Trading under ambiguity and the effect of learning

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Abstract

In this experimental study, I investigate the effect of ambiguity on investors’ willingness to trade under different information conditions. The results confirm the prediction of a wide set of theoretical models, that ambiguity aversion reduces willingness to trade in incomplete markets. Participants choose significantly wider bid-ask spreads when return distributions are ambiguous rather than objectively known. This effect also persists when subjects learn probabilities progressively. However, in the latter case, different information generates more divergent quotes. The more extreme quotes are consistent with a particular updating rule, which is conditional smooth preferences. These findings highlight the role of gradual information release for market under- and overreaction in ambiguous markets.

JEL-Classification: G02, G11, G14, C91, D81, D82
Keywords: learning, ambiguity, market participation, experiment, maximum likelihood updating, full Bayesian updating, conditional smooth preferences.

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1 Introduction

The link between ambiguity and trading volume is not obvious. Ambiguity is a fundamental feature of financial markets where objective probabilities of the states of nature are often unknown. The extent to which traders perceive ambiguity determines whether ambiguity begets more or less trading activity. Speculative trade increases, for instance, when ambiguity enables investors to develop subjective beliefs that are sufficiently heterogeneous. Several studies that find a positive correlation between trading volume and measures of uncertainty support this hypothesis (see [Karpoff, 1987] for an extensive survey of this literature).

In contrast, trading volume falls if investors dislike events with unknown probabilities. In that case, investors who already have state-invariant positions will not re-allocate their portfolio. In incomplete markets, staying out of the market might even be the sole possibility to avoid ambiguity. Diminished willingness to trade can therefore be rationalized with ambiguity-averse preferences. Various models of ambiguity aversion (e.g. Choquet expected utility (CEU), maxmin expected utility (MEU), $\alpha$-maxmin expected utility ($\alpha$-MEU)) depart from expected utility theory by modeling decision-makers who consider different distributions for opposite actions: one for going long and one for going short. The ambiguity-averse seller short-sells at higher prices, the ambiguity-averse buyer displays a lower willingness to pay. In between, there is a range of prices at which buyer and seller do not agree on trade. In line with this rationale, [Antoniou et al., 2015] find equity flows to be negatively related to ambiguity.

Thus, the different movements in trading volume can be reconciled if one acknowledges that only ambiguity perceived as such impedes trading ([Dimmock et al., 2016]). While ambiguity-averse traders conceive a set of probability distributions, traders with subjective beliefs do not perceive any ambiguity. The extent to which traders perceive ambiguity is, however, likely to vary with incoming information. As ambiguity often triggers enhanced information acquisition, understanding traders’ reactions requires first examining how they process information under ambiguity. The way investors
update ambiguous beliefs upon incoming information defines their perception of ambiguity and, hence, their willingness to trade.

This paper offers a systematic comparison of willingness to trade assets with ambiguous and unambiguous return distributions, in a stylized incomplete market with one uncertain asset and money. To examine the relevance of learning, the experiment studies investment decisions under ambiguity across two information conditions: one where investors base their decisions on given probabilities; and a second where investors receive additional information before investing. Two main updating rules serve as theoretical benchmarks: *full Bayesian updating* (henceforth FBU, Jaffray 1989; Pires 2002) and *maximum likelihood updating* (henceforth MLU, Gilboa and Schmeidler 1993). With FBU, subjects update a set of priors prior by prior and evaluate the resulting set of posteriors according to their ambiguity preferences. With MLU, on the other hand, subjects consider a subset of priors that maximizes *ex-ante* the probability of receiving the observed information. Additional information leads an agent to discard unlikely priors and to perceive substantially less ambiguity. Eventually, he will come to a single posterior belief and will not perceive any ambiguity at all. In this way, the arrival of information may generate a singleton posterior, which depends on the nature of the arrived information and, therefore, might be heterogeneous across agents.

The experiment is designed as an individual decision-making environment where subjects cannot learn from the market. In addition, any risk-sharing motive to trade is excluded because subjects start with a riskless position. A 2x2 design allows comparing decisions across two dimensions. The first dimension varies the degree of uncertainty by comparing decisions under risk versus ambiguity. The second dimension distinguishes between situations where information about return distributions is released at once and those where information is processed sequentially. The design is implemented with 2 treatments, such that the first dimension of variation is analyzed in a within-subject comparison and the second dimension between subjects. Treatment “No Learning” (NL) investigates the relation between ambiguity and investment decisions when belief updating is not required. Treatment “Learning” (L) examines the ambiguity effect when traders receive additional information.
information after prior distributions are specified.

