

Intersecting Brane Worlds on Tori and Orbifolds

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Abstract: We review the construction and phenomenology of intersecting brane worlds. The breaking of chiral and supersymmetry together with a reduction of gauge symmetry by introducing D-branes at generic angles into type I or type II string compactifications is taken as a starting point for the engineering of phenomenologically appealing four-dimensional vacua. Examples that come very close to Standard Model and GUT physics are considered and issues like the cancellation of anomalies and the emergence of global symmetries studied in some detail. Concerning the gravitational backreaction, the perturbative potential generated at the disc level is shown to lead to a run-away instability of geometric moduli within the purely toroidal compactifications. It can at least partly be avoided in certain orbifold vacua, where the relevant moduli are frozen. These more restricted models still possess sufficient freedom to find semi-realistic brane world orbifolds.

1 Introduction

The central issue in the construction of four-dimensional vacua with interesting low-energy dynamics from ten-dimensional string theory is the breaking or reduction of the various symmetries involved. Upon a Kaluza-Klein reduction on an internal toroidal space the ten-dimensional theories give rise to 16 or 32 supersymmetries, a gauge group of rank 16 and only nonchiral fermion spectra. As a novel approach to make contact with low energy phenomenology within type II or type I string theory, the concept of intersecting brane worlds has been developed. The gauge fields are confined to a three-dimensional subspace in the entirely six-dimensional internal space, the location of D6-branes, and in order to obtain four-dimensional vacua with chiral fermion spectra in semi-realistic gauge groups and with little or no supersymmetry these intersect each other in certain

patterns. The idea dates back, in a dual disguise, to the work [1] which was the first to realize explicitly, that certain magnetic background fields can break supersymmetry, together with chiral and gauge symmetry. It was then embedded into string theory for toroidal and orbifold background spaces [2, 3] and generalized to include also the NSNS B field modulus [4, 5] (see also [6] for a review). The background fluxes, when interpreted as a constant background for open string propagation, induce a modification of the standard Dirichlet boundary conditions, which maps to the boundary conditions of rotated D-branes [7] under T-duality. This duality also implies a mapping between commutative and noncommutative background spaces for open strings ending on Dp -branes with magnetic background fields and such ending on $D(p - 1)$ -branes located at relative angles as well as an identification of symmetric and asymmetric orbifolds [8].

In the resulting picture one deals with intersecting brane worlds on D6-branes, where the effective theory on the noncompact four-dimensional space common to all the D6-branes is determined by the intersection patterns on the internal torus or orbifold. A systematic analysis of the phenomenological properties and the perspectives to engineer a Standard Model in this approach was performed in [2, 9, 5, 10]. It is an interesting feature of these models that the standard model Higgs field can be possibly identified with an open string tachyonic field in a bifundamental representation. The general philosophy of breaking supersymmetry¹ already at the string scale follows the so-called large extra dimensions scenario [12].

In the absence of supersymmetry the cancellation of forces either among the branes themselves and also with respect to their effect onto the background geometry is no longer valid. As a first approximation to the true dynamics one can compute the leading perturbative contribution of the disc diagram to the potential of the background Kähler and complex structure moduli [10]. It turns out that in purely toroidal models the tension of the D6-branes always exceeds that of orientifold planes, if present at all, which implies the impossibility of partial supersymmetry breaking. The potential actually pushes the complex structure moduli to a degenerate limit where the torus is completely squashed. The generic strategy to deal with such situations would be to perform a shifting of the background in the way of a Fischler-Susskind mechanism [13] which is however very demanding and practically not tractable. A more modest method to evade the run-away behavior is given by freezing the relevant moduli through orbifolding. We follow this route by using a \mathbb{Z}_3 orbifold background together with the intersecting brane world approach, where the disc potential for all the moduli, Kähler, complex structure as well as twisted, is flat or fixed. The more severe restrictions which are imposed by the orbifolding still leave enough freedom to construct a class of models very close to the Standard Model, albeit with a number of problems in the Higgs sector.

