FREEZE-OUT IN
ULTRA-RELATIVISTIC HEAVY-ION COLLISIONS*

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I discuss the effects and quantities that influence the decoupling of particles from the fireball. The crucial role is played by the scattering rate. I show the results for the scattering rate at SPS and RHIC and discuss their implications.

Strongly interacting matter is produced in ultra-relativistic collisions of heavy atomic nuclei at SPS and RHIC. The system cools down quickly, however, and hadronic spectra are formed as soon as when interactions disappear. Hence, they carry no direct signals of the early hot, dense and deconfined phase. There are indirect signatures of collective behaviour of strongly interacting matter encoded in spectra, but it is important to disentangle them from features generated in the freeze-out process. Therefore, it is important to understand the freeze-out mechanism if one is interested in the early hot plasma phase.

1. Motivation

Modelling the freeze-out. A particle will decouple from the system when the density drops so low that it does not scatter anymore. One expects that all particles will decouple at roughly the same time, when the density drops below a critical value.

How big is the space-time region where particles are emitted? Cascade generators typically predict that hadrons are liberated from very early times on, during a time interval of up to 50 or 75 fm/c. In spite of that, the standard freeze-out prescription[1] in hydrodynamic simulations assumes all particles to be set free at a specified three-dimensional hyper-surface. I will discuss how good such an idealization is and under which conditions it works.

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In a hydrodynamic model one has to specify the freeze-out condition. A universal freeze-out criterion, valid for systems of all sizes at all energies is desirable. A constant particle freeze-out density has been observed in S-induced reactions at 200 AGeV projectile energy[2]. Its universality was disproved by measurements at different collision energies[3]. The pion phase-space density, also proposed to be universal in the past[4], shows a dramatic increase at RHIC[5]. A criterion using the critical mean free path[3] does not take into account the expansion of the fireball; I will discuss this below.

I approach the problem of a universal freeze-out criterion differently: I will not ask for a condition to be fulfilled by the system, but rather for circumstances under which a particle escapes. The freeze-out condition turns out to depend on the particle momentum. This is close to the cascade-generator understanding of the freeze-out.

Understanding spectra. Hadronic spectra are formed at the freeze-out but they still contain information about the earlier dynamics of the system. The collective transverse flow that develops throughout the whole history of a collision flattens the spectrum and may lead to its concave curvature.

On the other hand, a freeze-out mechanism in which high-$p_\perp$ particles are liberated before the low-$p_\perp$ ones can also make the spectrum concave, since high-$p_\perp$'s would come from a hotter region.

This shows that a good understanding of the freeze-out is necessary for the measurement of the collective flow from the spectra.

2. Freeze-out depends on . . .

. . . chemical composition. When calculating the scattering rate of a test particle, the density is multiplied by the cross-section for scattering on particles from the medium. Densities of different species are always multiplied by the corresponding cross-sections. Thus one concludes that freeze-out is characterized by a critical mean free path (here for pions)

$$1/\lambda_\pi = n_\pi \sigma_{\pi\pi} + n_N \sigma_{\pi N} + n_K \sigma_{\pi K} + \dots . \tag{1}$$

An estimate of the pion mean free path[3,7] at freeze-out gives a value of the order of 1 fm, maybe[7] 2–3 fm.

. . . expansion. If freeze-out was characterized merely by density and chemical composition, the mean free path would have to be comparable with the system size, which is about an order of magnitude larger than the observed λ_π. The expansion of the fireball is crucial[8], however. After the

pion travels the distance λ_π, the density drops and the system is too dilute for another scattering. The mean free path is rather short at the moment of the last scattering, but it is infinite a couple of fermis later.

This mechanism sets a dynamical condition for freeze-out[8]: it occurs when the rate of density decrease is faster than the scattering rate.

. . . particle momentum. The mean free path and the density decrease rate are quantities that characterize the medium. On the other hand, the dependence on momentum[9] is specific for every particle and leads to the construction of a freeze-out criterion that will treat individual particles.

Therefore, I can formulate the freeze-out criterion by saying that a particle decouples if its probability to escape without further scattering is reasonably large[10,11] (say, 0.5).

The sharp freeze-out along a three-dimensional hyper-surface is a special case, in which the escape probability of *all* particles quickly changes from 0 to 1. If, however, the escape probability grows slowly or has a pronounced momentum dependence, the sharp freeze-out approximation becomes questionable.

3. Formalism, calculation, and results

The probability of a particle with momentum p at the space-time point (τ, x) not to scatter anymore is given as[7,9]

$$P(x, \tau, p) = \exp\left(-\int_\tau^\infty d\tau'\, R(x + v\tau', p)\right). \tag{2}$$

The (opacity) integral of the scattering rate $R(x+v\tau', p)$ along the expected trajectory of the particle gives the average number of collisions the particle would suffer if it moved straight.