In the two treatments, participants submit a bid and an ask quote for an uncertain asset. In some rounds, participants learn the objective probability distribution of the asset’s value and invest, thus, in a risky asset. In other rounds, they receive imprecise information about the distribution, which makes the latter ambiguous. While in treatment NL information about the distribution is revealed at once, participants in treatment L learn the distribution across two stages: they first receive information about a prior distribution, and observe then an additional signal.

One main result is that participants express a lower willingness to trade by choosing significantly wider bid-ask spreads when returns have ambiguous distributions. The effect of ambiguity aversion even persists when subjects learn distributions progressively. This result adds to the evidence of ambiguity aversion found in a multitude of Ellsberg experiments (i.a. Chow and Sarin [2002], Halevy [2007], and Camerer and Weber [1992] for a review of the literature). It shows that ambiguity aversion manifests itself in spreads when portfolio re-allocation is not possible. The average ambiguity premium in long and short positions amounts to 20% and 16.4% of the expected value, respectively, and is in line with previous findings (Yates and Zukowski [1976], Bernasconi and Loomes [1992], and the references in Camerer and Weber [1992]). The ambiguity premium over and above the risk premium cuts down trade by, on average, 12 percentage points, and mean profits by 30%. These findings confirm that ambiguity aversion is well suited to model freezes in trading activity.

A second main result is that learning generates more extreme quotes. Yet, there is no evidence of subjects being predominantly MLU agents. MLU predicts small to zero spreads, but subjects choose the same average spread when the same ambiguous distribution is learned progressively. However, the evidence in favor of FBU is limited: bids and asks are significantly lower (higher) after the arrival of a low (high) signal. The chosen quotes are consistent with updating second-order beliefs about ambiguous probabilities. A bulk of 36.87% decisions for ambiguous prospects is centered around Bayesian updates of the mid-prior. The remainder of quotes discloses heterogeneity in
the way of updating ambiguous beliefs. One noticeable group is insensitive to additional information and refrains from trading. Another major group consists of extreme updaters who choose to trade at all prices.

In sum, the results identify a negative relation between ambiguity and willingness to trade that is robust to the information condition. The relation between ambiguity and trading volume, though, is not conclusive since a lower willingness to trade does not directly translate into lower trading volume. Despite ambiguity-averse trading preferences, differential information combined with Bayesian updating of recursive preferences may generate updated beliefs that are divergent enough to spawn speculative trading. This effect of learning matters, in particular, because the link between ambiguity and liquidity is characterized by feedback effects. Traders who do not perceive ambiguity nurture liquidity, which, in turn, encourages price discovery. In contrast, whenever ambiguity engenders a drop in liquidity, prices fail to aggregate information and are more prone to excess volatility (Dow and Wergeland 1992b; Guidolin and Rinaldi 2010). In that case, ambiguity produces market frictions that may persist over longer periods. The experimental data suggest that gradual information processing can mitigate ambiguity effects if quotes become sufficiently heterogeneous.

Moreover, subjects’ more extreme reactions with gradual information release have direct implications for discretionary disclosure policy. Miller (2002) and Kothari et al. (2009) find evidence for an asymmetric disclosure of bad and good news: while managers disclose good news immediately, they accumulate bad news before releasing them. The experimental findings indicate that the asymmetric disclosure has effects beyond the one of supporting managers’ careers: it possibly dampens negative, but fosters positive stock price reactions.