2 Intersecting D-branes

In this preliminary section we collect the ingredients to the construction of intersecting brane worlds without going into greater detail of their derivation. The starting point is given by the observation that the string spectrum at any intersection point of two D-

¹The techniques of intersecting brane worlds have also been employed to obtain vacua which still preserve $\mathcal{N} = 1$ supersymmetry in four dimensions [11]. The resulting compactifications are believed to have purely geometrical interpretation in eleven-dimensional M-theory given by manifolds with holonomy G_2 .

branes is given by a single fermion of some definite chirality, the ground state in the R sector, together with scalars from the NS sector, which are generically massive, but may also be tachyonic, i.e. of negative mass squared. The tachyonic scalars may be related to Higgs fields which mediate spontaneous symmetry breaking in the effective theory whereas the massive states decouple from the massless sector. All of these fields come in bifundamental representations of the gauge group $U(N_a) \times U(N_b)$ where the N_a and N_b are the respective numbers of branes at the intersection. In addition, on any brane a there is also the usual massless $\mathcal{N} = 4$ vectormultiplet in the adjoint of the $U(N_a)$ from the strings with both ends on that brane.

The classification of supersymmetric brane configurations can most easily be performed by evaluating the Killing spinor equations for any one brane and a second rotated one:

$$\Gamma_{0\dots 6}\epsilon = \tilde{\epsilon} \quad \& \quad R\Gamma_{0\dots 6}R^{-1}\epsilon = \tilde{\epsilon} \Rightarrow R^2 = 1$$

with $R = \exp(\varphi_{ab}\Gamma_{ab}/2)$ the rotation operator and φ_{ab} the relative angles of branes a and b . Supersymmetry is preserved whenever $R \in SU(3)$. This is in a one to one correspondence with the BPS no-force law

$$\mathcal{A}_{ab} = \int_0^\infty dl \langle B_a | e^{-l\mathcal{H}_{cl}} | B_b \rangle \sim (1_{\text{NSNS}} - 1_{\text{RR}}),$$

here written for the balance of attractive and repulsive forces in the annulus amplitude. Therefore, only supersymmetric configurations can lead to globally stable brane worlds, at least if no other background fields are being switched on. As mentioned already, in order to avoid the common problem of stabilizing the hierarchy of gravitational and gauge theoretical scales in the absence of supersymmetry, one needs to take reference to a large extra dimension scenario, where the internal space transverse to the D-branes is large compared to the space along the branes. As has been noticed in [2] in the purely toroidal approach, there is no overall internal transverse space that could be chosen large in order to realize such a situation, such that one needs to depart from the flat background to do so. We shall later mention the opportunity to blow up orbifold fixed points as one such possibility. In any case one still needs to consider the stability of the geometrical hierarchy of volumes, once fixed at tree level, when perturbations are taken into account, as well as the appearance of open string tachyons.²

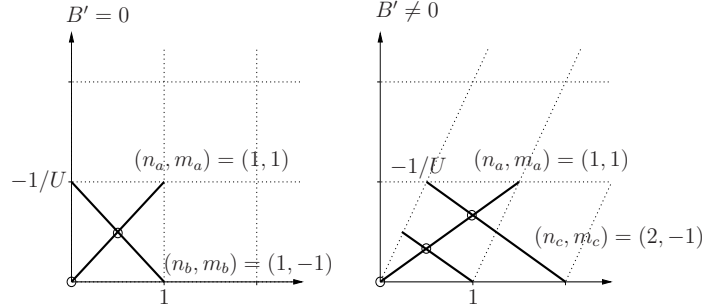
The above completely generic features of the construction get slightly modified and extended when turning to type I (or type I' after the T-duality) string theory. One needs to add orientifold O6-planes and gets additional contributions to the Euler characteristic zero amplitude via the Klein bottle and Möbius strip diagrams. As well, one needs to take into account that the world sheet parity reflects the magnetic background fields of type I, which translates into a geometric reflection symmetry $\Omega\mathcal{R}$ acting as $\varphi_a \longleftrightarrow -\varphi_a$ on the relative angles of the D-brane spectrum.

