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DETERMINATION OF SOIL STRENGTH CHARACTERISTICS PERFORMING THE PLATE BEARING TEST

Summary: In the international literature there is often confusion between the terms Modulus of Elasticity E_{Young} and Modulus of Deformation E_{Def} for homogenous, isotropic masses, which, sometimes, plainly referred as Modulus E , without any further explanation. In this paper a distinction between the E_{Young} and the E_{Def} is being done and the interest is concentrated in the Plate Bearing Test, which is commonly used in highway earthworks. Moreover, a soil classification diagram is given, in which the soils are classified by their shear strength parameters and their behavior, as totally elastic or elastoplastic, under specific loading applied by rigid circular plate. Finally, a method for the estimation of the soil parameters is proposed, using only the in-situ Plate Bearing Test. The study and the documentation were supported by a presentation of theoretical examples, based on the Finite Element Method.

1. INTRODUCTION

In highway earthworks the knowledge of the Modulus of Elasticity of the soils is very important, whether the projects are in the stage of design, construction or compaction check and this is required to be obtained by an easy manner. Due to the complexity and difficulty in determining the Modulus of Elasticity in the laboratory by the Triaxial Test, which is usually not available on the worksite, the common practice for the designer is to determine the Modulus of Deformation with the in situ and easier Plate Bearing Test. It must be mentioned that, though the soils under load generally have elastoplastic behavior, for the determination of the Modulus of Deformation the elastic theory of Boussinesq is used in every case.

2. BOUSSINESQ'S THEORY

The issue of the vertical displacement under a circular area carrying a uniform pressure on the surface of a semi-infinite, homogenous, isotropic mass is a classical problem in highway engineering. For the solution of this problem, analytical and numerical methods have been proposed. Boussinesq (1885) gave the first complete solution for the settlement of elastic masses'. Under a concentrated vertical load acting in its centerline, the rigid circular plate will settle uniformly (fig. 1). The distribution of the vertical and horizontal principal stresses in the centerline is found by integration, whence the compression stress along z axis of a cylindrical element in the centerline can be computed, thus

$$\sigma_z = \frac{1}{E} \cdot [\sigma_z - \nu \cdot (\sigma_x + \sigma_y)] = \frac{1}{E} \cdot (\sigma_z - 2 \cdot \nu \cdot \sigma_r)$$

and the displacement of a layer extending from $z=0$ to $z=z$ becomes

$$\Delta z = \frac{1}{E} \int_0^z (\sigma_z - 2\nu\sigma_r) dz$$

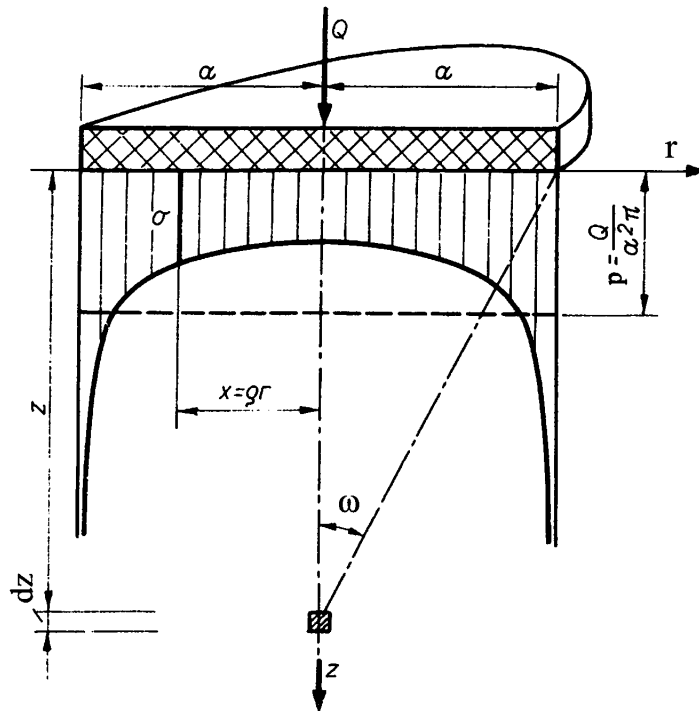


Fig. 1. Contact pressure distribution (p) below a rigid circular plate loaded by a vertical singular force (Q) at the center (Kezdi και Rethati 1988) [1]