This paper relates two strands of research. One strand examines the effect of ambiguity on market parameters. In theoretical models of market microstructure, for instance in Cao et al. (2005); Ui (2011) and Easley and Hara (2010), ambiguity aversion is used to model limited market participation. In line with this theoretical literature, this paper uses a stylized decision experiment to test the hypothesis that market participation decreases with
ambiguity. Two other experimental studies analyze the effects of ambiguity on financial decisions. Ahn et al. (2014) individual-decision experiment confirms the heterogeneity in ambiguity attitudes, providing evidence for subjective expected utility (SEU), ambiguity aversion as well as for pessimism. Bossaerts et al. (2010) show in their market experiment that heterogeneity in ambiguity attitudes does not only affect portfolio choices, but also asset prices. Standard price predictions do not hold and prices do not aggregate beliefs if only the least ambiguity-averse traders provide the total supply of ambiguous assets. In contrast, the design in the present experiment identifies ambiguity aversion not through portfolio allocation but with chosen spreads. It focuses on individual willingness to trade and, thus, extends the study of ambiguity aversion to markets that do not provide the opportunity to fully insure against ambiguous states. A related study is Sarin and Weber (1993). They find bids and the resulting market prices for ambiguous assets to be consistently lower in sealed-bid and oral double auctions, although ambiguous and unambiguous assets had identical expected payoffs. As they conclude, subjects are less willing to pay for ambiguous assets that they apparently consider as more risky. Another related work is the experimental study of Eisenberger and Weber (1995). They find no interaction between ambiguity and the buying/selling price ratio. As their focus lies on the buying/selling price ratio, willingness to pay and willingness to accept are elicited from different default positions. This study, in contrast, focuses on the individual willingness to trade by keeping the starting position constant and state-invariant. This allows for testing the prediction made in Dow and Werlang (1992a) under varying conditions. Another strand of the literature analyzes belief updating under ambiguity. The current work contrasts from Epstein and Schneider (2008), which models updating of ambiguous information. Instead, this research evaluates belief updating of ambiguous priors when information is precise. Cohen et al. (2000) use in this context a dynamic extension of the Ellsberg experiment to

\[\text{Note, in their oral double auctions subjects are endowed with assets. In that case, ambiguity-averse traders want to get rid of their uncertain endowment and drive down the offer price.}\]
differentiate between FBU and MLU behavior. They, too, find heterogeneity in updating behavior. The behavior of a non-negligible amount of subjects is consistent with MLU, but FBU seems to be the more predominant updating rule in their implementation of the Ellsberg-experiment. The current paper emphasizes the importance of these two updating rules for trading activity and provides another framework to distinguish between them. One related experiment also studies learning in ambiguous asset markets: [Baillon et al. (2013)] investigate learning with a natural source of uncertainty. In their individual decision-making design, subjects submitted ask prices for options on initial public offerings (IPOs). Using the neo-additive model ([Chateauneuf et al. 2007]), they find no evidence for pessimism (ambiguity aversion). Furthermore, whereas pessimism is not affected by the arrival of new information, sufficient information reduces likelihood insensitivity. The following experiment adds to this literature and contrasts markets with ambiguity shocks and ambiguous markets with gradual information release. Moreover, it compares learning in ambiguous markets to learning in risky markets to identify learning effects that are specific to ambiguity.

The paper is organized as follows. Section 2 presents the stylized decision model and the theoretical predictions. Section 3 describes the implementation in the experiment. The results are presented in Section 4. Section 5 discusses their implications and concludes.

2 The theoretical framework

2.1 Investing in ambiguous versus risky prospects

2.1.1 A stylized decision problem

Consider a simple investment opportunity in a market with two states and one risky asset. The investor may invest in one unit of the risky asset with value $V \in \{V_L, V_H\}$. The probability for the high-value state corresponds to $Pr(V = V_H) = \pi$.

The investor is endowed with cash $W_0$ and tenders both a bid quote, $b$, and an ask quote, $a$, before knowing the transaction price, $p$. The price $p$
is exogenous and is drawn from a uniform distribution, i.e. \( p \sim U[V_L, V_H] \).

The agent’s demand corresponds, thus, to:

\[
X = \begin{cases} 
+1 & \text{if } p \leq b \\
-1 & \text{if } p \geq a \\
0 & \text{otherwise}
\end{cases}
\]

The agent is a price taker: at the end, he will pay a price \( p \) that he cannot influence and that will possibly differ from his quotes \( b \) and \( a \). The quotes \( b \) and \( a \) merely determine the probability that a buy or a short-sale (henceforth sell) occurs. A higher bid \( b \), for instance, increases the probability to buy, as the random price \( p \) is more likely to fall below it. Notice that the agent will always trade whenever the bid equals the ask. The investor’s wealth at the end of the period is \( W_1 = W_0 + (V - p) \cdot X \).