3 Intersecting brane worlds

Upon compactifying a setting of intersecting D6-branes on a six-dimensional internal torus $\mathbb{T}^6 = \bigotimes_{I=1}^3 \mathbb{T}_I^2$, factorized into two-dimensional ones with Kähler moduli T^I and complex

²See also [14] in this context.

structure moduli U^I , any such D6-brane wraps a special Lagrangian 3-cycle given by a line on any single \mathbb{T}_I^2 . The homology class of the 3-cycle is then defined by combining those of the 1-cycles, i.e. by three sets of two integers, one set for each \mathbb{T}_I^2 , as being depicted for two examples of tori with different complex structures in the figure below.



The number of chiral fermions χ in the $(\mathbf{N}_a, \mathbf{N}_b)$ representation is given by the intersection number

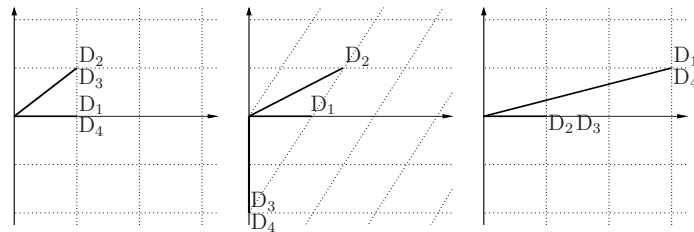
$$\chi \text{ in } (\mathbf{N}_a, \mathbf{N}_b) : \quad I_{ab} = \prod_a (m_a^I n_b^I - m_b^I n_a^I)$$

of the two respective cycles. A crucial step in the development of semi-realistic type I models of this kind was to realize that the intersection numbers depend on the value of $\Re(U^I) = 0, 1/2$, odd numbers being accessible only with non-vanishing $\Re(U^I)$ [5]. Along with the appearance of chiral fermions one needs to address the consistency requirements of anomaly cancellation in the effective theory, which lifts to the cancellation of RR charge in string theory. An explicit computation of the conditions that result in the different theories can be found in the references. The conditions which emerge for instance for type I strings on a purely toroidal background are given by [2]

$$\sum_a N_a n_a^1 n_a^2 n_a^3 = 32, \quad \sum_a N_a n_a^1 m_a^2 m_a^3 = \sum_a N_a m_a^1 n_a^2 m_a^3 = \sum_a N_a m_a^1 m_a^2 n_a^3 = 0.$$

They have been shown to imply the cancellation of all irreducible contributions to the anomalies in four dimensions.

As an example for a solution with interesting features, we present a unified extension of the Standard Model, a left-right symmetric Standard Model in a type I intersecting brane world. The brane setting can be read off from the following figure



The RR charges cancel for the choice $N_1 = 3$, $N_2 = N_3 = 2$, $N_4 = 1$ for the respective multiplicities of branes, leading to the gauge group

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1).$$

One needs to take care that of the originally four $U(1)$ factors only two are free of anomalies, whereas the other two decouple at the string scale, due to Green-Schwarz type interactions. The spectrum of massless fermions is tuned to produce the desired set of three

generations of quark and lepton doublets, as is usually considered within GUT models of this kind.

Many of the phenomenological properties of the effective theory can be discussed without explicit reference to a particular model. The gauge coupling constants can be obtained from ten dimensions by a straightforward dimensional reduction along the world volume of the respective brane from the string coupling, i.e.

$$\frac{1}{g_a^2} \sim \frac{1}{g_S} \frac{\text{vol}_a}{l_S^3},$$

where vol_a is the volume of the brane a . These extra relative factors may change the unification patterns compared to standard field theory results. The Yukawa couplings involve fields that live at different intersection loci on the internal space and therefore arise from string diagrams with world sheets that stretch between different branes. The coupling strength can then be estimated classically by the minimal area, e.g. $\exp(-A_{\bar{\psi}\psi H}/l_S^2)$. These interactions are suitable to generate mass terms for the nonchiral part of the matter spectrum, such as the fermions in the $\mathcal{N} = 4$ vectormultiplets, which gain masses

$$M_{\bar{\psi}\psi}^2 \sim g_{\text{Yu}} \langle H \rangle \sim l_S^{-2}$$

of the order of the string scale and decouple. It was argued in [9] that there may be a specific signature in this class of large volume compactifications that may be able distinguish them from other kinds of models, excitations of fields localized at the intersections:

$$(\alpha_{-\delta_{ab}})^n |0\rangle \quad \text{with} \quad M_n^2 \sim nq \langle \delta_{ab} \rangle = \frac{nq}{\pi} \langle \varphi_{ab} \rangle \stackrel{?}{\ll} l_S^{-2}.$$

