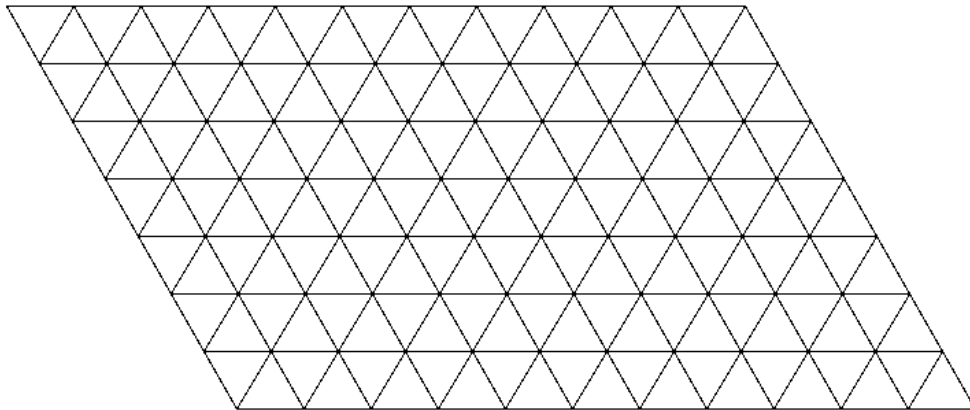


Figure 2: *Intrahemispheric connection pattern. Each point where lines intersect corresponds to a cortical element, and the six line segments radiating from it indicate its immediate neighbor elements.*

Sensory stimuli are applied to drive the self-organization of the bihemispheric cortex model. Self-organization proceeds according to the rule:

$$\Delta w_{ji} = \epsilon(a_i - w_{ji})a_j \quad (5)$$

By stimulating sensory elements at random according to a uniform probability distribution, the average response of each cortical element to stimuli from different sensory elements can be established. This allows one to define a *receptive field* for each cortical element, consisting of those sensory elements to which the given cortical element responds significantly. Each receptive field is defined by the horizontal ( $x$ ) and vertical ( $y$ ) offsets of its center relative to the expected location at the homotopically positioned sensory element, and its  $x$  and  $y$  radii. Dual to the notion of receptive field, we have for each sensory element an associated *response field*, consisting of those cortical elements which respond significantly to the given sensory element. Further details about the single hemisphere version of the model, including the activation dynamics, competitive activity distribution, and the rule used for self-organization, are available in [7] and [8].

## 1.2 Topographic Maps

The two-dimensional topology of the model sensory surface and hemispheres allows one to consider the topographic maps defined by association of cortical elements with the corresponding receptive field centers in the sensory surface. Figure 3 provides an example. One may also consider the “inverse maps” obtained by associating sensory elements with their

response field centers. For simplicity, the discussion below deals only with the first type of map, although clearly the statements may be applied to the inverse maps as well, essentially by interchanging the word “sensory” by the word “cortical”, and the word “receptive” by the word “response”.

We are interested here mainly in providing a quantitative measurement of the quality of the resulting maps and of hemispheric asymmetry. A very simple working definition of a topographic map suffices for our present needs. By a topographic map we mean a representation of the organization, or lack thereof, in our computational model of a single cortical hemisphere. We choose a rectangular matrix  $M_{i,j}$  whose entries consist of quadruplets  $(cx_{i,j}, cy_{i,j}, rx_{i,j}, ry_{i,j})$  indexed by the location coordinates  $i, j$  of the associated cortical elements, and indicating, respectively, the  $x$  and  $y$  offsets of the receptive field center of cortical node  $(i, j)$ , and the  $x$  and  $y$  radii of the receptive field. This is the information available to compute the desired measures of organization.

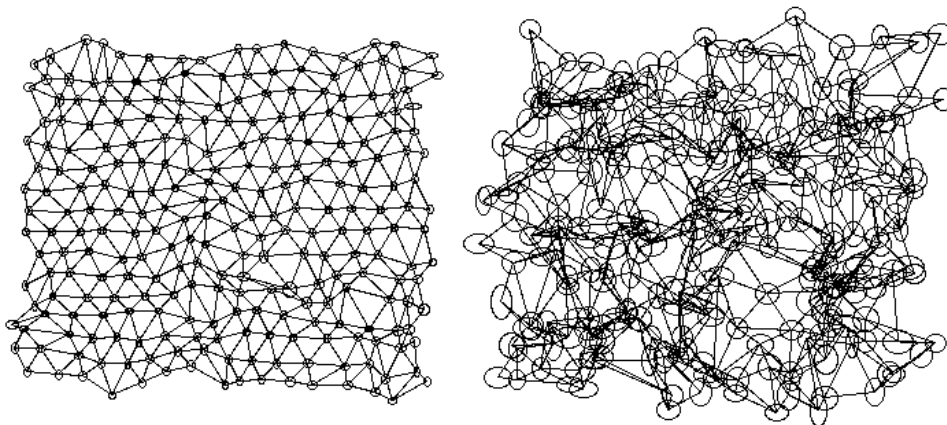


Figure 3: *Visual representation of topographic map formation in the bihemispheric computational model. The ellipses represent the receptive fields. The left hemisphere has developed a rather organized topographic map, while the right hemisphere remains fairly random, with large receptive fields and irregularly located receptive field centers.*

### 1.3 Measures to be computed

We now describe the measures that we wish to compute. The theoretical development leading to the actual definitions of the measures is undertaken in the next section.

For each map  $M$ :

- a family of entropy measures,  $||M||$ , defined as distances, in the abstract space of topographic maps, between the given topographic map  $M$  and an “ideal” or reference topographic map, the latter typically meaning a completely uniform topographic map.

Each entropy measure is a non-negative number, with the value 0 indicating complete agreement with the chosen reference map (“organization”) and larger values indicating an increasingly large deviation from the reference map (“disorganization”).

- an organization measure ranging between 0 and 1, with 0 representing no topographic map formation and 1 representing a very highly organized topographic map. The organization measure is obtained from the entropy by a simple transformation.
- the mean and variance of each of the attributes  $cx, cy, rx, ry, tr$ .

To compare hemispheres  $L$  and  $R$ :

- a family of lateralization metrics, which indicate the difference in the total level of organization (map formation) between the two hemispheres; their values range between  $-1$  for total left hemisphere dominance and  $+1$  for total right hemisphere dominance.
- an interhemispheric correlation measure, which assesses the degree of mirror symmetry of the topographic maps for the two hemispheres after normalizing them to have the same total level of organization; its values range between  $-1$  for total antisymmetry between the two hemispheres and  $+1$  for total mirror symmetry between the two hemispheres.
- the standard statistical correlation between like attributes  $cx, cy, rx, ry, tr$  in the two hemispheres.
- the difference in means between hemispheres for each of  $cx, cy, rx, ry, tr$ .

## 2 Theoretical framework

In this section we present a theoretical framework for the development of organization, lateralization, and correlation measures. There has been considerable research in morphometrics addressing the comparison of shapes by techniques such as Procrustean analysis [2], and it is natural to approach the problem of comparing the organization of the two hemispheres in this way, by selecting certain “landmarks” which define corresponding map locations in the hemispheres. This approach is promising, and we will pursue it below with one small caveat. A noteworthy difference between standard Procrustean analysis and our current problem of hemispheric comparison is that, for the purpose of measuring organization and lateralization, we do not wish to allow the figures being compared to be scaled before being matched, as this operation affects the overall organization levels. On the other hand, the measurement of interhemispheric correlation in the case in which the two hemispheres are of different sizes may require a preliminary step in which the hemispheres are scaled to a common size.

One is tempted to select the landmarks in such a way that the associated geometrical regions coincide approximately with the “organized regions” of the hemispheres. Indeed, our experiment with human test subjects suggests that humans assess organization level and hemispheric asymmetry of organization by first identifying these organized regions (see the

Experimental Comparison section below). This is reasonable in measuring the interhemispheric correlation. On the other hand, in measuring the net degree of lateralization the geometric similarities between figures defined by the organized regions are of little direct concern. It is easy to find pairs of topographic maps having approximately the same level of individual organization, and thus producing a bihemispheric map with no lateralization, which nonetheless are considerably different in appearance when viewed purely as geometric entities associated with the locations of the receptive field centers in the organized regions. For example, the shapes of the organized regions in the two maps might be radically different geometrically even though these regions occupy the same fraction of the total area in both cases. Thus, for the purpose of measuring total lateralization with Procrustean methods it is best to avoid restricting attention to the organized regions. As landmarks one could then use *all* receptive field centers in the hemispheres. The mean square error of the best fit between the landmark collections of the two hemispheres may then be used as a measure of total lateralization. This is one of the approaches described below.

Another way to measure lateral asymmetry is to first measure the performance of each hemisphere separately, then measure the performance of the bihemispheric system, and finally combine these measurements to obtain a measure of the relative difference in the level of performance between hemispheres. The following version of this idea has been used before [4]: abbreviating the left and right hemispheric performances as  $\rho_L$ ,  $\rho_R$ , and letting  $\rho_{L,R}$  measure the performance of the combined bihemispheric system, one can consider the number

$$\frac{\rho_R - \rho_L}{\rho_{L,R}} \quad (6)$$

to be a measure of lateralization. Negative values of this measure indicate dominance of the left hemisphere, and positive values indicate right dominance, assuming the value of  $\rho_{L,R}$  is non-negative.

This latter particular form has heuristic merit but it needs a solid theoretical foundation. Below we propose three properties that we believe should be satisfied by any good measures of lateralization. We then proceed to show that a certain general form for the lateralization metrics can be derived from these abstract properties. It turns out that the heuristic form of the metric mentioned above, with  $\rho_{L,R} = \rho_L + \rho_R$ , may be recovered from the general form by an appropriate choice of the functional parameter.

Since at the present stage we are interested in measuring asymmetries in topographic map formation only rather than actual asymmetries of function, we will assume that the desired lateralization metric depends only on performance measures  $\rho_L$ ,  $\rho_R$  for the individual hemispheres, and not on any correlational information involving both hemispheres simultaneously such as might potentially be incorporated in the measure  $\rho_{L,R}$ . A measure of bihemispheric correlation is, however, computed by our measurement program and is briefly described in the section following the current one.

We begin by proposing properties for the monohemispheric performance measures. Our measures actually judge the degree of hemispheric *disorganization*. By analogy with thermodynamics we therefore refer to them as *entropies*. These measures may be interpreted as Procrustean measures if we consider all receptive field centers in each hemisphere to be



landmarks. Motivated in part by the desire to allow meaningful comparisons to be made between hemispheres with unequal numbers of elements, we normalize the entropies with respect to the total number of processing elements in the measured hemisphere. A measure of *organization* on a scale from 0 to 1 is then computed from the entropy according to a formula described below.

## 2.1 Single hemisphere metrics

For theoretical reasons, we split the development of the monohemispheric measure into two stages. In the first stage we produce a disorganization measure, or *entropy*, which indicates the degree of disorganization *per processing element* of the given topographic map relative to a certain predefined reference organization pattern. For map  $M$ , we designate the value of this disorganization measure as  $\|M\|$ . The disorganization value 0 indicates a completely organized topographic map, while larger values indicate increasing values of disorganization. One reason for normalizing the entropy with respect to the total number of elements is to allow meaningful comparisons to be made between hemispheres with unequal numbers of elements.

The second stage in the construction of the monohemispheric measure consists simply of rescaling the entropy measure to yield an *organization* measure  $\text{org}(M)$  taking values between 0 and 1, with the organization value 0 indicating a totally disorganized map and the value 1 indicating a completely organized map.

In general, one may consider as the organization measure

$$\text{org}(M) = f(\|M\|) \tag{7}$$

where  $f$  is a decreasing function transforming the range of the given disorganization measure  $\| \cdot \|$  into the interval  $[0, 1]$ .

The information used in computing the disorganization measure is that which is coded in the measured hemisphere’s topographic map. Recall that our working representation of the organization (or lack thereof) in our computational model of a single cerebral hemisphere’s map is a 2-D matrix  $M$ . Each matrix entry  $M_{i,j}$  consists of a quadruplet  $(cx_{i,j}, cy_{i,j}, rx_{i,j}, ry_{i,j})$  indicating respectively the  $x$  and  $y$  offsets of the receptive field center of cortical node  $(i, j)$ , and the  $x$  and  $y$  radii of the receptive field center. Geometrically, each topographic map corresponds to a torus obtained by identifying opposite edges of the rectangular array of processing elements. Notice that, in accordance with our definition, two topographic maps may be added by adding the corresponding matrices.

### 2.1.1 Unnormalized versions of the disorganization metrics.

The following are desirable properties for the monohemispheric disorganization metric  $\| \cdot \|$ . We use the term “reference location” to refer to the coordinates  $(i, j)$  of node  $M_{i,j}$ .

1. *size independence*: only the disorganization per processing element is measured, so that one controls for differing hemispheric sizes.

2. *spatial homogeneity*: If a map  $M$  is given and a new map  $M'$  is obtained from  $M$  by any distance-preserving geometric transformation of the torus associated with the reference locations of the elements of  $M$ , then  $M$  and  $M'$  have the same unit disorganization measure.
3. *metric property*: the disorganization measure behaves like a distance function, i.e. it satisfies the triangle inequality

$$\|M - M'\| \leq \|M - M''\| + \|M'' - M'\| \quad (8)$$

Next, three specific options are given for the unit disorganization measure.