2.1.2 Predictions

Denote \( \Pi^* \) as the agent’s subjective set of beliefs about \( \pi \), the probability for the high-value state.

For the benchmark analysis of expected utility, assume that the agent holds a single probability belief \( \pi \), i.e. \( \Pi^* \) is a singleton. If the agent is risk-neutral, then he buys at prices below his expected valuation, sells at prices above it, and therefore sets \( a^* = b^* = E[V] \). A risk-averse agent, on the other hand, chooses a strictly positive spread between bid and ask, with \( b^* < E[V] \) and \( a^* > E[V] \) (the simple proof is in the Appendix B.1).

Optimal values of bid and ask may change when the agent perceives ambiguity about \( \pi \). That is, if he contemplates an interval of probabilities \( \Pi^* = [\pi_l, \pi_h] \), bid and ask quotes adjust to his ambiguity preferences. Different models of ambiguity aversion will then predict different trading quotes. The present argumentation follows Dow and Werlang (1992a), but uses the intuitive model of maxmin expected utility (MEU - Gilboa and Schmeidler 1989) instead of Choquet expected utility.

An MEU agent evaluates different actions with different probability dis-
tributions. He considers the worst possible expected outcome, which differs between the cases where he buys and sells. A risk-neutral MEU agent buys if

\[ p \leq \min_{\forall \pi \in [\pi_L, \pi_H]} E[V|\pi]. \]

He sells if

\[ p \geq \max_{\forall \pi \in [\pi_L, \pi_H]} E[V|\pi]. \]

Thus, the expected payoff functions of ambiguity-averse buying and selling strategies are shifted downwards, relative to the case of expected utility (see Figure 1). Due to the fact that willingness to buy and willingness to sell do not intersect at a single strictly positive price, there is a region of prices at which zero holding of the asset is optimal (Dow and Werlang).
Given risk-neutrality, models of ambiguity aversion with kinked preferences predict wider spreads for ambiguous than for unambiguous prospects. Predictions under risk aversion, however, depend on the preferences and can vary widely. Importantly, for smooth ambiguity preferences like those studied in Klibanoff et al. (2005) (henceforth KMM), the spread converges to the spread of an expected utility maximizer when ambiguity aversion converges to neutrality. The validity of smooth preferences is discussed in Section 4.3. The main objective of the experiment is not to identify kinked from smooth preferences, but to generally compare spreads for ambiguous and unambiguous assets. Differences in spreads are used to test whether ambiguity leads to a premium that is, on average, larger than the risk premium.

2.2 Introducing information

Consider now an environment where the agent receives an informative signal prior to investing. The signal $s \in \{\vartheta_L, \vartheta_H\}$ is binary, symmetric and correct with probability $q = P(s = \vartheta_L|V = V_L) = P(s = \vartheta_H|V = V_H)$. Henceforth, the prior and posterior beliefs are denoted with $Pr(V = V_H) =: \mu$ and $Pr(V = V_H|s, \mu) =: \rho$, respectively.

For exposition, predictions are presented for risk-neutral EU and MEU agents. The difference in the predictions hold under risk aversion as well.

2.2.1 Bayesian updating

A rational agent who has a single prior belief $\mu$ applies Bayes’ rule, then quotes a bid and an ask $b = a = E[V|s]$. That is, the risk-neutral EU agent adjusts the quotes to information, but holds a zero spread before and after information. A risk-averse EU agent holds the same non-zero spread for the
same belief value, regardless of the belief being a prior or a posterior belief.

In contrast, if the prior is ambiguous, optimal quotes depend on the way
the agent updates ambiguous beliefs. It is still an open question how agents
update ambiguous beliefs. The literature has proposed various updating rules
(see Epstein and Schneider 2007, Gilboa and Schmeidler 1989, Hanany and
Klibanoff 2007; Jaffray 1989; Klibanoff et al. 2009). Here we focus on two
main concepts that do not require any specific preference model. Moreover,
the two paradigms make maximum opposite predictions with respect to the
spread.