The density and the chemical composition enter into the calculation of the scattering rate. The expansion dynamics determines its time dependence through the time depenednce of the density.

I show results[7] for the scattering rate at the lower bound of the opacity integral and compare values for SPS and RHIC. Data on the pion freeze-out phase-space density at RHIC[5] show an increase by factor of 2 with respect to the SPS, so it is interesting to compare the two systems and ask how it is possible that pions cease to interact at RHIC in a system denser than that at the SPS?

In calculating[7] the results, a thermal distribution of momenta was assumed and the abundances of individual species were tuned in order to
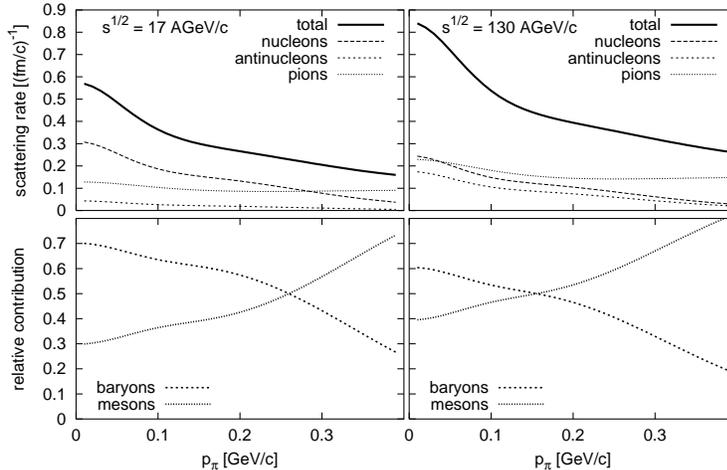
Figure 1. Scattering rate as a function of pion momentum with respect to the medium with $T = 100$ MeV. Results are obtained for SPS (left) and RHIC (right). Contributions to the scattering rate from scattering on pions, nucleons and antinucleons are indicated. The two lower panels shows the baryonic and mesonic relative contributions.

reproduce the observed phase-space densities and ratios of mid-rapidity yields.

Typically, the nucleon contribution to pion scattering is smaller at RHIC as there is less baryon stopping, but it is roughly replaced by scattering on antinucleons (Fig. 1). Scattering on pions is stronger at RHIC than at the SPS, because of the increase in phase-space density, but in neither case does it clearly dominate the scattering rate because of the small $\pi\pi$ cross-section. The dramatic effect of high phase-space density thus has little influence on the freeze-out.

All results are summarized in Figure 2. It is seen that generally the scattering rate strongly depends on the momentum. High-$p_\perp$ particles decouple more easily from the system and may be able to escape earlier when the system is still rather dense. Hence, there is *time ordering* in particle production and it does not seem to be a good approximation to assume that *all* particles are produced at a single freeze-out hyper-surface (as done in many hydrodynamic simulations).

The scattering rate also increases with the temperature, even if the density is kept constant. The escape probabilities for realistic density decrease rates were estimated[7] from (2); it was found that for a temperature of
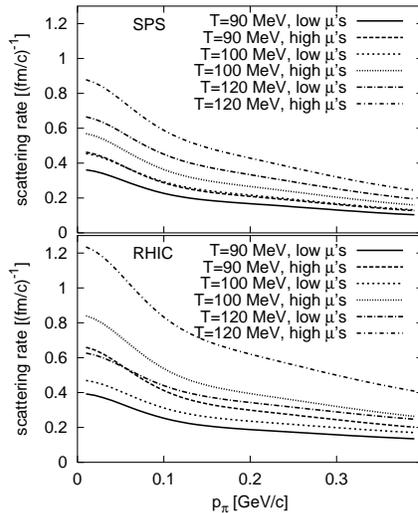
Figure 2. The pion scattering rates as functions of momentum with respect to the medium, calculated for different temperatures and sets of chemical potentials allowed by data. Details can be found in the original paper[7].

120 MeV the chance to escape is about 10% for a particle with momentum above 250 MeV. In order to obtain a reasonable escape probability, say 30–50%, the temperature must drop to about 100 MeV. This suggests a freeze-out at a low temperature.

4. Conclusions

Time ordering of the emission of different momenta may have observable effects on the $K_\perp$ dependence of HBT radii, because high-$p_\perp$ particles would come from a smaller source than the low-$p_\perp$ ones. The bulk of the pions seem to freeze-out rather late, at a temperature of about 100 MeV.

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