**Basic root mean square measure.** We choose the unit disorganization metric  $\| \cdot \|$  to be an average over all the processing elements of a distance function  $|(cx, cy, rx, ry)|$  in the space  $\mathbb{R}^4$  of node entries  $(cx, cy, rx, ry)$ . We have chosen a standard weighted Euclidean, or root mean square, distance for  $| \cdot |$ :

$$|(cx, cy, rx, ry)| = \sqrt{a_{cx}cx^2 + a_{cy}cy^2 + a_{rx}rx^2 + a_{ry}ry^2} \quad (9)$$

The entropy measure becomes

$$\|M\|_{\text{RMS1}} = \sqrt{\frac{1}{N} \sum_{\text{all M-nodes}} a_{cx}cx^2 + a_{cy}cy^2 + a_{rx}rx^2 + a_{ry}ry^2} \quad (10)$$

**Shift-independent root mean square measure.** As an improvement, we consider an entropy measure based on the statistical variances of the node entries:

$$\|M\|_{\text{RMS}} = \sqrt{a_{cx}\sigma^2(cx) + a_{cy}\sigma^2(cy) + a_{rx}\sigma^2(rx) + a_{ry}\sigma^2(ry)} \quad (11)$$

This measure is obtained from the first version of the root mean square measure  $\| \cdot \|_{\text{RMS1}}$  defined above in Eq.( 10), by eliminating the dependence on mean shifts of the nodal attribute values. In the remainder of this paper, except where noted explicitly, we will use the term “root mean square measure” to refer to the shift-independent version  $\|M\|_{\text{RMS}}$  defined here, and not to the measure  $\|M\|_{\text{RMS1}}$  defined above.

**Differential root mean square measure.** To reduce the effect of slowly spatially varying map deformations, such as slight curving of the maps, the following measure is considered. Notice that mean shifts in the various attributes are also eliminated.

We define the *differential square distance* between neighboring nodes  $n, n'$  to be:

$$|(n, n')|_{\text{diffRMS}}^2 = a_{cx}(cx(n') - cx(n))^2 + a_{cy}(cy(n') - cy(n))^2 + a_{rx}(rx(n') - rx(n))^2 + a_{ry}(ry(n') - ry(n))^2 \quad (12)$$

and then we define the differential root mean square measure as follows. The *square* of the measure is given below for typographical convenience.

$$\|M\|_{diffRMS}^2 = \frac{1}{N} \sum_{\text{all M-nodes } n} \sum_{\text{all neighbors } n' \text{ of } n} |(n, n')|_{diffRMS}^2$$

Various parameter choices  $(a_{cx}, a_{cy}, a_{rx}, a_{ry})$  bring out different aspects of hemispheric activity with the measures defined above. For example, the choice  $(1, 1, 0, 0)$  considers only the location  $cx, cy$  of the receptive field centers, while  $(0, 0, 1, 1)$  causes the measurements to depend exclusively on the receptive field radii  $rx, ry$ . We have obtained good results with the parameter vector choice  $(1, 1, 1, 1)$ .

**Proposition 2.1.** *The disorganization measures  $\| \cdot \|_{RMS1}$ ,  $\| \cdot \|_{RMS}$ , and  $\| \cdot \|_{diffRMS}$  defined above satisfy the properties 1)–3).*

*Proof.* Size independence is enforced in the definitions of the measures. The first and third measures involve an explicit division by  $N$ , and the second is defined in terms of the variances, which are size independent as well. Spatial homogeneity follows from the fact that the measures involve only the values of the various attributes at individual elements of the measured topographic map. Any permutation of the elements leaves these measures unchanged. The metric property is a consequence of the Cauchy-Schwarz-Buniakowski inequality.  $\square$

**Standard statistical measures.** Finally, we compute means and standard deviations of the attributes  $cx, cy, rx, ry$  in a given topographic map  $M$ . These measures are standard; they are defined as follows, where  $X$  denotes any one of the attributes:

$$\mu_M(X) = \frac{1}{N} \sum_{\text{all nodes } n} (X(M, n)) \quad (13)$$

$$\sigma_M(X) = \sqrt{\frac{1}{N-1} \sum_{\text{all nodes } n} (X(M, n) - \mu_M(X))^2} \quad (14)$$

The above measures provide alternative information sources for organization assessments. Highly organized topographic maps are associated with small values for the variances of the various attributes.

### 2.1.2 Normalizing the disorganization metrics.

Having identified three potentially useful measures  $\| \cdot \|_{RMS1}$ ,  $\| \cdot \|_{RMS}$  and  $\| \cdot \|_{diffRMS}$ , we proceed to normalize them to a 0 to 1 range which is more convenient for the computation of the corresponding organization measures.

The transformation from unnormalized to normalized measures is performed by the sigmoid, or logistic, function defined by

$$\text{sigmoid}_{\tau,s}(x) = \frac{1}{2} (1 + \tanh(s(x - \tau))) = \frac{1}{1 + e^{-2s(x-\tau)}}, \quad (15)$$

Applying the sigmoid function to each unnormalized entropy measure  $|| \cdot ||$  defined in the preceding section, we obtain the corresponding normalized measure

$$||M||_{\text{sigmoid}} = \text{sigmoid}_{\tau,s} (||M||) \quad (16)$$

The sigmoid function involves two parameters  $\tau$  and  $s$ , the *threshold* and the *steepness*, respectively. Increasing the value of  $\tau$  increases the hemispheric disorganization level required to produce a given value of the disorganization measurement. Increasing  $s$  decreases the width of the transition region allowed by the measure between organization and disorganization. As  $s$  approaches  $\infty$ , the sigmoid function approaches a simple threshold detector, which returns the value 1 for inputs greater than the threshold  $\tau$ , and the value 0 for inputs less than this threshold. Finite values of  $s$  yield continuous approximations to this threshold detecting behavior.

### 2.1.3 Sigmoid activation measures.

Motivated by the desire to mimic humans' lateralization judgments more closely (see the Experimental Comparison section below), we have considered the following two additional ways to compute entropy functions in addition to the three described above. The additional measures are normalized from the outset, so no additional normalizing transformation is needed. The difference between the new measures and those described above is that in the new measures, the sigmoid function is applied locally, at each node, and the result is averaged over all nodes, while in the measures defined above the sigmoid is applied after the average has been computed.

#### Sigmoid RMS measure.

$$||M||_{\text{sigmoidRMS}} = \frac{1}{N} \sum_{\text{all M-nodes}} \text{sigmoid}_{\tau,s} (|(cx, cy, rx, ry)|) \quad (17)$$

Here,  $| \cdot |$  is the weighted Euclidean distance (Eq. 9) described above when considering the unnormalized versions of the entropy measures, and the sigmoid function is as in Eq. 15. <sup>2</sup>

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<sup>2</sup>One observes that the function  $\text{sigmoid}(x)$  is commonly used in neural modelling to compute nodal activation from total nodal input. Thus, the sigmoid-based entropy may be interpreted as the output produced by a neural network having the nodes of the measured topographic map as inputs, and one sigmoid processing element per input connected to a common linear output node via identical weights equal to  $1/N$ . This suggests the idea that a neural network might be constructed which can learn to compute an organization measure producing results similar to those given by the test subjects, using, e.g., error backpropagation. We have not yet attempted to implement this idea.

**Sigmoiddiff measure.** By combining the differential RMS measure Eq. 13 with the sigmoid as in Eq. 17, the following measure is obtained.

$$\|M\|_{\text{sigmoiddiff}} = \frac{1}{N} \sum_{\text{all M-nodes } n} \text{sigmoid}_{\tau,s} \left( \sum_{\text{all neighbors } n' \text{ of } n} |(n, n')|_{\text{diffRMS}}^2 \right)^{\frac{1}{2}}$$

#### 2.1.4 Organization from normalized disorganization.

Given any one of the measures  $\| \cdot \|_x$  of disorganization defined above, which has been scaled to take values between 0 and 1, we compute the associated organization measure  $\text{org}_x$  as:

$$\text{org}_x(M) = 1 - \|M\|_x \quad (18)$$

#### 2.1.5 Organized area measure.

Another approach to the measurement of organization is to measure the fraction of the sensory area that is covered by a well-formed topographic map. The above sigmoid-based organization measures may be viewed as producing only an approximation to this area. Such a measure can be obtained by computing, for each sensory node, the perimeter of the triangles formed by the arcs connecting the corresponding cortical node with its immediate neighbors in the topographic map. If this perimeter is below a certain threshold, and if the node's receptive field radii are less than other threshold values, and if the node's total response is above a third threshold, then the given node is considered to form part of the organized region; otherwise, it is considered to lie outside this region. The resulting organization measure is

$$\text{org}_{\text{area}}(M) = \sum_{\text{all } M\text{-triangles}} \mathbb{I}\{\text{perimeter} < \tau_p, \quad rx < \tau_{rx}, \quad ry < \tau_{ry}, \quad tr > \tau_{tr}\} a(\text{triangle}) \quad (19)$$

Here, the numbers  $\tau_p$ ,  $\tau_{rx}$ ,  $\tau_{ry}$ ,  $\tau_{tr}$  are the *thresholds* referred to above, which are used to decide, for each triangle, if it belongs to the organized region or not. The symbol  $a(\text{triangle})$  denotes the sensory area of the given triangle. The sum includes one term for each sensory triangle whose vertices are all immediate neighbors of each other. The function  $\mathbb{I}(q)$  equals 1 if the condition  $q$  holds and 0 if  $q$  fails. Thus, only triangles which simultaneously have small perimeter, small receptive field radii, and a significant total response effectively contribute to the sum.

**Proposition 2.2.** *The size independence and spatial homogeneity properties for monohemispheric measures hold for the sigmoid-based entropies and for the organized area measure as well. However, the metric property does not hold in the form stated.*

*Proof.* The named measures are explicitly defined as averages over all nodes of the measured topographic map. This guarantees size independence. Spatial homogeneity follows from the fact that the expression which is averaged is independent of the node's location, depending

only on the values of the attributes  $cx, cy, rx, ry$ . The failure of the metric property for the sigmoid and organized area measures is roughly due to their threshold-detecting behavior.  $\square$

One should note that while the motivation underlying the organized area measure is to measure the “organized area”, the choice of small perimeter as a criterion to identify the organized regions is rather ad hoc, based on observations of receptive field map behavior which suggest that map regularity is correlated with small perimeter. Indeed, the performance of this measure for receptive field maps is reasonably good, as shown in the Examples in the Appendix. However, in the case of the “inverse maps” corresponding to response fields instead of receptive fields, situations arise in which the most regular regions in a map have the triangles with the largest perimeter; see the first two Examples. In other examples, the measure yields better results. The organized area measure thus does not produce consistent results for response field maps.

## 2.2 Lateralization metrics

We now address the issue of extracting lateral asymmetry information from the organization values for the two hemispheres’ maps. We seek a function  $lat(org(L), org(R))$  measuring the degree of directional preference in the total degree of organization (map formation) of the bihemispheric system, with a value of  $-1$  indicating total left dominance, and  $+1$  indicating total right dominance. We assume the measurements  $org(L), org(R)$  are organization values indicating the average degree of organization per processing element of the respective left and right hemispheres as described in the preceding section. If these organization values have been obtained from disorganization metrics, or entropies,  $||L||, ||R||$ , then we will refer to the quantities  $\log ||L||, \log ||R||$  as “log entropies”; their use will simplify the analysis to follow in one of the subsections below.

### 2.2.1 Simple difference lateralization.

One can simply subtract the organization values for the two hemispheres to obtain a lateralization value between  $-1$  and  $+1$ . The resulting measure is given by:

$$lat(org(L), org(R)) = org(R) - org(L) \tag{20}$$

In fact, this simple idea has been used by other authors. However, after some experimentation and comparison with human lateralization assessments, it has become clear to us that the lateralization values so obtained fail to agree with the intuitively satisfying judgements made by the human subjects (see the Experimental Comparison section). This has motivated us to define improved lateralization metrics, which we will now describe.

### 2.2.2 Lateralization via steepening.

We can improve the simple difference lateralization measure by applying a *steepening* transformation  $f$ :

$$lat\_steep(org(L), org(R)) = f(org(R) - org(L)) \tag{21}$$

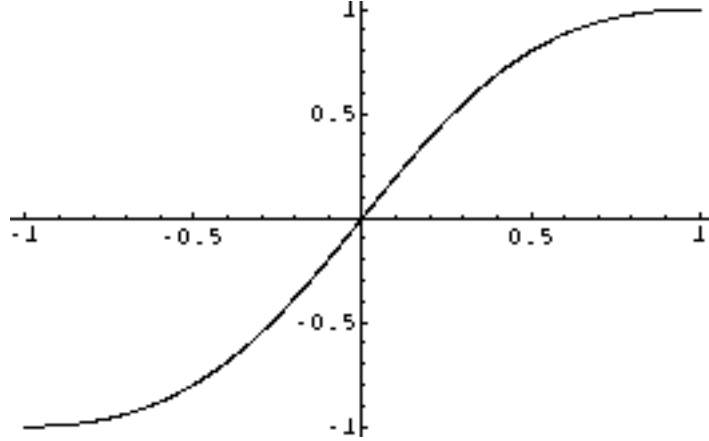


Figure 4: *Graph of the steepening function, Eq. 23*

where

$$f : [-1, 1] \rightarrow [-1, 1], \quad (22)$$

is defined by

$$f(x) = \frac{(1+x)^s - (1-x)^s}{(1+x)^s + (1-x)^s} \quad (23)$$

A graph of the function  $f$  for  $s = 2$  is shown here. In this way, we obtain the following steepened lateralization measure:

$$\text{lat\_steep}(\text{org}(L), \text{org}(R)) = \frac{(1 + \text{org}(R) - \text{org}(L))^s - (1 - (\text{org}(R) - \text{org}(L)))^s}{(1 + \text{org}(R) - \text{org}(L))^s + (1 - (\text{org}(R) - \text{org}(L)))^s} \quad (24)$$

### 2.2.3 Lateralization via an intermediate unbounding transformation.

In this subsection we obtain an alternative measure of lateralization by first transforming any given organization measures into unnormalized measures of disorganization, and then seeking a measure of lateralization which can be computed from these unnormalized disorganization metrics. Some of the entropies discussed above were derived from unnormalized versions, so the latter may be used in the formation of lateralization measures below. The unnormalizing procedure may seem circuitous, and in fact the lateralization measure obtained via this route involves only the original normalized organization measures, so the unnormalizing procedure is really just an intermediate technical stage which allows us to scale the resulting metrics by arbitrary numbers without leaving the allowed range.