The appearance of anomalous $U(1)$ symmetries, cured by a suitable version of the Green-Schwarz mechanism, presents a convenient source for global symmetries. For instance, in type I string theory, there are four axionic scalars that derive from the RR forms $C^{(p)}$:

$$4d \text{ axions} \quad a^I = \int_{\mathbb{T}_I^2} C^{(2)}, \quad a^0 = \int_{\mathbb{T}^6} C^{(6)}.$$

The four-dimensional couplings relevant for the GS mechanism descend from $C^{(2)} \wedge \text{tr } F^4$ and $C^{(6)} \wedge \text{tr } F^2$, leading to an anomalous contribution schematically given by

$$N_a I_{ab} \sum_{\alpha} (F \wedge *a^{\alpha}) \times (a^{\alpha} F \wedge F).$$

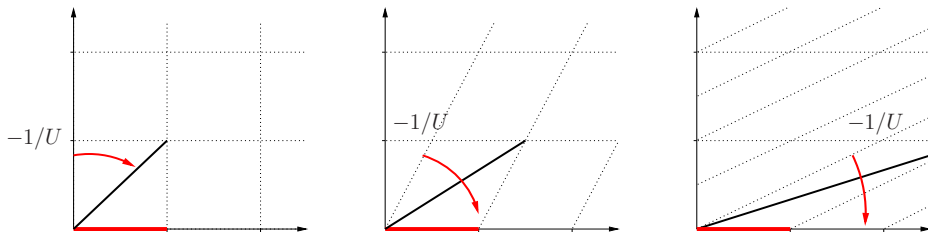
The coefficient is just correct to match up with the contributions of the chiral fermions in order to make the GS mechanism work out. Those gauge bosons which belong to anomalous $U(1)$ gauge symmetries acquire mass terms of the order of the string scale, i.e. up to four abelian factors decouple, but they survive as global symmetries of the theory. Actually, one needs to be even more careful here and distinguish the abelian and mixed anomalies and also regard that non-anomalous gauge fields may get massive. In any case, this set of global symmetries may rather generically contain baryon- and/or lepton-number conservation, which prevents proton decay, otherwise mediated via dimension six four-fermion terms $l_S^2 (\bar{u}_L^c u_L) (\bar{e}_L^+ d_L)$ in the effective action. In scenarios with low string scale, these would not be sufficiently suppressed, which usually is a major problem.

4 Gravitational stability and stabilization

So far, we have ignored the problem of the stability of our brane worlds. Now we proceed to consider the backreaction of the torus when the branes on top are arranged in a nonsupersymmetric way [10]. From the annulus diagram one can obtain the disc tadpoles $\langle\phi\rangle_{\text{disc}}$ and $\langle U^I\rangle_{\text{disc}}$ of the dilaton and complex structure moduli fields by taking the limit of an infinitely long and thin tube. This allows to deduce the scalar potential to this order of perturbation theory:

$$V(\phi, U^I) = e^{-\phi} \left(\sum_a N_a \prod_I \sqrt{(n_a^I)^2 \Im(U^I) + \frac{(m_a^I + \Re(U^I)n_a^I)^2}{\Im(U^I)}} - 16 \prod_I \sqrt{\Im(U^I)} \right).$$