Since in the case of rigid, circular plate (Kezdi και Rethati, 1988):

$$\sigma_z - 2 \cdot \nu \cdot \sigma_r = \frac{1}{2}(1 + \nu)p \sin^2 \omega \cdot (1 - 2\nu + 2 \cos^2 \omega) \quad \text{and}$$

$$z = \alpha \cdot \cot \omega,$$

$$dz = -\alpha \cdot d\omega / \sin^2 \omega$$

Whence

$$\Delta z = \frac{1}{2}(1 + \nu) \frac{pa}{E} \int_{-a}^{-\pi/2} (1 - 2\nu - 2 \cos^2 \omega) d\omega =$$

$$(1 - \nu) \frac{pa}{2E} \left[2(1 - \nu) \left(\frac{\pi}{2} - \omega \right) - \sin \omega \cos \omega \right]$$

For semi-infinite, elastic mass, $z \rightarrow \infty$ also $\omega \rightarrow 0$, the total settlement becomes:

$$\Delta z_o = \frac{\pi \cdot p \cdot a}{2E} (1 - \nu^2) \quad (1)$$

where, p : uniform pressure that is applied on the circular rigid plate ($p=Q/\pi\alpha^2$),
 α : the radius of the plate, ν : Poisson ratio and E : the Modulus of Elasticity

3. MODULUS OF ELASTICITY (E_{Young}) AND MODULUS OF DEFORMATION (E_{Def})

The modulus of elasticity, for homogenous, isotropic masses, is given by the general equation $E=\sigma/\epsilon$. The form of this equation is either a matrix, in the case of three-dimensional loading or more simply $E=\sigma_1/\epsilon_1$ in the case of unconfined compression load. When normal axle load is applied on the pavement, soil deformations are not totally elastic but elastoplastic.

The Modulus of Elasticity, which is an index of the material stiffness and a fundamental material constant, can be graphically defined by the slope of the tangent in a stress-strain diagram (tangent modulus), in the area where the material has totally elastic behavior and the displacements are resilient. In Figure 2 the Modulus of Elasticity is the slope of the tangent OA.

An important remark is that the Modulus of Deformation, as it is defined and shown in fig. 2, is dependent on the magnitude of the loading. Graphically it is defined according to the slope of line OB (secant modulus).

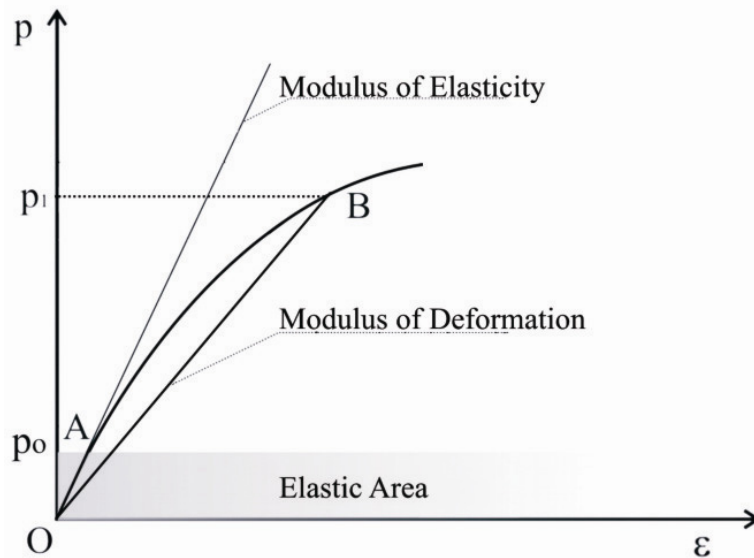


Fig. 2. The Modulus of Elasticity and the Modulus of Deformation [2]