**Lateralization in terms of unnormalized disorganization metrics.** We assume to begin that the unnormalizing procedure has been completed, and we seek a lateralization metric computable from unnormalized disorganization metrics. Thus, we assume that we

are given an unnormalized version  $\| \cdot \|$  of a disorganization measure, and we seek to produce a satisfactory lateralization metric  $lat(\|L\|, \|R\|)$ . We propose the following desirable properties for the lateralization metric  $lat(\|L\|, \|R\|)$ :

1. *scale invariance*: laterally symmetric scale changes do not affect the metric, i.e.

$$lat(ax, ay) = lat(x, y) \text{ for all scalars } a \quad (25)$$

2. *antisymmetry*: exchanging the hemispheres only changes the sign of the metric, i.e.

$$lat(y, x) = -lat(x, y) \quad (26)$$

3. *monotonicity*: measurably greater relative degrees of disorganization of the left hemisphere yield measurably greater values for the metric, i.e.

$$lat(x, y) \text{ is an increasing function of } x \text{ if } y \text{ remains fixed.} \quad (27)$$

4. *metric property*: the absolute value of the lateralization metric behaves like a distance function, i.e. it satisfies the triangle inequality

$$|lat(x, z)| \leq |lat(x, y)| + |lat(y, z)| \quad (28)$$

**Proposition 2.3.** *In order to satisfy properties 25– 28, it is necessary and sufficient that the lateralization metric  $lat$  be of the form  $lat(\|L\|, \|R\|) = h(\log(\frac{\|L\|}{\|R\|}))$ , where  $h$  is a strictly increasing odd positively subadditive function from the real numbers onto the interval  $(-1, 1)$ .*

*Proof.* Choosing  $a = \frac{1}{y}$  in property 1), we obtain  $lat(x, y) = lat(\frac{x}{y}, 1)$ . Thus,  $lat(x, y) = g(x/y)$  for some function  $g$ . We now write the candidate lateralization metric in terms of the log entropies  $\log \|L\|$ ,  $\log \|R\|$ :

$$lat(\|L\|, \|R\|) = g(\|L\|/\|R\|) = h\left(\log\left(\frac{\|L\|}{\|R\|}\right)\right) \quad (29)$$

This does not constitute a restriction on the metric  $lat$ , since any function  $g$  may be expressed as  $g(x) = g(\exp(\log x))$ , so that one may take  $h(y) = g(\exp(y))$ . Property 1) therefore says that our metric  $lat$  is a function ( $h$ ) of the difference in log entropies between the hemispheres. Property 2) translates to the following statement about  $h$ :

$$h(\log x - \log y) = -h(\log y - \log x) \quad (30)$$

This is equivalent to saying that  $h$  is an “odd” function, satisfying  $h(-u) = -h(u)$ . Property 3) simply says that  $g(x/y)$  is a strictly increasing function of  $x/y$ , or, equivalently,  $h$  is a strictly increasing function. Property 4) requires  $g$  to satisfy

$$|g(x/z)| \leq |g(x/y)| + |g(y/z)| \quad (31)$$



In terms of  $h$ , this requirement becomes

$$|h(\log x - \log z)| \leq |h(\log x - \log y)| + |h(\log y - \log z)| \quad (32)$$

By property 2) we can rewrite this as

$$h(|\log x - \log z|) \leq h(|\log x - \log y|) + h(|\log y - \log z|) \quad (33)$$

Thus, if we are given positive numbers  $a, b$ , we can define  $x = \exp(a), z = \exp(-b), y = 1$ , so that then  $a = \log x, b = -\log z, 0 = \log y$ , and we will conclude that we have the positive subadditivity property:

$$h(a + b) \leq h(a) + h(b) \quad \text{whenever } a, b \geq 0 \quad (34)$$

Conversely, if positive subadditivity holds then given arbitrary positive numbers  $x, y, z$  we have using the monotonicity property 3)

$$\begin{aligned} h(|\log x - \log z|) &= h(|(\log x - \log y) + (\log y - \log z)|) \\ &\leq h(|\log x - \log y| + |\log y - \log z|) \end{aligned}$$

and by subadditivity we obtain the desired property 4).  $\square$

Our current choice for  $h$  is

$$h(y) = \tanh((s/2)y) \quad (35)$$

Thus, the chosen lateralization metric is given by

$$\text{lat\_unbdd}(\|L\|, \|R\|) = \tanh\left(\frac{s}{2} \log\left(\frac{\|L\|}{\|R\|}\right)\right) \quad (36)$$

Since

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

and since

$$e^{\frac{s}{2} \log \frac{\|L\|}{\|R\|}} = \left(\frac{\|L\|}{\|R\|}\right)^{\frac{s}{2}},$$

the lateralization metric in Eq.( 36) can also be expressed as:

$$\text{lat\_unbdd}(\|L\|, \|R\|) = \frac{\|L\|^s - \|R\|^s}{\|L\|^s + \|R\|^s} \quad (37)$$

Larger values of  $s$  in the lateralization metric yield measurements which are more sensitive to lateral asymmetry, while smaller values of  $s$  decrease the sensitivity.

Notice that our current choice reduces to a version of the lateralization metric given in Eq. 6, where we interpret  $\rho_L$  as  $\|L\|^{-s}$ ,  $\rho_R$  as  $\|R\|^{-s}$ , and  $\rho_{L,R}$  as  $\|L\|^{-s} + \|R\|^{-s}$ .

**Unnormalized disorganization from normalized metrics.** In order to express our conclusions concerning lateralization measures in terms of normalized organization or disorganization metrics, we must transform the normalized metrics to obtain unnormalized metrics, and then apply the previously chosen lateralization formula to the resulting metrics. The simplest way to obtain an unnormalized disorganization metric from a given normalized disorganization value  $\|M\|$  is to take the reciprocal of the corresponding organization value:

$$\|M\|_{\text{unnormalized}} = \frac{1}{\text{org}(M)} \quad (38)$$

By substituting the resulting unnormalized disorganization into the expression for lateralization from Eq. (37), one obtains the corresponding lateralization metric:

$$\text{lat\_unbdd}(\|L\|, \|R\|) = \frac{\|L\|_{\text{unnormalized}}^s - \|R\|_{\text{unnormalized}}^s}{\|L\|_{\text{unnormalized}}^s + \|R\|_{\text{unnormalized}}^s} = \frac{\text{org}(\text{R})^s - \text{org}(\text{L})^s}{\text{org}(\text{R})^s + \text{org}(\text{L})^s} \quad (39)$$

As usual, the parameter  $s$  controls the steepness of the metric. Observe that the resulting lateralization metric depends only on the organization values, and therefore may be used independently of any intermediate disorganization metrics.

## 2.3 Correlation metrics

In addition to a lateralization metric, which measures the difference in the *total level* of organization between the hemispheres, we also seek a measure of the similarity in their *patterns* of organization. Similarity may be interpreted as referring to the degree of coincidence of the organized regions in the hemispheres being compared, or, in a more abstract sense, to the degree of parallelism of the vectors representing the hemispheric maps. We call the measure of similarity the *bihemispheric correlation*. The correlation takes values between  $-1$  and  $+1$ , where  $-1$  indicates total bilateral symmetry in the organization patterns, and  $+1$  indicates total antisymmetry. We let  $c(M, M')$  denote the correlation value associated with the maps  $M, M'$ .

The desired properties of the correlation are the following. It is assumed here that the two hemispheres whose correlation is to be computed have the same number of nodes. The case of hemispheres of different sizes is addressed briefly in the final subsection of the current section on correlation metrics.

1. *lateral symmetry*: The correlation is invariant under left–right interchange, i.e.

$$c(M, N) = c(N, M) \quad (40)$$

2. *scale invariance*: Only the direction of organization matters:

$$c(M, \mu N) = c(M, N) \quad \text{for all positive scalars } \mu. \quad (41)$$

3. *monohemispheric directional antisymmetry*: If  $N'$  is obtained from  $N$  by changing the sign of the entries  $cx, cy$  at all nodes of  $N$ , then:

$$c(M, N') = -c(M, N) \quad (42)$$

4. *normalization*:

$$c(M, M) = 1 \quad (43)$$

5. *random decorrelation*: If  $M$  and  $N$  are independent random maps (variables), then the expected value of their correlation is zero:

$$E[c(M, N)] = 0 \quad (44)$$

### 2.3.1 Root mean square correlation.

We may easily define an interhemispheric correlation measure by simple geometrical considerations. Namely, the inner product associated with the weighted Euclidean metric used to construct the organization measure  $\| \cdot \|_{\text{RMS1}}$  in the preceding section produces an abstract geometrical structure in the space of topographic maps, and we use this abstract geometry to measure the angle  $\theta$  between the vectors representing the given topographic maps. By the parallelogram rule, the cosine of this angle is given by:

$$c_{\text{RMS}}(L, R) = \frac{\|L\|^2 + \|R\|^2 - \|L - R\|^2}{2\|L\|\|R\|} \quad (45)$$

(Recall that the difference between two topographic maps is simply the difference between two matrices. This requires further explanation if the two hemispheres have different numbers of nodes; this is addressed below in a separate subsection.) This is our basic root mean square correlation measure. Its values vary between  $-1$  and  $+1$ , with  $-1$  indicating total antisymmetry in the topographic maps being compared, and  $+1$  indicating total mirror symmetry. Observe that the disorganization measure used to construct the root mean square correlation is *not* the usual root mean square measure  $\| \cdot \|_{\text{RMS}}$ , but rather the shift-dependent version  $\| \cdot \|_{\text{RMS1}}$ .

**Proposition 2.4.** *The measure of correlation defined above satisfies properties 1), 2), 4), 5), and property 3) also if one chooses the parameter vector  $a = (1, 1, 0, 0)$  in the definition of the weighted Euclidean distance in Eq. 9.*

*Proof.* All properties follow from the fact that the correlation as defined is the cosine of the angle between the abstract vectors representing the hemispheric maps  $L, R$ . Property 1) is clear, as the angle between two vectors does not depend on the order in which they are considered. Property 2) follows from the fact that the angle is independent of the lengths of the vectors. Property 3) is simply the trigonometric identity  $\cos(\theta + \pi) = -\cos(\theta)$ , property 4) says  $\cos(0) = 1$ , and property 5) is the standard definition of independence used in probability and statistics.  $\square$

### 2.3.2 Steepened root mean square correlation.

A variant of the basic root mean square measure is obtained by introducing a steepness parameter as for the lateralization metrics described above. The resulting root mean square correlation measure is given as follows. We refer to this measure as the *steepened root mean square correlation*, abbreviated as *sRMS correlation*.

$$c_{\text{sRMS}}(L, R) = f \left( \frac{\|L\|^2 + \|R\|^2 - \|L - R\|^2}{2\|L\|\|R\|} \right) \quad (46)$$

where the steepening function  $f$  is as in Eq.( 23).

### 2.3.3 Map overlap symmetry measure.

In terms of the sensory regions  $\text{oreg}(M)$  and  $\text{oreg}(M')$  corresponding to the organized regions in the individual hemispheric maps, and in terms of the set-theoretic operation  $\Delta$  of symmetric difference, we define a measure of correlation, which we refer to as the *map overlap measure*, as follows:

$$\text{map\_overlap}(L, R) = \text{area}(\text{oreg}(M) \cap \text{oreg}(M')) - \text{area}(\text{oreg}(M) \Delta \text{oreg}(M')) \quad (47)$$

The intersection term measures the overlap of the organized regions, while the symmetric difference term measures the discrepancy between these regions. The map overlap measure associates the maximum value 1 to a bihemispheric map if and only if each hemisphere covers the entire sensory surface with a well-formed map. The value  $-1$  corresponds to the case in which the two sensory regions covered by the two hemispheric maps are completely disjoint and together cover the entire sensory surface.

A variant of the above measure is obtained by dividing by the area of the union of the organized regions. The resulting measure is the *normalized map overlap* measure. The extreme values  $+1, -1$  for the normalized map overlap measure are attained, respectively, when the two maps cover the same sensory region, and when they cover disjoint regions. It is no longer relevant whether or not the entire sensory surface is covered by each map or by their union, as in the case of the unnormalized map overlap measure.

## 2.4 Standard statistical measures.

As we did in the case of organization measures, we may now compute means and standard deviations of the *differences* in the attributes  $cx, cy, rx, ry$  between the two hemispheres. Letting  $X$  denote any one of the attributes:

$$\mu_{LR}(X) = \frac{1}{N} \sum_{\text{all nodes } n} (X(L, n) - X(R, n)) \quad (48)$$

$$\sigma_{LR}(X) = \sqrt{\frac{1}{N-1} \sum_{\text{all nodes } n} (X(L, n) - X(R, n) - \mu_{LR}(X))^2} \quad (49)$$

These measures provide alternative information sources for organization and symmetry assessments. Highly organized topographic maps are associated with small values for the variances of the various attributes. Small values of the variance indicate a high degree of transhemispheric symmetry in the associated attribute.