2.2.2 Full Bayesian updating

Agents with multiple priors apply FBU when they update prior by prior to
end up with a set of posteriors. In the case where an agent considers solely
the support of prior probabilities (without having second-order beliefs over
priors), he will update the two extreme priors to two extreme posteriors.
Therefore, unless \( q = 1 \), FBU does not fully eliminate ambiguity. The choice
of the relevant posterior and hence the evaluation of an action depend then
on the ambiguity preferences. For instance, an MEU agent with a high signal
\((s = \vartheta_H)\) buys an asset if

\[
p \leq \min_{\mu \in [\mu_l, \mu_h]} E[V | s = \vartheta_H, \mu].
\]

He therefore bids \( b = E[V | s = \vartheta_H, \mu_l] \). Analogously, his ask corresponds to
\( a = E[V | s = \vartheta_H, \mu_h] \), with \( b < E[V | s = \vartheta_H, \frac{\mu_l + \mu_h}{2}] < a \).

Hence, the ambiguity-averse trader chooses a non-zero spread both before
and after the updating. Its value depends on \( \Pi^* \), the set of probabilities that
the trader considers as possible.

2.2.3 Maximum likelihood updating

With MLU, the information received pins down the prior that will be up-
dated. The prior that has \( ex-ante \) the highest probability to generate the
informational event is given \( ex-post \) the highest likelihood. In our specific
setting, an agent observing a high signal \((s = \vartheta_H)\) assigns the highest likelihood to the highest prior \(\mu_h\). The agent therefore postulates a single posterior whenever a single prior maximizes the likelihood of having generated the informative event. If so, the signal completely eliminates the perception of ambiguity. The agent adjusts his belief to one of the two extremes, depending on the signal being high or low.

The optimal bid satisfies then:

\[
p \leq E[V|\mu^*, s] \quad \text{with} \quad \mu^* = \arg\max_{\mu \in [\mu_l, \mu_h]} \ell(\mu|s),
\]

where \(\ell(\mu)\) represents the likelihood of a prior. The same prior \(\mu^*\) satisfies the likelihood in the condition for the optimal ask:

\[
p \geq E[V|\mu^*, s] \quad \text{with} \quad \mu^* = \arg\max_{\mu \in [\mu_l, \mu_h]} \ell(\mu|s),
\]

Hence, a risk-neutral MLU trader with \((s = \vartheta_H)\) and a unique posterior belief \(\rho(\mu^*, s = \vartheta_H)\) chooses equal bid and ask \(b = a = E[V|\mu^*, s = \vartheta_H]\).

A fundamental difference between FBU and MLU in this setting is, therefore, that the ranking of states is determined by different factors. When an agent applies FBU, the ranking of states is determined by the long or short position [Mukerji and Tallon, 2001], while an agent using MLU ranks the states according to his information.

### 2.3 Hypothesis and treatment effect

As shown in Section 2.1.2, under the assumption of risk-neutrality, ambiguity aversion introduces a bid-ask spread. In the case of risk-averse preferences, ambiguity aversion leads to wider spreads than the spread chosen at the mid-probability. Furthermore, the analysis of ambiguity aversion goes beyond any spread increase that can be explained by subjective expected utility. Consider, for instance, an ambiguous set of probabilities \([\pi_l, \pi_h]\) that encompasses the probability \(\pi = .50\), at which theory predicts a maximum spread with
risk-averse utility functions. If the mid-probability of the set differs from 50\% (i.e. \(\frac{\pi_l + \pi_h}{2} \neq .50\)), a subjective belief of \(\Pi^* = .50\) can rationalize a wider spread than the spread chosen at the mid-probability. In contrast, subjective beliefs fail to rationalize spreads that are wider than any chosen spread at every unambiguous probability \(\pi \in [\pi_l, \pi_h]\). In this context, the experiment targets evidence in favor of ambiguity aversion that cannot be simultaneously explained by subjective expected utility.

**Hypothesis 1** Ambiguous probabilities induce wider bid-ask spreads than unambiguous probabilities:

\[
E[a - b|\pi \in [\pi_l, \pi_h]] > E[a - b|\pi], \quad \forall \pi \in [\pi_l, \pi_h].
\]

(1)

Therefore, bid-ask pairs for an ambiguous set \([\pi_l, \pi_h]\) that are more divergent than bid-ask pairs chosen at any \(\pi \in [\pi_l, \pi_h]\), i.e. at all unambiguous probability values in the same set, are interpreted as evidence in favor of ambiguity aversion.