4. PLATE BEARING TEST

The plate bearing test (fig. 3) is intended for use in earthworks and foundation engineering, as well as in the construction of roads and airfields. It specifies a method which is used to check the compactness of soil and provides data for use in the design of road and airfield pavements and foundations. The test permits the relationship between load and settlement (load-settlement curve) to be determined. The aim is the assessment of the deformation and strength characteristics of soil. The plate bearing test is a test in which a load is applied in increments to the soil using a circular loading plate and a loading device, released in decrements and the entire process is repeated. The most common plates that are used for this test in the field are 300, 420, 600 or 760mm in diameter.

Most commonly, the soils under loading have elastoplastic behavior, thus the Modulus E that is derived from the plate bearing test, is the Modulus of Deformation (E_{Def}) and not the Modulus of Elasticity. It is possible for some soils, e.g. for chemically stabilized soils and for small values of loadings, the Modulus of Deformation to be equal to the Modulus of Elasticity ($E_{Def}=E_{Young}$).

The computational part of the plate bearing test uses an equation similar to the equation (1), which is widely used in highway earthworks, in the stage of design or for compaction and deformation check.



Fig. 3. The Plate Bearing Test

5. CLASSIFICATION OF SOILS ACCORDING TO THEIR BEHAVIOR UNDER SPECIFIC LOADING CONDITIONS

The behavior of a soil under specific load conditions (loading p uniformly distributed to the subgrade by a rigid or flexible plate with radius a) is depending mainly on its shear strength values, the cohesion c and the angle of internal friction ϕ . This soil behavior under the above specified conditions can either be totally elastic or more commonly elastoplastic.

In fig. 4 (cohesion – internal friction diagram) the soils are classified in three categories for the loading of 250kPa, distributed by a rigid bearing plate with 300mm diameter. The soils of “Area Ø” of the diagram have ultimate bearing capacity less than 250kPa, those of “Area A” have -for the same loading- elastoplastic behavior and finally those of “Area B” (or alternatively named as “Boussinesq’s Area”), for $p=250\text{kPa}$ the displacements are totally elastic and consequently the Boussinesq’s theory stands.

The “lower curve” of the diagram (fig. 4) corresponds to soils that have ultimate bearing capacity $q_u=250\text{kPa}$. This curve can be easily derived by inserting to the Terzaghi’s bearing capacity Equation [3] $q_u=250\text{kPa}$ and solving this for different values of angle of internal friction ϕ , in order to find the respective cohesion c values.

More recently, De Beer (1970) [4] has proposed empirical shape factors for the calculation of the ultimate bearing capacity, based on extensive laboratory tests in the case of rigid circular plate. The equation has the following form:

$$q_u = c \cdot N_c \cdot \left(1 + \frac{N_q}{N_c}\right) + \gamma \cdot D_f \cdot N_q \cdot (1 + tg\phi) + \frac{1}{2} \cdot \gamma \cdot B \cdot N_\gamma \cdot 0.6 \quad (2)$$

Where, c =cohesion, ϕ =the angle of internal friction, B = diameter of circular rigid plate, D_f = the foundation depth, γ = unit weight of soil, N_γ , N_c , N_q =bearing capacity factors

For highway earthworks the second part of the equation equals zero ($D_f = 0$) and also, the third part has little influence on the ultimate bearing capacity. So, the ultimate bearing capacity depends mainly on the cohesion and the internal friction angle.

The “upper curve” classifies the soils, according to their shear strength values c , ϕ , to those which have totally elastic behavior from those with elastoplastic behavior, always, under the same bearing conditions.

Regarding a soil with shear strength values which are on the “lower curve”, when the load reaches the value of its ultimate bearing capacity, the Mohr circle touches the Mohr-Coulomb criterion and the shear failure of the soil occurs.