## 2.5 Correlation measures in the case of unequal hemispheric sizes.

If the two hemispheres have different numbers of nodes, then the arrays  $L$  and  $R$  representing the hemispheric maps have different sizes. This precludes consideration of the standard matrix difference  $L - R$ , as required, for example, in Eq.( 45). The standard statistical measures, as in Eqs. ( 48), ( 49), and in the definition of the random decorrelation property at the beginning of the current section, are also undefined unless the hemispheric sizes coincide.

One way to deal with this difficulty is to first represent the map  $S$  of the smaller hemisphere as a larger map  $S'$  of size equal to the size of the larger hemisphere. This rescaling process is performed in such a way that the resulting map  $S'$  represents the original smaller map  $S$  as faithfully as possible. Once the rescaled map has been obtained, the measures of correlation defined above are used to compare the rescaled map with the original map of the larger hemisphere.

Thus, given a measure of correlation  $c(L, R)$  defined for the case in which the sizes of  $L$  and  $R$  coincide, the measure is extended to apply in the case of unequal hemispheric sizes by letting

$$c(L, R) := c(L', R'),$$

where the smaller hemispheric map has been scaled as described above, and the larger hemispheric map has been left unchanged.

The rescaling process itself is simple. If the smaller hemisphere has size  $n \times m$ , and if the larger hemisphere has size  $N \times M$ , where we assume  $n \leq N$  and  $m \leq M$ , then given “large” coordinates  $(I, J)$ , with  $1 \leq I \leq N$  and  $1 \leq J \leq M$ , we obtain corresponding “small” coordinates  $(i, j)$  by letting

$$\begin{aligned} i &= \lfloor I n/N \rfloor \\ j &= \lfloor J m/M \rfloor \end{aligned}$$

The rescaled map  $S'$  is then obtained from the original map  $S$  by using the rescaled indices:

$$S'(I, J) := S(i, j)$$

The above rescaling idea is not without problems, however. As a direct consequence of the rescaling process, the resulting map  $S'$  is constant on blocks of average size  $\lfloor N/n \rfloor \times \lfloor M/m \rfloor$ . This tends to produce significant cancellations in the sums involved in computing the correlation measures and therefore adversely affects the measurement values.

### 3 Experimental Comparison

We asked 8 researchers (mostly graduate students) at the University of Maryland familiar with the concept of a topographic map to participate in a simple experiment. Each subject was asked to give his or her subjective measurements of organization and lateralization of representative pictures of topographic map formation obtained in our simulations using the bihemispheric cortex model. We discuss the results below. See the Appendix for graphical views of the maps used, and for complete lists of the measurement values given by a program implementing the measures described above as well as those provided by the human test subjects.

Comparing the root mean square and sigmoid organization measures, we see that the latter produces values closer to the human values. This is most noticeable in the random starting frames in Examples 1 and 3, where the root mean square measure yields positive organization values. The starting frame from Example 1 is shown below, together with the measurement values produced by the program and the human subjects.

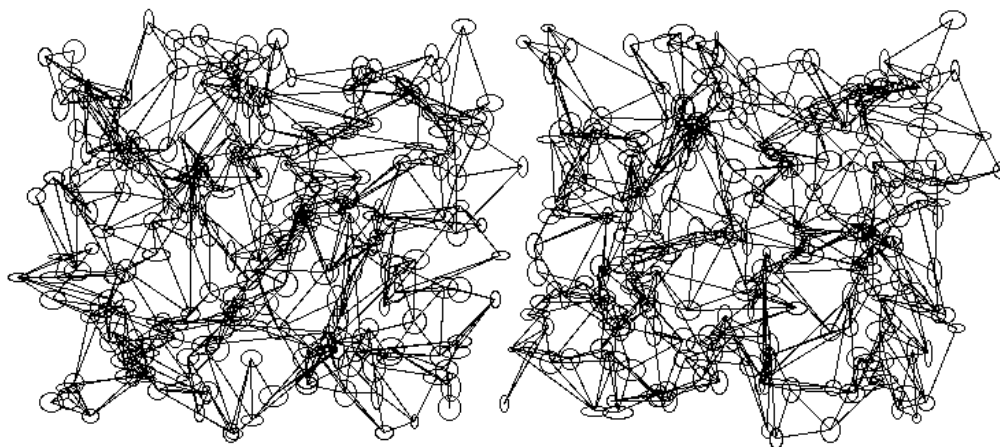


Figure 5: Example 1, frame #1

frame # 1 (receptive field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.11	0.13	0.02	0.03	0.12
sigmoidRMS	0.00	0.00	-0.00	-0.00	0.00
diffRMS	0.04	0.06	0.01	0.03	0.30
sigmoiddiff	0.01	0.01	0.00	0.00	0.00
area	0.04	0.04	-0.00	-0.00	0.00
subjective	0.01	0.01	0.00	0.00	0.00

subjective lateralization	0.00 +- .00
c_RMS	-0.06
c_sRMS	-0.11
map_overlap	-0.08
normalized_map_overlap	-0.99
subjective correlation	0.00 +- .00

The “reason” for this behavior for the random frames is that the root mean square measure requires an *infinite* variance in order to produce zero as the organization value, while human subjects assign the value 0 to even moderate variances. People seem to mentally divide a graphical view of a given topographic map into two parts: an “organized” part and a “disorganized” part, irrespective of moderate quantitative variations within these parts, and then attempt to assess the fraction of the total area which corresponds to the organized part. Root mean square measures simply average local deviations from uniformity, and are quite sensitive to moderate quantitative variations, i.e. they do care about the quantitative degree of disorganization present in a given region, and not just whether or not it is disorganized in some qualitative way. The sigmoid and sigmoiddiff measures, on the other hand, seem better able to mimic humans’ apparent establishment of a disorganization threshold which separates organization from disorganization in a given topographic map.

The various measures agree reasonably well with the test subjects in all of the examples with the following exceptions. In Example 4 (see Appendix), several of the organization measures produce values that are considerably smaller than those given by the subjects. Only the area and sigmoiddiff measures come close to the human ability to focus on the regularity of the map, ignoring the curvature of the lines joining receptive field centers. Frame #5 from Example 4 is shown below, together with the corresponding measurement values.

frame # 5 (receptive field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.32	0.35	0.03	0.06	0.07
sigmoidRMS	0.31	0.35	0.04	0.09	0.09
diffRMS	0.88	0.86	-0.02	-0.03	-0.02
sigmoiddiff	0.82	0.80	-0.02	-0.04	-0.02
area	1.00	0.95	-0.05	-0.09	-0.04
subjective	0.88	0.83	-0.05	-0.10	-0.04

subjective lateralization	-0.04 +- .05
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c_RMS	-0.82
c_sRMS	-0.98

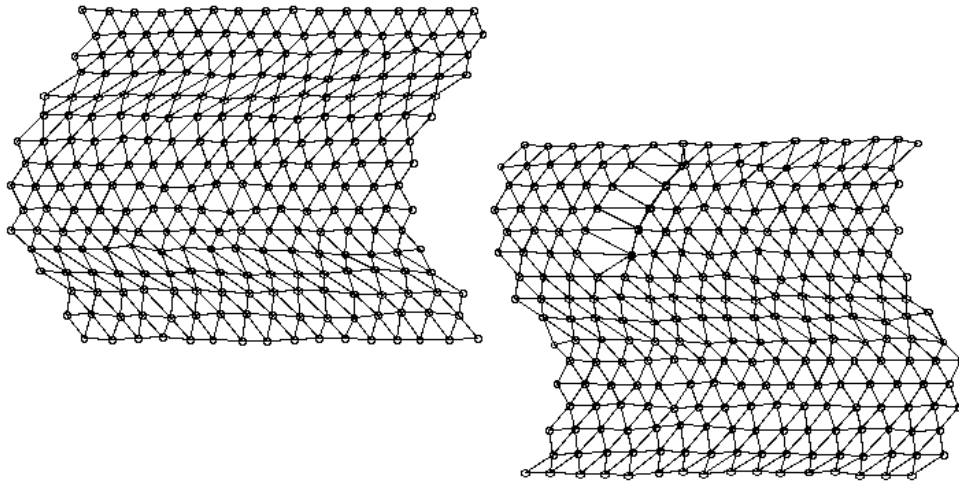


Figure 6: Example 4, frame #5

map_overlap	0.91
normalized_map_overlap	0.91
subjective correlation	-.83 +- .05

The area measure, however, overestimates the level of organization in Example 3a. Thus, overall, the best performance is obtained with the sigmoiddiff measure.

As for lateralization, we observe that the simple difference measure `lat` produces values that are not satisfactory from the point of view of the test subjects. The steepened lateralization measure `lat_steep` provides a considerable improvement, as does the logarithmic measure `lat_log`. The latter measure performs counter-intuitively in situations such as frame 3 of Example 3, when both hemispheres are highly organized. This is because `log_lat` is sensitive only to the *relative* magnitudes of the disorganization values, and so produces large lateralization values when both hemispheres have organization values close to 1, if the organization values of the two hemispheres differ from 1 by amounts which, though small, have a ratio which is not close to unity.

Our limited human correlation assessments suggest that the values produced by the current correlation measures are quite appropriate. The best correlation values are produced by the steepened root mean square measure. We are aware of the fact that the small sample size used to obtain the subjective measurements of organization, lateralization, and mirror symmetry makes it difficult to obtain good confidence limits for the resulting estimates of means and variances. We merely wish to present examples of human judgments of the various measures. Assuming normally distributed measurements,  $\chi$ -squared and Student  $t$  analysis shows that with a probability of at least .95, the true population means and variances are within approximately 40% of the sample values.



## 4 Discussion

In summary, we have developed measures of organization and lateralization which are supported by a good theoretical foundation and whose values agree well with test subjects' assessments based on graphical views of bihemispheric topographic map formation. We have also defined satisfactory measures of mirror symmetry or bihemispheric correlation. Our measures are normalized with respect to hemispheric size, so that hemispheres of different sizes can be meaningfully compared. Also, the measures correct for various distortions of the topographic maps, such as mean shifts and global curving, which should not greatly affect the associated measurement values.

Simple root mean square and sigmoid organization measures attempt to determine the deviation of the measured topographic maps relative to completely uniform, hexagonal lattice topographic maps, while human subjects are less restrictive in their judgment of organization. Another option suggested here is to have a neural network find an appropriate measure via supervised learning using error backpropagation. The current sigmoid-based organization measure corresponds to a single layer network, and already yields a considerable improvement over the root mean square measure. In our opinion the best performance has been obtained with the differential root mean square and sigmoiddiff measures. These measures allow significant global distortions for a given value of organization, focussing on local regularity of maps, as humans seem to do.

In work in progress, we are simulating the bihemispheric cortex model with various parameter values to gain insight into the factors affecting the development of organization and asymmetry.

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# Appendix

Results obtained in our simulations with the bihemispheric cortex model are presented here, and comparisons are provided between the output produced by a program implementing the measures we have developed and the assessments made by human test subjects.

In all of the examples, the attribute parameter vector is  $(1, 1, 0, 0)$  for the root mean square (RMS) correlation measurement, and  $(1, 1, 1, 1)$  for the RMS, differential RMS, sigmoid, and sigmoiddiff organization measurements, as well as for the sigmoiddiff correlation. In other words, RMS correlation is judged solely on the basis of the displacements of the receptive field centers, while the remaining judgments take the receptive field radii into account as well. The threshold and steepness for the sigmoid function are  $\tau = 1.15$  and  $s = 5$ , respectively, in the case of the sigmoid organization measure, and  $\tau = 1.05$  and  $s = 2.5$  for the sigmoiddiff measure.

First, a tabular summary of results is given, followed by graphical views of the topographic maps used as examples, together with detailed measurement values given by both the program and by humans.

## Measures computed

In the examples below, the values listed correspond to the measures defined above, as indicated in the following list of equation numbers:

- root mean square (RMS) organization:  $org_{RMS}$ , with the choice  $a = (1, 1, 1, 1)$  in Eq.( 11)
- sigmoid organization:  $org_{sigmoidRMS}$
- differential RMS organization:  $org_{diffRMS}$ , with the choice  $a = (1, 1, 1, 1)$  in Eq.( 13)
- sigmoiddiff organization:  $org_{sigmoiddiff}$ , with  $|| ||_{sigmoid}$  replaced by  $|| ||_{sigmoiddiff}$  from Eq.( 18)
- difference lateralization:  $lat$ , Eq.( 20) with the appropriate disorganization metric  $|| ||_{diffRMS}$  and  $|| ||_{sigmoiddiff}$  in each case
- steepened lateralization:  $lat_{steep}$ , Eq.( 24), with  $s = 2$  and the appropriate disorganization metric  $|| ||_{diffRMS}$  and  $|| ||_{sigmoiddiff}$  in each case
- lateralization via unbounded disorganization:  $lat_{unbdd}$ , Eq.( 39), with  $s = 1.5$  and the appropriate disorganization metric  $|| ||_{diffRMS}$  and  $|| ||_{sigmoiddiff}$  in each case
- RMS correlation:  $c_{RMS}$ , Eq.( 45), with the choice  $a = (1, 1, 0, 0)$  in Eq.( 10)
- sRMS correlation:  $c_{sRMS}$ , Eq.( 46), with the choice  $a = (1, 1, 0, 0)$  in Eq.( 10) and the steepness value  $s = 2$  in Eq.( 23)

- map overlap symmetry measure: `map_overlap`, Eq.( 47)
- normalized map overlap symmetry measure: as in Eq.( 47), dividing the right hand side by the area of the union of the organized regions in the two hemispheres

## Summary of the results

In the table of organization values below, only the means of the test subjects' measurements are given. The lateralization table includes columns for mean  $\pm$  standard deviation values as well as minimum and maximum values. Graphical views of selected frames from receptive field data files used to produce the preceding data tables are presented following the tables, together with the measurement program output and the means of the human measurements.