If subjects are ambiguity-averse, changes in their perception of ambiguity can translate in variation of the spread. In a second step, differences in quotes are used to assess how gradual information processing affects the perception of ambiguity.

The experiment is designed such that full Bayesian updaters would quote the same bid-ask pairs for ambiguous prospects in the two treatments NL and L. In contrast, maximum likelihood updaters would perceive substantially less ambiguity and choose smaller spreads in treatment L. To this effect, the comparison across treatments focuses on rounds with identical sets of marginal and FBU probabilities. Identical spreads in the two treatments indicate that subjects perceive the same support of probabilities, which is evidence in favor of FBU:

**Under FBU:** \(E[a - b|\rho \in [\rho_l^{FBU}, \rho_h^{FBU}]] = E[a - b|\pi \in [\pi_l, \pi_h]]\)

with \([\rho_l^{FBU}, \rho_h^{FBU}] = [\pi_l, \pi_h]\).
However, smaller observed spreads in treatment L are more consistent with beliefs resulting from MLU than from FBU, suggesting that subjects react more strongly to information and perceive less ambiguity than would an FBU agent:

\[ E[a - b|\rho \in [\rho_{l}^{FU}, \rho_{h}^{FU}]] < E[a - b|\pi \in [\pi_{l}, \pi_{h}]] \]
with \([\rho_{l}^{FU}, \rho_{h}^{FU}] = [\pi_{l}, \pi_{h}]\).

Thus, comparing the average spread between treatments NL and L for the same support of marginal and FBU probabilities allows for differentiating between FBU or MLU.

3 Experimental design

3.1 Treatment No Learning (NL)

Treatment NL consists of 20 rounds. In each round, subjects start with an endowment of cash \(W_{0}\) and tender both a bid and an ask (\(b, a \in [V_{L}, V_{H}], b < a\)). At the beginning of each round, subjects receive information about the uncertainty of the investment. At that stage, they learn whether \(\pi\) is ambiguous or not. The uncertainty in the asset’s value is visualized by displaying “urn A” that contains 100 balls in a mixture of red and blue balls. To determine the asset value, the computer draws a ball (henceforth “value ball”) from urn A: if a red ball is drawn, the asset takes the value \(V_{L}\). The asset takes the value \(V_{H}\), if the value ball is blue.

The proportion of red and blue balls in urn A varies across rounds (see Table 1 for the chosen parameters) and is shown to the subjects. That is, subjects learn \(\pi\) for risky prospects by observing the exact number of red and blue balls in urn A. When the distribution is ambiguous, the exact proportion of red and blue balls is not disclosed: instead, subjects observe a minimum

\(^{3}\)The submission of two separate quotes allows subjects to reflect on a buy and a sell separately, as presumed in models with kinked preferences.
number of red and a minimum number of blue balls. The remaining balls in urn A are depicted as grey. Thus, subjects learn an interval range for $\pi$ (e.g. $\pi \in [.15, .85]$), but they do not know its exact value (see Figures A1 in Appendix A for examples of urn A with unambiguous and ambiguous distributions).

To implement the payoff in an ambiguous round, the computer chooses with equal probability a value in $[\pi_l, \pi_h]$ with uniform probabilities. Subjects, however, did not receive any information about how the true composition of urn A is determined when $\pi$ is ambiguous.

Subjects quote then bid and ask on a second, separate screen.

### 3.2 Treatment Learning (L)

Treatment L is almost identical to treatment NL, except that it contains an interim, second stage in which subjects are given an additional signal about the asset value.

In the first stage, subjects receive information about the prior $\mu$. Like the subjects in treatment NL, they observe the composition of urn A, which is ambiguous or unambiguous, depending on the round of the experiment.

In a second stage, they receive an additional signal. They observe the color of another ball (henceforth “signal ball”) that is drawn from a second urn. The choice of the second urn sets the correlation between the signal and the asset value: if the value ball is red, i.e. the asset has value $V_L$, the signal ball is drawn from “urn L” that consists of 75 pink and 25 green balls. If the value ball is blue, the signal is drawn from “urn H” that consists, in turn, of 75 green and 25 pink balls. Hence, the signal is correct, i.e. a pink (green) ball is drawn when the value ball is red (blue), with a probability of 75%.