It is just the difference of the internal volumes of all the D6-branes and that of the orientifold O6-planes, which is also what one would have expected as the result of the Dirac-Born-Infeld effective action, evaluated for the present constant background. There is no potential for the Kähler moduli at this order, which is consistent with the fact that they do not couple to the boundary state that describes the D6-branes in the world sheet CFT. First of all, one can easily realize that there are no supersymmetric vacua where the potential vanishes together with its derivative. The volume of the D-branes is always larger than that of the orientifold planes. In fact, the potential does not allow for any nontrivial minima at all, but pushes the complex structure moduli $\Im(U^I)$ to infinity, while $\Re(U^I)$ is frozen anyway. Intuitively speaking, the D-branes are dragged towards the orientifold planes, and pull the entire torus to the degenerate limit, where the ratio of the radii diverges at constant volume. In the original T-dual type I picture this is the decompactification limit of the internal space.



The most generic strategy to deal with the tadpoles for the dilaton and the moduli would be to follow the Fischler-Susskind mechanism [13], which is however not practicable and very few examples are known where even only the first step can be performed [15]. A more modest approach consists in freezing the relevant moduli by imposing a certain discrete symmetry on the background via orbifolding. More precisely, a \mathbb{Z}_3 orbifold of type I theory allows only the two distinct values $U^I = (1 + i\sqrt{3})/2$ or $(1 - i\sqrt{3})/2$ for the complex structure modulus. The orientifold compactification relevant here is based on a type IIA orbifold and has been constructed in [16]. Interestingly, there are no twisted tadpoles generated and therefore the potential is entirely flat except for the dilaton that runs away to zero coupling:

$$V(\phi) = e^{-\phi} \cdot \text{const.}$$

The fact that the branes do not couple to the twisted moduli that parametrize the blowing-up of the singularities suggests a nice realization of a large extra dimension scenario, where the internal volume transverse to the branes appears when such a singularity is blown up. Of course, one has to expect a potential for these moduli from higher loop

corrections and, thus, this argument remains rather heuristic. In an explicit construction of such models³ one now needs to employ D-branes intersecting in patterns invariant under the additional \mathbb{Z}_3 rotation. Thus, any single D-brane is a representative of an entire orbit of equivalent branes, generically of length six, sometimes only of length three. One can very efficiently redo the former toroidal analysis of the chiral spectrum and the RR charge cancellation constraints by introducing effective winding numbers (z_a, y_a) which summarize the contributions of the individual branes into brane orbits. The only condition one finds is

$$\sum_a N_a z_a - 2 = 0.$$

It actually does not prevent a rank of the gauge group larger than two, as opposed to the supersymmetric solutions of [16]. A promising example for a model of this type that resembles the Standard Model with respect to gauge group and spectrum very closely is defined by three stacks of D6-branes with $(z_1, y_1) = (z_2, y_2) = (1/2, 3)$, $(z_3, y_3) = (-1/2, 3)$. It has gauge group

$$SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

after applying the GS mechanism and a fermion spectrum identical to the Standard Model with right-handed neutrinos. The Higgs fields, necessary to break $U(1)_{B-L}$ as well as $SU(2)_L \times U(1)_Y$, should correspond to open string tachyons with the correct gauge quantum numbers. By a systematic computer search among all the $4 \cdot 36^3$ solutions with these characteristics, which we were able to find, there were a couple of hundred models having either a standard model tachyonic Higgs in the $(\mathbf{1}, \mathbf{2})$ representation or a Higgs in the ‘symmetric’ representation of $U(1)_{B-L}$. But no model contains both Higgs fields. As an additional problem the global symmetry, which originates from the single anomalous and decoupled $U(1)$ gauge factor prevents the coupling $H \overline{Q}_L u_R$ needed to give mass to some of the quarks during the electroweak symmetry breaking phase transition. Therefore, we can finally only refer to some version of an exotic Higgs mechanism possibly involving a composite field that takes over the role of the missing elementary scalar.

In any case, the intersecting brane world approach offers a rich phenomenology with convenient tools for engineering vacua with many attractive features. We have also shown here that it is possible to address the very serious issue of stabilizing nonsupersymmetric brane world vacua by a perturbative CFT approach based on orbifold techniques. This work is surely not complete and many more perspectives remain for the future.

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³For proper definitions and a presentation of the technical details of this subsection see [10].

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