### Organization

#	org_RMS		org_sigmoidRMS		org_area		org_diffRMS		org_sigmoiddiff		people	
1a	.11	.13	.00	.00	.04	.04	.04	.06	.01	.01	0.01	0.01
1b	.34	.26	.39	.37	.26	.22	.35	.22	.35	.27	0.44	0.38
1c	.48	.46	.60	.58	.38	.36	.62	.54	.62	.64	0.60	0.60
2b	.33	.75	.41	.75	.06	.53	.29	.74	.19	.70	0.16	0.79
2c	.19	.67	.26	.77	.20	.67	.46	.79	.47	.78	0.39	0.71
3a	.66	.69	.00	.00	.51	.64	.56	.59	.29	.32	0.29	0.28
3b	.95	.96	.96	.96	1.0	1.0	.97	.97	.97	.97	0.93	0.93
3c	.96	.98	.96	.97	1.0	1.0	.99	.99	.98	.99	0.99	0.99
4b	.15	.09	.37	.15	.73	.57	.27	.24	.59	.52	0.79	0.74
4c	.32	.35	.31	.35	1.0	.95	.88	.86	.82	.80	0.88	0.83
5b	.86	.36	.79	.00	.96	.00	.86	.22	.83	.03	0.83	0.07
5c	.97	.28	.97	.00	1.0	.00	.98	.17	.98	.02	0.98	0.03

### Unbounded Lateralization

#	lat_RMS	lat_area	lat_diffRMS	lat_sigmoiddiff	from_human	people	people(min,max)
1a	.12	.00	.30	.00	.00	0.00 +- .00	.00 .00
1b	-.20	-.12	-.33	-.19	-.11	-0.16 +- .21	-.60 .00
1c	-.03	-.04	-.10	.02	.00	-0.03 +- .05	-.15 .00
2b	.55	.93	.61	.75	.83	0.79 +- .12	.60 .95
2c	.74	.72	.38	.36	.42	0.44 +- .20	.10 .75
3a	.03	.17	.04	.07	-.03	0.01 +- .03	.00 .10
3b	.01	.00	.00	.00	.00	0.00 +- .00	.00 .00
3c	.02	.00	.00	.01	.00	0.00 +- .00	.00 .00
4b	-.37	-.18	-.09	-.09	-.05	-0.07 +- .05	-.15 .00
4c	.07	-.04	-.02	-.02	-.04	-0.04 +- .05	-.10 .00
5b	-.57	-1.0	-.77	-.99	-.95	-0.92 +- .07	-1.0 -.80
5c	-.73	-1.0	-.87	-.99	-.99	-1.00 +- .00	-1.0 -1.0

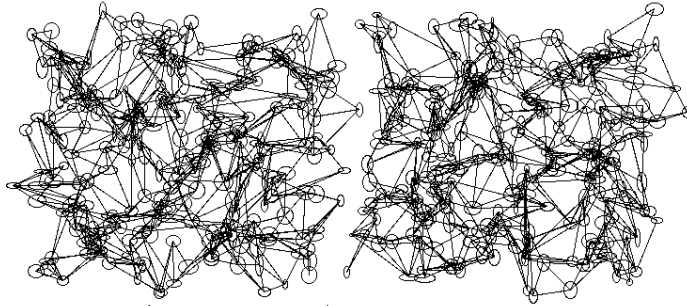
Steepened Lateralization

#	lat_RMS	lat_area	lat_diffRMS	lat_sigmoiddiff	from_human	people	people(min,max)
1a	.03	-.00	.03	.00	.00	0.00 +- .00	.00 .00
1b	-.15	-.09	-.26	-.15	-.12	-0.16 +- .21	-.60 .00
1c	-.05	-.03	-.17	.02	.00	-0.03 +- .05	-.15 .00
2b	.71	.77	.74	.81	.90	0.79 +- .12	.60 .95
2c	.78	.77	.60	.56	.58	0.44 +- .20	.10 .75
3a	.07	.26	.06	.07	-.02	0.01 +- .03	.00 .10
3b	.02	-.00	-.00	-.00	.00	0.00 +- .00	.00 .00
3c	.03	-.00	.00	.00	.00	0.00 +- .00	.00 .00
4b	-.11	-.32	-.05	-.13	-.10	-0.07 +- .05	-.15 .00
4c	.06	-.09	-.03	-.04	-.10	-0.04 +- .05	-.10 .00
5b	-.80	-1.0	-.91	-.97	-.96	-0.92 +- .07	-1.0 -.80
5c	-.94	-1.0	-.98	-1.0	-1.0	-1.00 +- .00	-1.0 -1.0

Correlation and Map Overlap

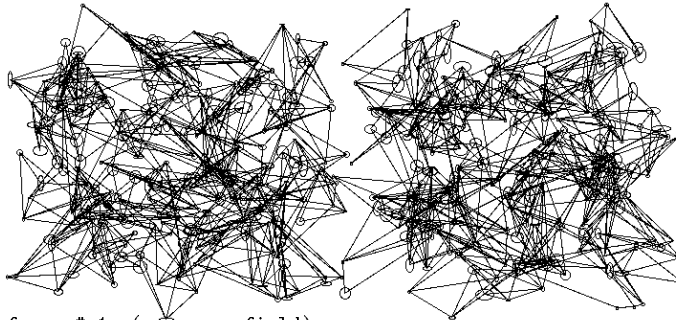
#	c_RMS	c_sRMS	map_overlap	map_overlap (normalized)	people	people(min,max)
1a	-.06	-.11	-.08	-.99	.00 +- .00	.00 .00
1b	-.29	-.54	-.48	-1.0	-.77 +- .19	-.90 -.50
1c	-.52	-.82	-.74	-1.0	-.92 +- .08	-1.0 -.80
2b	-.17	-.34	-.59	-1.0	-.80 +- .08	-.90 -.70
2c	-.54	-.84	-.86	-1.0	-.90 +- .00	-.90 -.90
3a	.03	.05	-.21	-.25	.30 +- .16	.10 .50
3b	.40	.69	1.0	1.0	.87 +- .09	.80 1.0
3c	.67	.93	1.0	1.0	.97 +- .05	.90 1.0
4b	-.72	-.95	-.21	-.22	-.63 +- .12	-.80 -.50
4c	-.82	-.98	.91	.91	-.83 +- .05	-.90 -.80
5b	.01	.01	-.96	-1.0	.03 +- .05	.00 .10
5c	-.05	-.10	-1.0	-1.0	.00 +- .00	.00 .00

Example 1



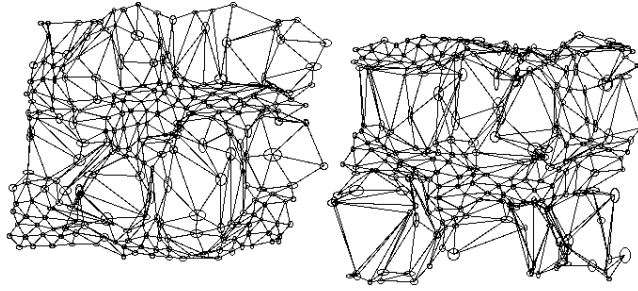
frame # 1 (receptive field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.11	0.13	0.02	0.03	0.12
sigmoidRMS	0.00	0.00	-0.00	-0.00	0.00
diffRMS	0.04	0.06	0.01	0.03	0.30
sigmoiddiff	0.01	0.01	0.00	0.00	0.00
area	0.04	0.04	-0.00	-0.00	0.00
subjective	0.01	0.01	0.00	0.00	0.00
subjective lateralization			0.00 +- .00		
c_RMS			-0.06		
c_sRMS			-0.11		
map_overlap			-0.08		
normalized_map_overlap			-0.99		
subjective correlation			0.00 +- .00		



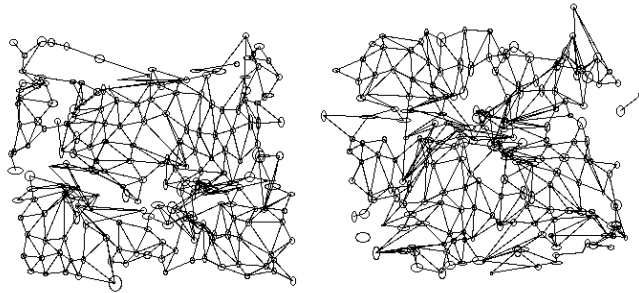
frame # 1 (response field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.01	0.01	0.00	0.00	0.00
sigmoidRMS	0.02	0.04	0.02	0.03	0.48
diffRMS	0.00	0.00	0.00	0.00	0.00
sigmoiddiff	0.00	0.00	0.00	0.00	0.00
area	0.00	0.00	0.00	0.00	0.00
c_RMS			-0.75		
c_sRMS			-0.96		
map_overlap			-0.00		
normalized_map_overlap			-1.00		



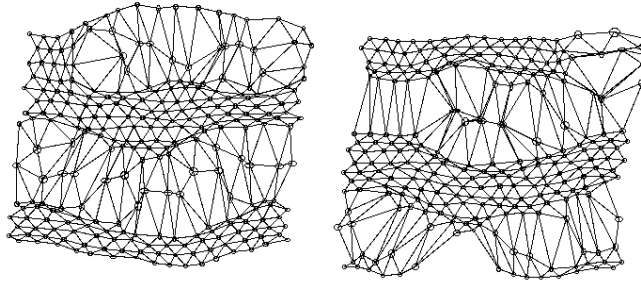
frame # 2 (receptive field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.34	0.26	-0.08	-0.15	-0.20
sigmoidRMS	0.39	0.37	-0.02	-0.05	-0.04
diffRMS	0.35	0.22	-0.13	-0.26	-0.33
sigmoiddiff	0.35	0.27	-0.08	-0.15	-0.19
area	0.26	0.22	-0.05	-0.09	-0.12
subjective	0.44	0.38	-0.06	-0.12	-0.11
subjective lateralization			-0.16	+- .21	
c_RMS			-0.29		
c_sRMS			-0.54		
map_overlap			-0.48		
normalized_map_overlap			-1.00		
subjective correlation			-.77	+- .19	



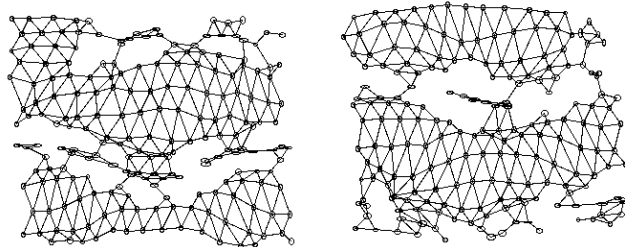
frame # 2 (response field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.29	0.27	-0.02	-0.04	-0.05
sigmoidRMS	0.41	0.39	-0.02	-0.03	-0.04
diffRMS	0.18	0.14	-0.04	-0.07	-0.19
sigmoiddiff	0.14	0.10	-0.04	-0.07	-0.25
area	0.07	0.04	-0.03	-0.07	-0.40
c_RMS			-0.25		
c_sRMS			-0.47		
map_overlap			-0.11		
normalized_map_overlap			-1.00		



frame # 9 (receptive field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.48	0.46	-0.03	-0.05	-0.03
sigmoidRMS	0.60	0.58	-0.02	-0.03	-0.03
diffRMS	0.62	0.54	-0.08	-0.17	-0.10
sigmoiddiff	0.62	0.64	0.01	0.02	0.02
area	0.38	0.36	-0.02	-0.03	-0.04
subjective	0.60	0.60	0.00	0.00	0.00
subjective lateralization			-0.03	+- .05	
c_RMS			-0.52		
c_sRMS			-0.82		
map_overlap			-0.74		
normalized_map_overlap			-1.00		
subjective correlation			-.92	+- .08	

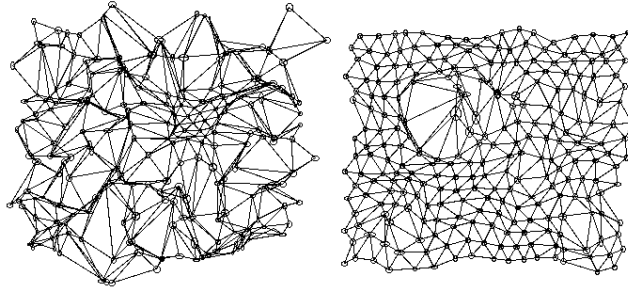


frame # 9 (response field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.64	0.63	-0.01	-0.02	-0.01
sigmoidRMS	0.70	0.70	-0.00	-0.00	-0.00
diffRMS	0.63	0.64	0.01	0.02	0.01
sigmoiddiff	0.43	0.44	0.01	0.02	0.02
area	0.07	0.00	-0.07	-0.14	-1.00
c_RMS			-0.38		
c_sRMS			-0.66		
map_overlap			-0.07		
normalized_map_overlap			-0.96		