Subjects observe the color of the signal ball (pink or green), but they do not know whether the signal ball is drawn from urn L or urn H (in other words, they do not know whether the asset has value $V_L$ or $V_H$). Figure A2 in Appendix A depicts an example of the screen at the second stage.
3.3 Experimental Procedures

The computerized experiment was run in the laboratory of Technical University Berlin. In total, 67 and 66 students participated in treatments NL and L, respectively. Each treatment was run with 3 sessions of ca. 22 subjects.

The trading game started once all participants read the instructions and answered an understanding test correctly. After all subjects completed the trading game, control measures of general attitudes towards risk, uncertainty and ambiguity were elicited.

The asset can take either the value \( V_L = 0 \) or \( V_H = 100 \). Subjects start each round with a cash endowment \( W_0 = 100 \).

The set of possible probability values is chosen to be parsimonious to have enough observations for the comparison between treatments. Each treatment consists of 14 rounds with unambiguous as well as 6 rounds with ambiguous probabilities, amounting to 20 rounds in total. The variation in the unambiguous probabilities \( \pi \) and \( \mu \) is identical in both treatments NL and L. The ambiguous rounds, on the other hand, differ between the two treatments: in L, the set of priors is fixed to \([.15; .85]\) (see Table 1). There, the variation in beliefs comes from the signal’s value that implies either a low range for the set of FBU posteriors \( \rho(s = \vartheta_l) \in [.05; .65] \) or a high range \( \rho(s = \vartheta_h) \in [.35; .95] \). As described in Subsection 2.3, the two set of probabilities \([.05; .65]\) and \([.35; .95]\) in NL were chosen to equal the set of posterior beliefs under FBU in L. This enables to compare bids and asks for the same dispersion in probabilities, when information on the distribution is provided immediately or sequentially.

Within each treatment, participants made their decisions in alternating blocks of 7 consecutive risky and 3 consecutive ambiguous rounds. Within each block, probabilities were ordered in increasing or decreasing order for less confusion (Vieider et al., 2015). In one out of the three sessions (per treatment), the ordering of blocks were reversed. In addition, subjects played 4 trial rounds with different parameter values. Two of the trial rounds had

\[4\text{The experimental interface was programmed with the software z-tree (Fischbacher, 2007). Participants were recruited with the ORSEE database (Greiner, 2004).}\]
Table 1: Chosen values for the probability $\pi$ and the prior $\mu$ with corresponding Bayesian posterior $\rho$

<table>
<thead>
<tr>
<th></th>
<th>No Learning</th>
<th>Learning</th>
<th>$\rho(s = \vartheta_L)$</th>
<th>$\rho(s = \vartheta_H)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk</td>
<td>$\pi = .05$</td>
<td>$\mu = .05$</td>
<td>$\rho = .02$</td>
<td>$\rho = .14$</td>
</tr>
<tr>
<td></td>
<td>$\pi = .15$</td>
<td>$\mu = .15$</td>
<td>$\rho = .05$</td>
<td>$\rho = .35$</td>
</tr>
<tr>
<td></td>
<td>$\pi = .35$</td>
<td>$\mu = .35$</td>
<td>$\rho = .15$</td>
<td>$\rho = .62$</td>
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<td></td>
<td>$\pi = .50$</td>
<td>$\mu = .50$</td>
<td>$\rho = .25$</td>
<td>$\rho = .75$</td>
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<td>$\pi = .65$</td>
<td>$\mu = .65$</td>
<td>$\rho = .38$</td>
<td>$\rho = .85$</td>
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<td>$\rho = .65$</td>
<td>$\rho = .95$</td>
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<tr>
<td></td>
<td>$\pi = .95$</td>
<td>$\mu = .95$</td>
<td>$\rho = .86$</td>
<td>$\rho = .98$</td>
</tr>
</tbody>
</table>

$T_R = 7 \times 2 = 14$  
$T_{RI} = 7 \times 2 = 14$

<table>
<thead>
<tr>
<th>Prior</th>
<th>Posterior (with FBU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambiguity</td>
<td>$\pi \in [.05; .65]$</td>
</tr>
<tr>
<td></td>
<td>$\pi \in [.15; .85]$</td>
</tr>
<tr>
<td></td>
<td>$\pi \in [.35; .95]$</td>
</tr>
</tbody>
</table>

$T_A = 3 \times 2 = 6$  
$T_{AI} = 1 \times 6 = 6$

| Total | $T_{NL} = 20$ | $T_L = 20$ |

Note: Subjects in treatment L are informed about the prior $\mu$ and the signal, but not about the Bayesian posterior $\rho$. Posterior probabilities are rounded to two decimal places. The parameter $T$ denotes the number of rounds. Each parameter value occurs in two rounds, except for the ambiguous prior in L: the 6 ambiguous rounds start with the same set [.15, .85].
ambiguous probabilities.