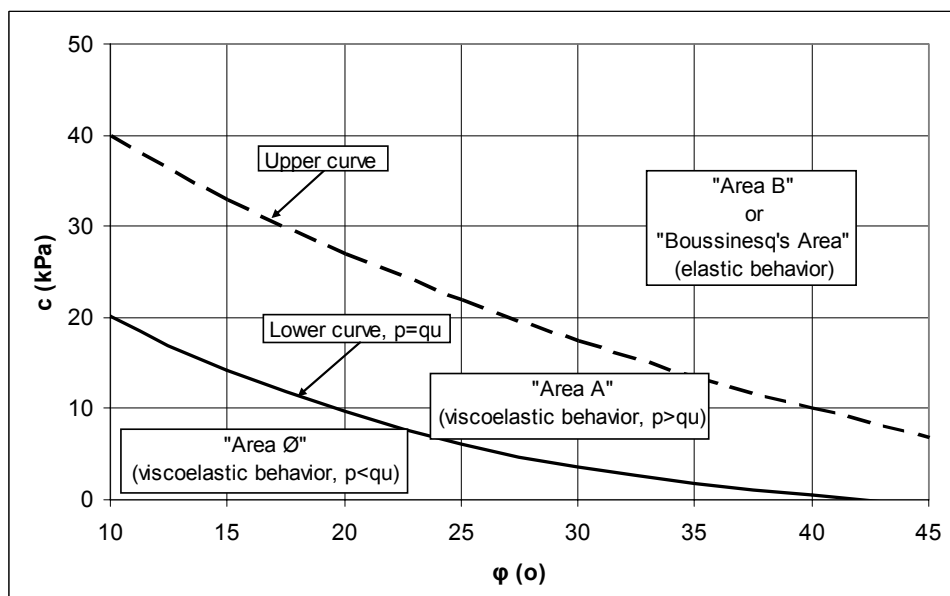


Fig. 4. Classification of soils in three categories (Ø, A and B), for $p=250\text{kPa}$ and for rigid circular loading plate with diameter of $2a=300\text{mm}$

For $p=250\text{kPa}$, a soil with shear strength values $(c, \phi) = (30, 17.5)$, as it is shown in Figure 4, appears to have marginally elastic behavior. Its behavior for $p > 250\text{kPa}$ will be elastoplastic. For the same loading of 250kPa a soil with $(c, \phi) =$

(11, 20) will have high elastoplastic behavior and for a little load increment the shear failure occurs.

An Organic Clay, with shear strength values $c=4\text{kPa}$ και $\varphi=22^\circ$, according to Terzaghi's equation, has ultimate bearing capacity $q_u=127.8\text{kPa}$, for the plate with 300mm diameter. As it is clear from the Figure 4, this soil belongs to "Area Ø".

6. CORRELATION BETWEEN THE E_{Young} AND THE E_{Def} . DETERMINATION OF THE STRENGTH PARAMETERS OF THE SOIL USING THE PLATE BEARING TEST

Usually, the value of the Modulus of Elasticity of a soil is greater than the respective value of the Modulus of Deformation ($E_{\text{Young}} > E_{\text{Def}}$), but in some special cases the two moduli can be equal, e.g. when low loads are applied on chemically stabilized soils.

A theoretical study with a suitable finite element software led to a correlation between the Modulus of Elasticity and the Modulus of Deformation. This correlation is given with the introduction of the coefficient I_L , which is a factor of the shear strength parameters of the soil (c, φ), the radius α of the bearing plate and the magnitude of the load p . Thus,

$$E_{\text{Young}} = E_{\text{Def}} \cdot I_L \quad (3)$$

where $I_L = I_L(c, \varphi, p, \alpha)$ and $I_L \geq 1$

When a soil has elastoplastic behavior, its E_{Def} , is given by the equation:

$$E_{\text{Def}} = \frac{\pi \cdot p \cdot \alpha}{2 \cdot dz} \cdot (1 - \nu^2) \quad (4)$$

If for two successive loading steps p_1, p_2 is $E_{\text{Def}}^{p=p_1} = E_{\text{Def}}^{p=p_2}$ then $E_{\text{Young}} = E_{\text{Def}}^{p=p_1} = E_{\text{Def}}^{p=p_2}$ (e.g. in fig. 2, for $p_1=p_0$ and $p_2=p_0/2$). That is, in this case is $I_L=1$.