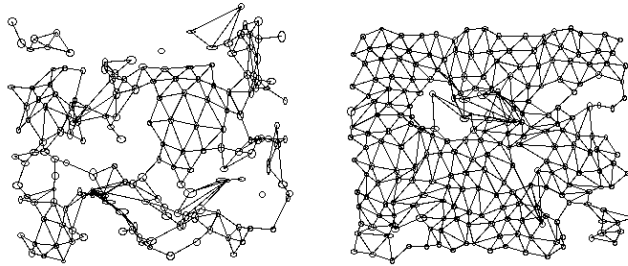


## Example 2



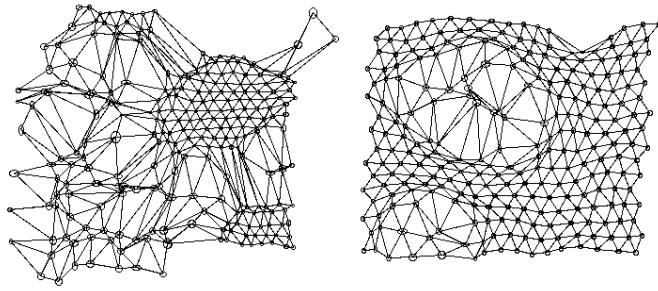
frame # 2 (receptive field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.33	0.75	0.42	0.71	0.55
sigmoidRMS	0.41	0.75	0.34	0.61	0.42
diffRMS	0.29	0.74	0.44	0.74	0.61
sigmoiddiff	0.19	0.70	0.51	0.81	0.75
area	0.06	0.53	0.47	0.77	0.93
subjective	0.16	0.79	0.63	0.90	0.83
subjective lateralization			0.79 +- .12		
c_RMS			-0.17		
c_sRMS			-0.34		
map_overlap			-0.59		
normalized_map_overlap			-1.00		
subjective correlation			-0.80 +- .08		



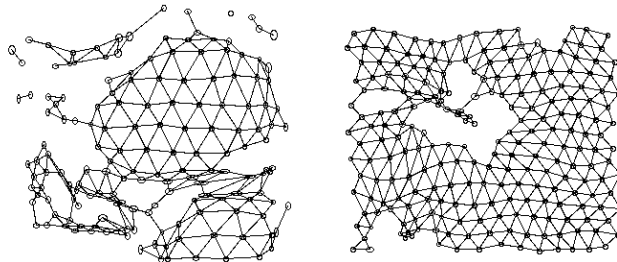
frame # 2 (response field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.47	0.70	0.23	0.43	0.29
sigmoidRMS	0.53	0.77	0.25	0.47	0.27
diffRMS	0.34	0.61	0.27	0.51	0.41
sigmoiddiff	0.17	0.54	0.37	0.65	0.70
area	0.01	0.34	0.32	0.58	0.99
c_RMS			-0.08		
c_sRMS			-0.16		
map_overlap			-0.35		
normalized_map_overlap			-0.99		



frame # 5 (receptive field)

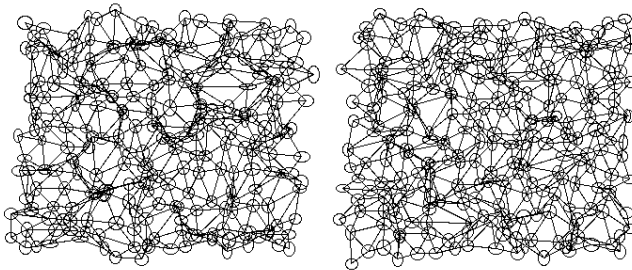
Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.19	0.67	0.48	0.78	0.74
sigmoidRMS	0.26	0.77	0.51	0.81	0.67
diffRMS	0.46	0.79	0.33	0.60	0.38
sigmoiddiff	0.47	0.78	0.31	0.56	0.36
area	0.20	0.67	0.47	0.77	0.72
subjective	0.39	0.71	0.32	0.58	0.42
subjective lateralization			0.44 +- .20		
c_RMS			-0.54		
c_sRMS			-0.84		
map_overlap			-0.86		
normalized_map_overlap			-1.00		
subjective correlation			-.90 +- .00		



frame # 5 (response field)

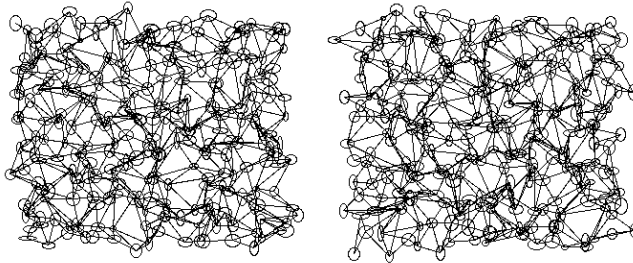
Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.37	0.75	0.38	0.67	0.49
sigmoidRMS	0.49	0.84	0.35	0.63	0.38
diffRMS	0.42	0.80	0.38	0.67	0.45
sigmoiddiff	0.25	0.74	0.49	0.79	0.67
area	0.00	0.43	0.43	0.72	1.00
c_RMS			-0.44		
c_sRMS			-0.74		
map_overlap			-0.43		
normalized_map_overlap			-1.00		

### Example 3



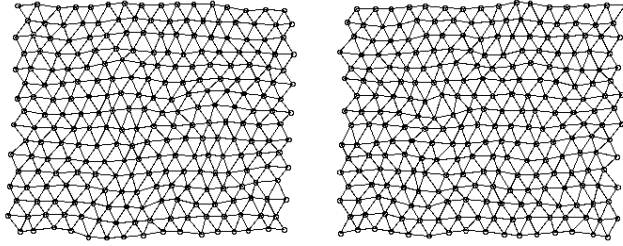
frame # 1 (receptive field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.66	0.69	0.04	0.07	0.03
sigmoidRMS	0.00	0.00	-0.00	-0.00	0.00
diffRMS	0.56	0.59	0.03	0.06	0.04
sigmoiddiff	0.29	0.32	0.03	0.07	0.07
area	0.51	0.64	0.13	0.26	0.17
subjective	0.29	0.28	-0.01	-0.02	-0.03
subjective lateralization			0.01 +- .03		
c_RMS			0.03		
c_sRMS			0.05		
map_overlap			-0.21		
normalized_map_overlap			-0.25		
subjective correlation			0.30 +- .16		



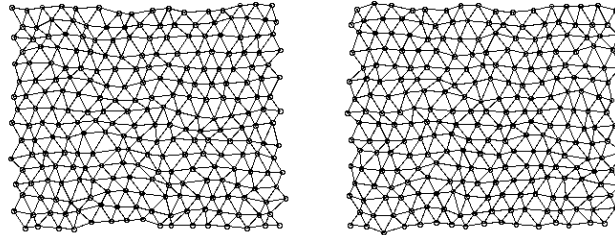
frame # 1 (response field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.47	0.47	-0.00	-0.00	-0.00
sigmoidRMS	0.01	0.01	-0.00	-0.00	-0.00
diffRMS	0.27	0.25	-0.02	-0.03	-0.06
sigmoiddiff	0.05	0.05	0.01	0.01	0.00
area	0.17	0.18	0.01	0.01	0.04
c_RMS			-0.04		
c_sRMS			-0.08		
map_overlap			-0.30		
normalized_map_overlap			-0.89		



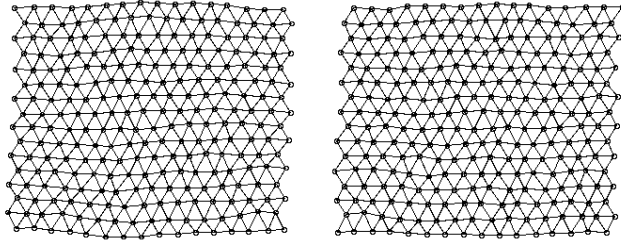
frame # 2 (receptive field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.95	0.96	0.01	0.02	0.01
sigmoidRMS	0.96	0.96	0.00	0.00	0.00
diffRMS	0.97	0.97	-0.00	-0.00	0.00
sigmoiddiff	0.97	0.97	-0.00	-0.00	0.00
area	1.00	1.00	-0.00	-0.00	0.00
subjective	0.93	0.93	0.00	0.00	0.00
subjective lateralization			0.00 +- .00		
c_RMS			0.40		
c_sRMS			0.69		
map_overlap			1.00		
normalized_map_overlap			1.00		
subjective correlation			0.87 +- .09		



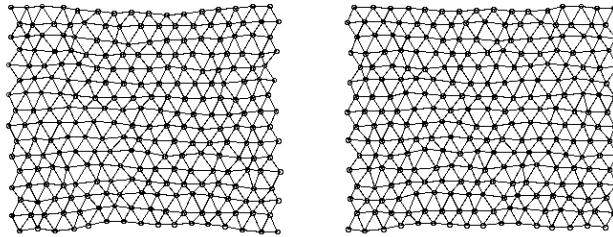
frame # 2 (response field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.93	0.94	0.01	0.03	0.01
sigmoidRMS	0.96	0.96	0.00	0.00	0.00
diffRMS	0.94	0.94	0.00	0.00	0.00
sigmoiddiff	0.95	0.95	0.00	0.00	0.00
area	1.00	1.00	0.00	0.00	0.00
c_RMS			0.42		
c_sRMS			0.71		
map_overlap			1.00		
normalized_map_overlap			1.00		



frame # 6 (receptive field)

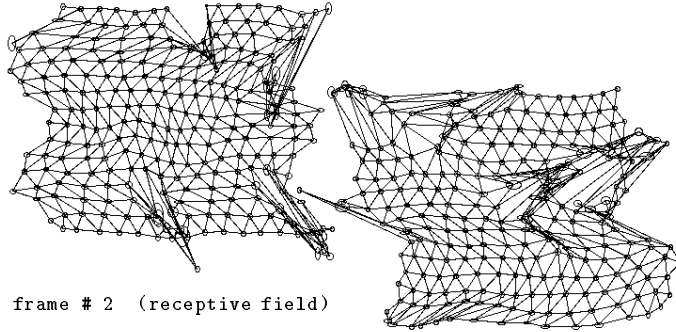
Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.96	0.98	0.02	0.03	0.02
sigmoidRMS	0.96	0.97	0.00	0.01	0.01
diffRMS	0.99	0.99	0.00	0.00	0.00
sigmoiddiff	0.98	0.99	0.00	0.00	0.01
area	1.00	1.00	-0.00	-0.00	-0.00
subjective	0.99	0.99	0.00	0.00	0.00
subjective lateralization			0.00 +- .00		
c_RMS			0.67		
c_sRMS			0.93		
map_overlap			1.00		
normalized_map_overlap			1.00		
subjective correlation			0.97 +- .05		



frame # 6 (response field)

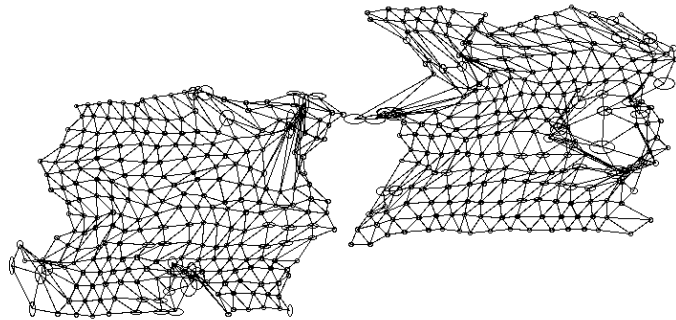
Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.96	0.97	0.02	0.03	0.01
sigmoidRMS	0.96	0.97	0.00	0.01	0.01
diffRMS	0.98	0.98	0.00	0.00	0.00
sigmoiddiff	0.98	0.98	0.00	0.00	0.00
area	1.00	1.00	0.00	0.00	0.00
c_RMS			0.71		
c_sRMS			0.94		
map_overlap			1.00		
normalized_map_overlap			1.00		

Example 4



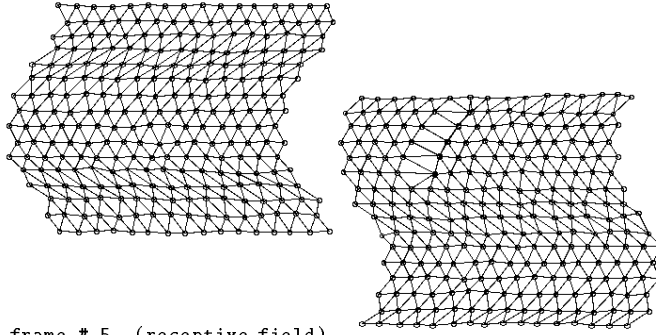
frame # 2 (receptive field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.15	0.09	-0.05	-0.11	-0.37
sigmoidRMS	0.37	0.15	-0.21	-0.41	-0.59
diffRMS	0.27	0.24	-0.03	-0.05	-0.09
sigmoiddiff	0.59	0.52	-0.06	-0.13	-0.09
area	0.73	0.57	-0.16	-0.32	-0.18
subjective	0.79	0.74	-0.05	-0.10	-0.05
subjective lateralization			-0.07 +- .05		
c_RMS			-0.72		
c_sRMS			-0.95		
map_overlap			-0.21		
normalized_map_overlap			-0.22		
subjective correlation			-.63 +- .12		



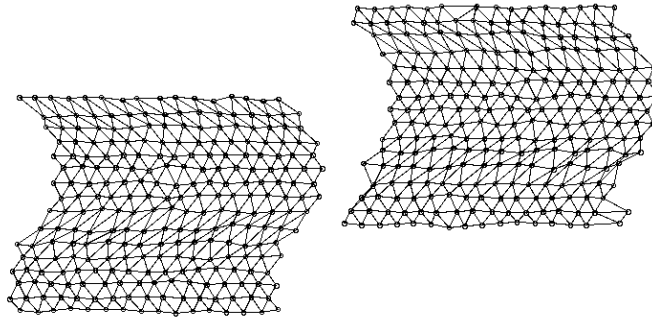
frame # 2 (response field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.23	0.13	-0.10	-0.20	-0.40
sigmoidRMS	0.41	0.19	-0.21	-0.41	-0.52
diffRMS	0.44	0.36	-0.08	-0.17	-0.15
sigmoiddiff	0.56	0.45	-0.11	-0.23	-0.16
area	0.66	0.49	-0.16	-0.32	-0.22
c_RMS			-0.87		
c_sRMS			-0.99		
map_overlap			-0.38		
normalized_map_overlap			-0.43		



frame # 5 (receptive field)

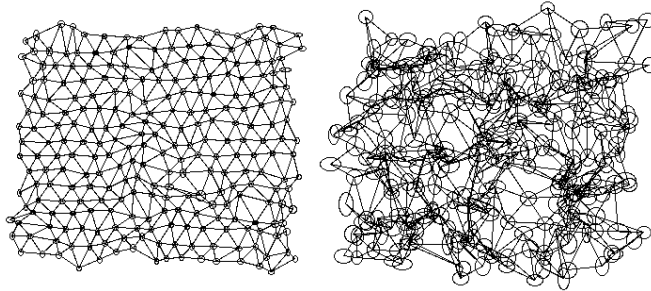
Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.32	0.35	0.03	0.06	0.07
sigmoidRMS	0.31	0.35	0.04	0.09	0.09
diffRMS	0.88	0.86	-0.02	-0.03	-0.02
sigmoiddiff	0.82	0.80	-0.02	-0.04	-0.02
area	1.00	0.95	-0.05	-0.09	-0.04
subjective	0.88	0.83	-0.05	-0.10	-0.04
subjective lateralization			-0.04 +- .05		
c_RMS			-0.82		
c_sRMS			-0.98		
map_overlap			0.91		
normalized_map_overlap			0.91		
subjective correlation			-.83 +- .05		



frame # 5 (response field)

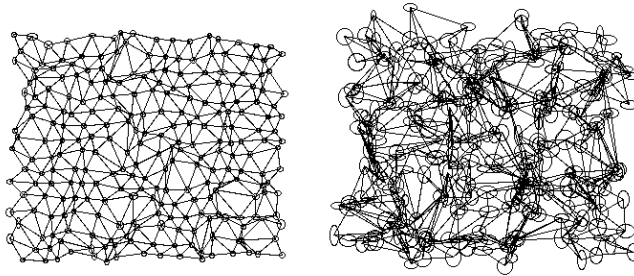
Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.30	0.33	0.03	0.06	0.07
sigmoidRMS	0.28	0.34	0.06	0.11	0.14
diffRMS	0.85	0.85	-0.01	-0.02	0.00
sigmoiddiff	0.80	0.78	-0.02	-0.04	-0.02
area	0.98	0.97	-0.01	-0.02	-0.01
c_RMS			-0.97		
c_sRMS			-1.00		
map_overlap			0.90		
normalized_map_overlap			0.90		

Example 5



frame # 2 (receptive field)

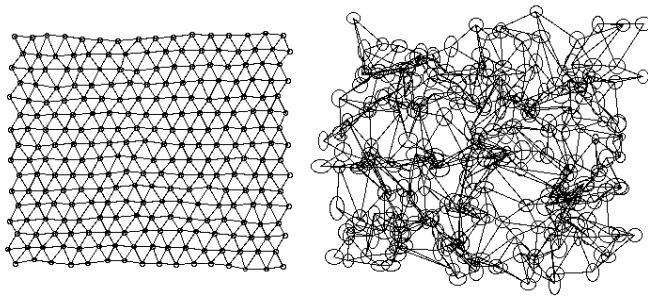
Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.86	0.36	-0.50	-0.80	-0.57
sigmoidRMS	0.79	0.00	-0.79	-0.97	-1.00
diffRMS	0.86	0.22	-0.64	-0.91	-0.77
sigmoiddiff	0.83	0.03	-0.80	-0.97	-0.99
area	0.96	0.00	-0.96	-1.00	-1.00
subjective	0.83	0.07	-0.76	-0.96	-0.95
subjective lateralization			-0.92 +- .07		
c_RMS			0.01		
c_sRMS			0.01		
map_overlap			-0.96		
normalized_map_overlap			-1.00		
subjective correlation			0.03 +- .05		



frame # 2 (response field)

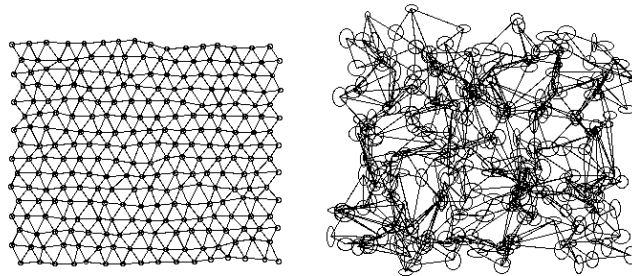
Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.83	0.19	-0.64	-0.91	-0.80
sigmoidRMS	0.80	0.00	-0.80	-0.98	-1.00
diffRMS	0.78	0.05	-0.73	-0.95	-0.97
sigmoiddiff	0.69	0.00	-0.68	-0.93	-1.00
area	0.86	0.00	-0.86	-0.99	-1.00
c_RMS			-0.02		
c_sRMS			-0.03		
map_overlap			-0.86		
normalized_map_overlap			-1.00		





frame # 11 (receptive field)

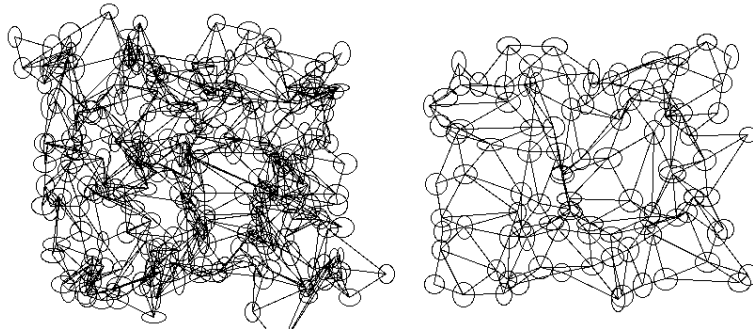
Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.97	0.28	-0.69	-0.94	-0.73
sigmoidRMS	0.97	0.00	-0.97	-1.00	-1.00
diffRMS	0.98	0.17	-0.82	-0.98	-0.87
sigmoiddiff	0.98	0.02	-0.96	-1.00	-0.99
area	1.00	0.00	-1.00	-1.00	-1.00
subjective	0.98	0.03	-0.95	-1.00	-0.99
subjective lateralization			-1.00	+- .00	
c_RMS			-0.05		
c_sRMS			-0.10		
map_overlap			-1.00		
normalized_map_overlap			-1.00		
subjective correlation			0.00	+- .00	



frame # 11 (response field)

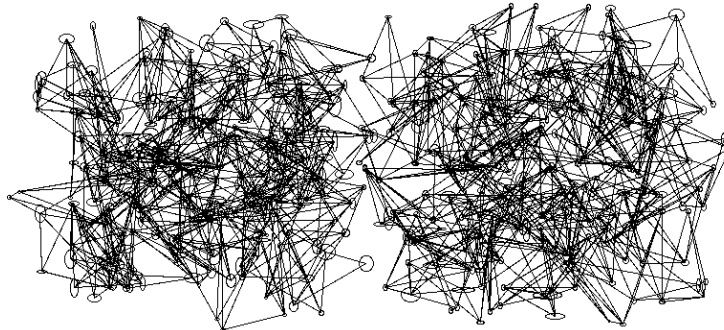
Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.96	0.19	-0.78	-0.97	-0.84
sigmoidRMS	0.97	0.00	-0.96	-1.00	-1.00
diffRMS	0.97	0.06	-0.91	-1.00	-0.97
sigmoiddiff	0.97	0.01	-0.97	-1.00	-1.00
area	1.00	0.00	-1.00	-1.00	-1.00
c_RMS			-0.10		
c_sRMS			-0.19		
map_overlap			-1.00		
normalized_map_overlap			-1.00		

Example 6



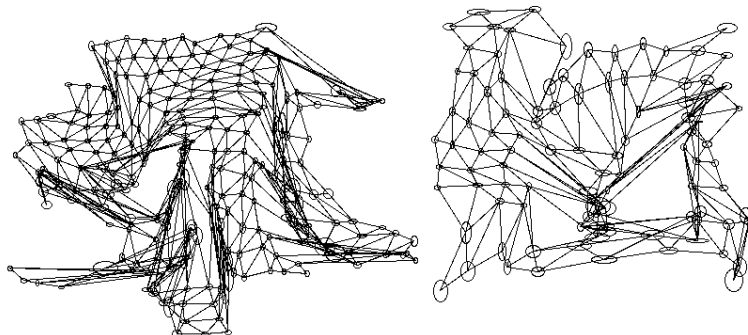
frame # 1 (receptive field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.14	0.60	0.46	0.76	0.80
sigmoidRMS	0.00	0.00	0.00	0.00	0.00
diffRMS	0.07	0.15	0.08	0.17	0.52
sigmoiddiff	0.01	0.15	0.14	0.27	0.97
area	0.01	0.56	0.55	0.84	1.00
c_RMS			0.00		
c_sRMS			0.00		
map_overlap			-0.56		
normalized_map_overlap			-0.99		



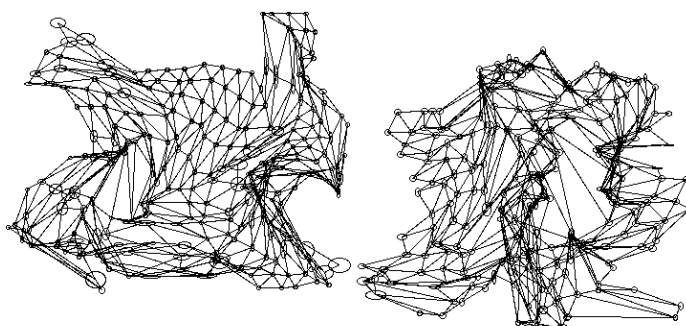
frame # 1 (response field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.00	0.00	-0.00	-0.00	-0.00
sigmoidRMS	0.00	0.00	0.00	0.00	0.00
diffRMS	0.00	0.00	-0.00	-0.00	-0.00
sigmoiddiff	0.00	0.00	-0.00	-0.00	-0.00
area	0.00	0.01	0.01	0.02	1.00
c_RMS			-0.80		
c_sRMS			-0.98		
map_overlap			-0.01		
normalized_map_overlap			-1.00		



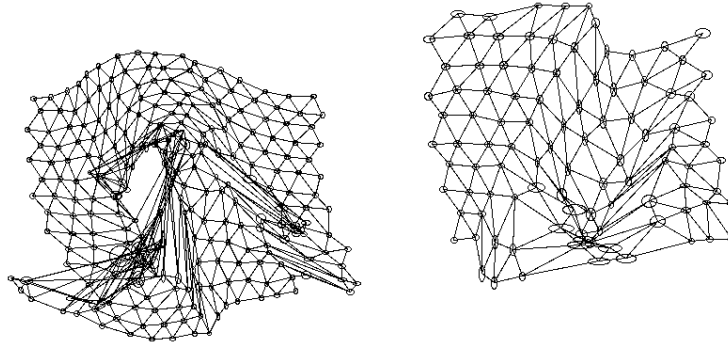
frame # 3 (receptive field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.00	0.12	0.12	0.24	1.00
sigmoidRMS	0.00	0.11	0.11	0.22	1.00
diffRMS	0.02	0.02	0.00	0.01	0.00
sigmoiddiff	0.25	0.22	-0.03	-0.07	-0.10
area	0.34	0.23	-0.11	-0.22	-0.29
c_RMS			-0.34		
c_sRMS			-0.61		
map_overlap			-0.34		
normalized_map_overlap			-0.69		



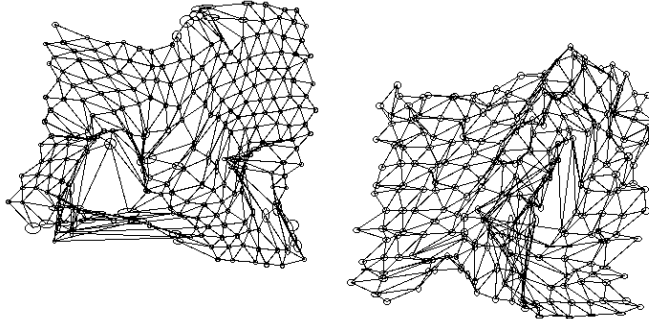
frame # 3 (response field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.00	0.00	0.00	0.00	0.00
sigmoidRMS	0.00	0.00	-0.00	-0.00	-0.00
diffRMS	0.08	0.09	0.01	0.01	0.09
sigmoiddiff	0.16	0.12	-0.03	-0.07	-0.21
area	0.32	0.19	-0.12	-0.25	-0.37
c_RMS			-0.90		
c_sRMS			-0.99		
map_overlap			-0.31		
normalized_map_overlap			-0.70		



frame # 10 (receptive field)

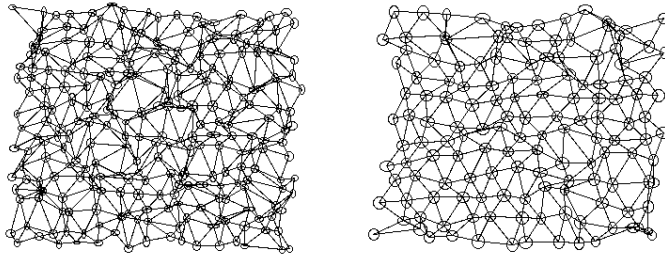
Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.01	0.35	0.34	0.61	0.99
sigmoidRMS	0.06	0.46	0.40	0.69	0.91
diffRMS	0.11	0.27	0.16	0.31	0.59
sigmoiddiff	0.52	0.55	0.04	0.07	0.04
area	0.58	0.44	-0.14	-0.27	-0.20
c_RMS			-0.60		
c_sRMS			-0.88		
map_overlap			-0.16		
normalized_map_overlap			-0.21		



frame # 9 (response field)