The coefficient I_L is given in Table 1, for the bearing plate with the diameter of $2\alpha=300\text{mm}$ and for loads from $p=200\text{kPa}$ up to 500kPa per 50kPa . Since for small loads ($p < 200\text{kPa}$) the measurements with the bearing plate has not good accuracy, the I_L was calculated and shall be used for $p \geq 200\text{kPa}$. If, for very small loads, the accuracy of the measurements were good, the graphical determination of the E_{Young} could be possible, inserting into the Equation 1 a pair of values (p, dz) from the elastic part of the load-settlement curve.

Table 1. The I_L coefficient for the bearing plate with diameter $2\alpha=300\text{mm}$

p (kPa)	$10 < \varphi \leq 23$			$23 < \varphi \leq 45$		
	$I_L = \frac{k}{(\sin \phi)^l \cdot \left(\frac{c - c_o}{c_o} \right)^m}$			$I_L = \frac{k}{(\sin \phi)^l \cdot \left(\frac{c}{c_o} \right)^m}$		
	k	l	m	k	l	m
200	0.296	1.954	$2.037-4.034 \cdot \sin \varphi$	0.535	2.242	$2.464-3.176 \cdot \sin \varphi$
250	0.224	2.234	$2.406-4.938 \cdot \sin \varphi$	0.488	2.417	$2.723-3.475 \cdot \sin \varphi$
300	0.195	2.357	$2.619-5.616 \cdot \sin \varphi$	0.488	2.451	$2.575-3.236 \cdot \sin \varphi$
350	0.216	2.257	$2.507-5.381 \cdot \sin \varphi$	0.480	2.496	$2.647-3.285 \cdot \sin \varphi$
400	0.327	1.898	$1.979-3.686 \cdot \sin \varphi$	0.510	2.376	$2.319-2.786 \cdot \sin \varphi$
450	0.186	2.399	$2.594-5.502 \cdot \sin \varphi$	0.505	2.391	$2.416-2.888 \cdot \sin \varphi$
500	0.146	2.622	$2.773-6.045 \cdot \sin \varphi$	0.483	2.477	$2.443-2.859 \cdot \sin \varphi$
$c_o = 0.03 \cdot p$ and $c_{\min} = 1 \text{ kPa}$						

Since the Modulus of Deformation is strongly dependent on the magnitude of the applied load, different values of Modulus of Deformation, which are derived from different standards, might not be comparable.

The Equation (3) has four unknown parameters, the Modulus of Elasticity E_{Young} , the cohesion c , the angle of internal friction φ and the Poisson ratio ν . If no data are available for the Poisson ratio ν , the value of 0.35 can be assumed. A non-linear system with three equations and three unknowns can be defined, inserting three pair of values (p , dz) from the load-settlement curve. The solution of the system leads to the determination of the unknown parameters. For the definition of the system the proposal method requires the ultimate bearing capacity q_u of the soil to be greater or equal to 300kPa.

If $q_u \geq 350 \text{ kPa}$, various systems can be defined doing combinations with the equations. In this case for more accuracy, the E_{Young} , c , φ values can be the average of the values from some solutions.

Both cases, for $10 < \varphi \leq 23$ and $23 < \varphi \leq 45$, must be examined by the designer and it must be checked whether the value of φ , that has determined, is between that limits. In the opposite circumstance the solution must be rejected.

In Table 2 are presented three examples of determination the strength parameters of three different soils (OL, CL and CH). Moreover, the Modulus of Deformation has been calculated according to the standard DIN 18134 [5]. The DIN 18134 proposed that in order to determine the Modulus of Deformation with the rigid bearing plate of 300mm the load shall be increased until a settlement of 5mm or a normal stress below the plate of 500kPa is reached.