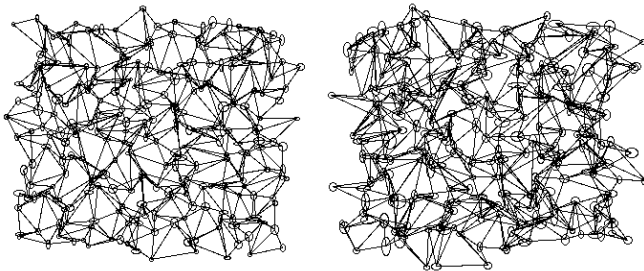
Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.02	0.03	0.01	0.02	0.30
sigmoidRMS	0.13	0.06	-0.07	-0.15	-0.52
diffRMS	0.25	0.31	0.06	0.11	0.16
sigmoiddiff	0.44	0.27	-0.17	-0.33	-0.35
area	0.52	0.27	-0.25	-0.47	-0.46
c_RMS			-0.94		
c_sRMS			-1.00		
map_overlap			-0.41		
normalized_map_overlap			-0.62		

Example 7



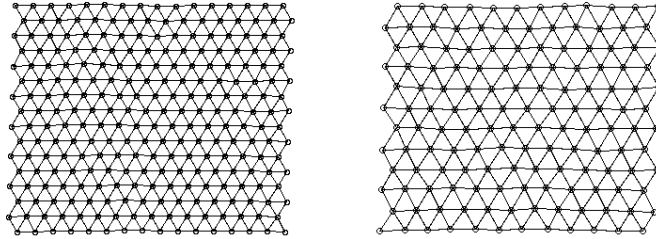
frame # 1 (receptive field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.74	0.85	0.11	0.22	0.10
sigmoidRMS	0.15	0.12	-0.02	-0.05	-0.17
diffRMS	0.63	0.64	0.02	0.04	0.01
sigmoiddiff	0.38	0.70	0.33	0.59	0.43
area	0.51	0.75	0.24	0.46	0.28
c_RMS			-0.01		
c_sRMS			-0.01		
map_overlap			-0.17		
normalized_map_overlap			-0.19		



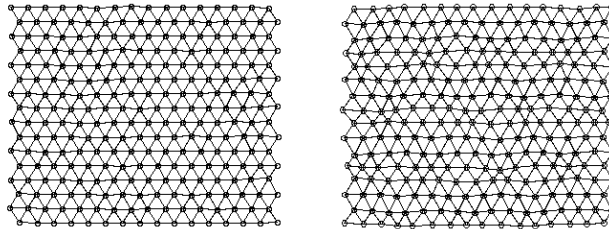
frame # 1 (response field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.48	0.25	-0.23	-0.43	-0.45
sigmoidRMS	0.24	0.05	-0.19	-0.37	-0.83
diffRMS	0.26	0.08	-0.18	-0.35	-0.71
sigmoiddiff	0.05	0.00	-0.04	-0.09	-1.00
area	0.16	0.01	-0.15	-0.29	-0.97
c_RMS			0.00		
c_sRMS			0.00		
map_overlap			-0.17		
normalized_map_overlap			-0.99		



frame # 7 (receptive field)

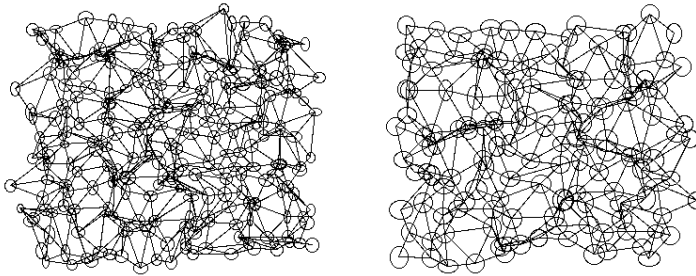
Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	1.00	1.00	-0.00	-0.00	-0.00
sigmoidRMS	0.96	0.99	0.02	0.04	0.02
diffRMS	1.00	0.99	-0.00	-0.01	-0.01
sigmoiddiff	0.99	1.00	0.01	0.01	0.01
area	1.00	1.00	0.00	0.00	0.00
c_RMS			0.34		
c_sRMS			0.61		
map_overlap			1.00		
normalized_map_overlap			1.00		



frame # 7 (response field)

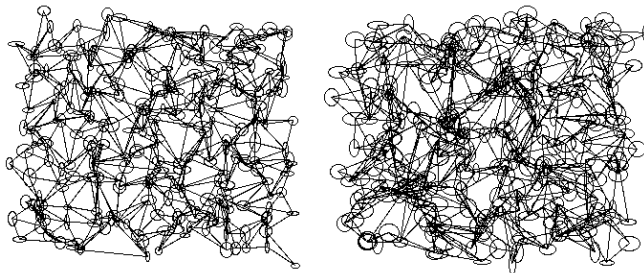
Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	1.00	0.98	-0.01	-0.03	-0.02
sigmoidRMS	0.96	0.86	-0.11	-0.21	-0.08
diffRMS	1.00	0.98	-0.02	-0.03	-0.02
sigmoiddiff	0.99	0.98	-0.01	-0.02	-0.01
area	1.00	1.00	-0.00	-0.00	-0.00
c_RMS			0.29		
c_sRMS			0.53		
map_overlap			1.00		
normalized_map_overlap			1.00		

Example 8



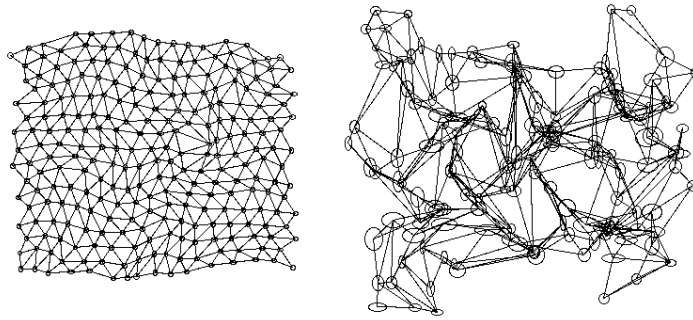
frame # 1 (receptive field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.57	0.71	0.14	0.27	0.16
sigmoidRMS	0.00	0.00	-0.00	-0.01	0.00
diffRMS	0.42	0.39	-0.03	-0.05	-0.06
sigmoiddiff	0.18	0.32	0.14	0.27	0.41
area	0.47	0.56	0.08	0.16	0.13
c_RMS			-0.08		
c_sRMS			-0.16		
map_overlap			-0.26		
normalized_map_overlap			-0.33		



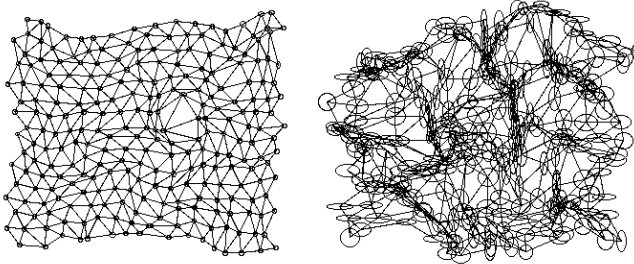
frame # 1 (response field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.34	0.15	-0.19	-0.37	-0.55
sigmoidRMS	0.04	0.00	-0.03	-0.07	-1.00
diffRMS	0.13	0.03	-0.10	-0.20	-0.80
sigmoiddiff	0.01	0.01	-0.00	-0.01	0.00
area	0.21	0.01	-0.20	-0.39	-0.98
c_RMS			-0.29		
c_sRMS			-0.54		
map_overlap			-0.21		
normalized_map_overlap			-1.00		



frame # 11 (receptive field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.80	0.07	-0.74	-0.96	-0.95
sigmoidRMS	0.88	0.00	-0.88	-0.99	-1.00
diffRMS	0.88	0.01	-0.88	-0.99	-1.00
sigmoiddiff	0.86	0.02	-0.84	-0.99	-0.99
area	0.98	0.02	-0.96	-1.00	-0.99
c_RMS			-0.52		
c_sRMS			-0.82		
map_overlap			-0.98		
normalized_map_overlap			-0.99		

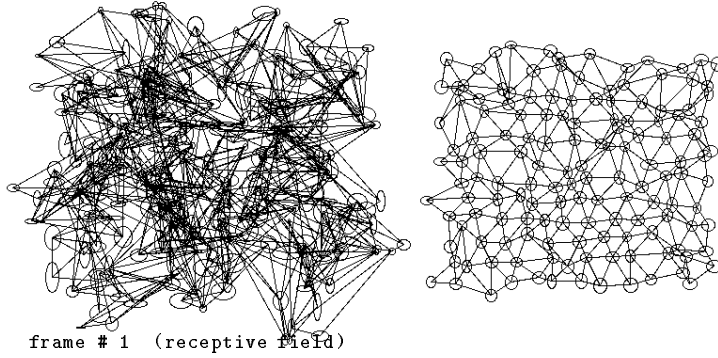


frame # 11 (response field)

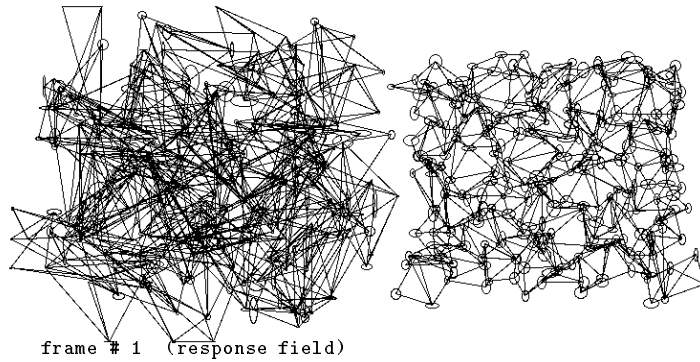
Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.78	0.14	-0.64	-0.91	-0.86
sigmoidRMS	0.87	0.00	-0.87	-0.99	-1.00
diffRMS	0.85	0.26	-0.58	-0.87	-0.71
sigmoiddiff	0.80	0.07	-0.73	-0.95	-0.95
area	0.96	0.00	-0.96	-1.00	-1.00
c_RMS			-0.55		
c_sRMS			-0.85		
map_overlap			-0.96		
normalized_map_overlap			-1.00		



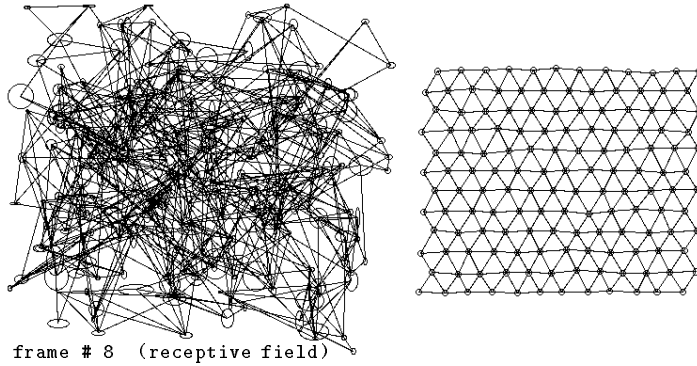
Example 9



Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.00	0.85	0.85	0.99	1.00
sigmoidRMS	0.00	0.07	0.07	0.14	1.00
diffRMS	0.00	0.64	0.64	0.91	1.00
sigmoiddiff	0.00	0.69	0.69	0.94	1.00
area	0.00	0.94	0.94	1.00	1.00
c_RMS			-0.03		
c_sRMS			-0.05		
map_overlap			-0.94		
normalized_map_overlap			-1.00		

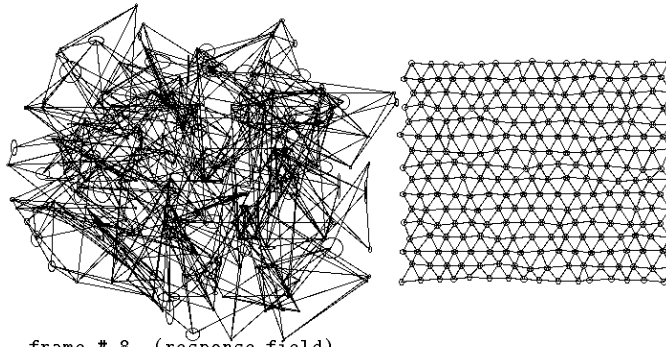


Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.00	0.32	0.32	0.58	1.00
sigmoidRMS	0.02	0.04	0.02	0.04	0.48
diffRMS	0.00	0.12	0.12	0.24	1.00
sigmoiddiff	0.00	0.02	0.02	0.04	1.00
area	0.00	0.02	0.02	0.03	1.00
c_RMS			-0.35		
c_sRMS			-0.62		
map_overlap			-0.02		
normalized_map_overlap			-1.00		



frame # 8 (receptive field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.00	1.00	1.00	1.00	1.00
sigmoidRMS	0.00	0.98	0.98	1.00	1.00
diffRMS	0.00	0.99	0.99	1.00	1.00
sigmoiddiff	0.00	1.00	1.00	1.00	1.00
area	0.00	1.00	1.00	1.00	1.00
c_RMS			0.07		
c_sRMS			0.13		
map_overlap			-1.00		
normalized_map_overlap			-1.00		



frame # 8 (response field)

Measure	org		lat	lat_steep	lat_unbdd
	L	R			
RMS	0.00	0.98	0.98	1.00	1.00
sigmoidRMS	0.00	0.82	0.82	0.98	1.00
diffRMS	0.00	0.97	0.97	1.00	1.00
sigmoiddiff	0.00	0.97	0.97	1.00	1.00
area	0.00	1.00	1.00	1.00	1.00
c_RMS			-0.31		
c_sRMS			-0.56		
map_overlap			-1.00		
normalized_map_overlap			-1.00		