It is clear from the table that the E_{Def} values are much smaller to the respective E_{Young} data values and also, the calculated values of E_{Young} with the proposal method are very close to the data values. That means that this method gives a good approach of the Modulus of Elasticity. The method was based on theoretical examples with suitable finite element software. A small deviation is presumable, but it is expected not too serious. In every case, according to the writer, it gives a better approximation for the real behavior of the soil from the load-dependent E_{Def} . The method also gives an estimation of the c , ϕ values.

Table 2. Examples of calculation the strength parameters for various soils

Soil	Data Values			E_{Def} (MPa) DIN 18134	Calculated Values		
	E_{Young} (MPa)	c (kPa)	ϕ (°)		E_{Young} (MPa)	c (kPa)	ϕ (°)
OL	37.0	10.0	22.0	11.0	35.0	14.2	18.7
CL	55.0	15.0	25.0	18.7	51.7	24.3	19.9
CH	25.0	20.0	15.0	11.2	23.4	32.5	8.6

In the case of determination the Modulus of Elasticity at the worksite, the available data for the designer is the load-settlement curve, derived from the plate bearing test. The designer, in order to verify whether the calculated values are reliable or not, a reproduction of the curve is required, inserting the calculated values into the Equation 3, calculating the displacements and comparing these with the respective data values. Theoretically, the reproduced curve shall be identified with the data curve (fig. 5).

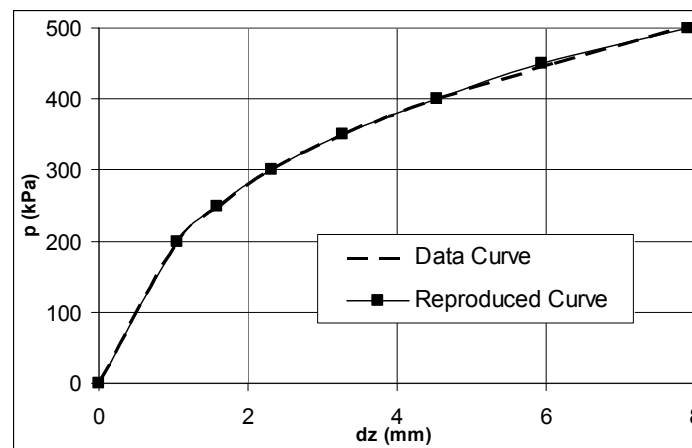


Fig. 5. Verification of the calculated values for the soil of CL (Table 2)

For cohesive soils, which have high sensitivity, the performance of the Triaxial Test in the laboratory with disturbed specimens can lead to totally wrong outcomes regarding the Modulus of Elasticity. So, in every case the proposal method could give a good approach of the Modulus of Elasticity performing the simple Plate Bearing Test.

7. CONCLUSIONS

The accurate determination of the Modulus of Elasticity is necessary for a consistent design and quality control of highway earthworks, since the Modulus of Deformation is strongly dependent on the magnitude of the applied load. Thus, different values of Modulus of Deformation, derived from different standards, might not be comparable. The common practice in the worksite is the determination of the Modulus of Deformation using the Plate Bearing Test, instead of the Modulus of Elasticity by use of the more complex laboratory Triaxial Test. Usually, the Modulus of Elasticity value is greater than the respective value of the Modulus of Deformation ($E_{Young} > E_{Def}$), but in some special cases the two moduli can be equal, e.g. when low loads are applied on chemically stabilized soils. A theoretical study with a suitable finite element software led to a correlation between the Modulus of Elasticity and the Modulus of Deformation. This correlation is given by introducing the coefficient I_L , which is a factor related to the shear strength parameters of the soil (c and ϕ), the radius α of the rigid bearing plate and the magnitude of the load p , that is, $I_L = I_L(c, \phi, p, \alpha)$. The correlation has the form $E_{Young} = E_{Def} \cdot I_L$. A good estimation of the Modulus of Elasticity can be achieved using the above equation with the data that derived from the Plate Bearing Test (load-settlement curve). Additionally, a soil classification has been attempted according to the shear strength parameters of soils and their elastic or elastoplastic behavior, under specific load conditions.

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