Decisions were incentivized with a random incentive system. To encourage subjects to consider each decision problem in isolation, the payoff-relevant round was chosen at the beginning of the trading game (Baillon et al., 2015). For this purpose, subjects threw a twenty-sided dice after the trial rounds, but before playing the 20 rounds. That is, they were aware that the payoff-relevant round was fixed during the experiment, but learned which round was chosen only at the end of the trading game.

Earnings consist of a show up fee (5 EUR), plus two-third of the randomly drawn round in the trading game plus one-third of a randomly chosen task for the elicitation of preferences. The exchange rate was 0.13 EUR per experimental currency units (ECU). Minimum and maximum earnings were 5 EUR and 28.84 EUR, respectively. Subjects earned, on average, 19.50 EUR for approximately 100 minutes.

4 Results

4.1 Treatment NL

Decisions for risky prospects. Subjects make mostly risk-averse choices: a majority of bid-ask pairs have a non-zero spread. Since the distribution of spreads is highly right-skewed, analyses focus mainly on quantiles. The median spread matches the risk of investing: it is hump-shaped in the probability, with a maximum at a probability of 50% (see Figure 3a). Furthermore, the spread is asymmetric around the probability, reflecting that increasing the bid (the ask) becomes more (less) risky with an increasing probability (see Figure 3b). Buying and selling are not equally risky as long as the low-value and high-value states are not equally probable. When the expected value is high, bidding is more risky than asking the expected value: a high bid entails the risk to pay a high price for a low-value asset, whereas a high

---

5 The instructions as well as the computer screen emphasized accordingly that hedging across rounds makes no sense once the payoff-relevant round is determined.

6 Most analyses yield even more significant results for mean values.
ask price limits the risk of selling a high-value asset. The reverse holds when
the expected value is low.
Overall, subjects choose a median spread of 5 ECU.

Table 2: Median and mean spread for various ranges of ambiguous
and unambiguous probabilities.

<table>
<thead>
<tr>
<th>π</th>
<th>[5%–65%]</th>
<th>[15%–85%]</th>
<th>[35%–95%]</th>
<th>Total obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Median</td>
<td>Mean</td>
<td>Median</td>
<td></td>
</tr>
<tr>
<td>Risk</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>18.50(.825)</td>
</tr>
<tr>
<td>Amb.</td>
<td>20</td>
<td>28</td>
<td>20</td>
<td>29.23(1.464)</td>
</tr>
<tr>
<td>Diff.</td>
<td>-11***</td>
<td>-18***</td>
<td>-10**</td>
<td>-10.73***</td>
</tr>
<tr>
<td>N</td>
<td>804</td>
<td>804</td>
<td>804</td>
<td>1340</td>
</tr>
</tbody>
</table>

Note: Median test (and two-sample test in means): *: p-value<.1, **: p-value<.05, ***: p-value<.01. Robust standard errors clustered at subject level (CRSE) in parentheses. The variable Amb. represents the indicator variable for rounds with an ambiguous probability.

Decisions for ambiguous prospects. Ambiguity about the probability re-

Subjects are more risk-averse in buying than in selling. I thank Marina Agranov
for pointing to me that this finding is consistent with recent evidence showing that the
willingness to sell is better at reflecting market beliefs, whereas willingness to buy reflects
more personal preferences.
duces significantly subjects’ willingness to trade. The median bid is shifted downwards, the median ask increases, leading to significantly wider spreads for ambiguous prospects (see Table A3 in Appendix C.1.2). Median spreads for prospects with ambiguous probabilities are three times as high as for unambiguous probabilities (see Table 2). Despite the asymmetry in risk premia for long and short positions, the ambiguity premium is almost symmetric. Subjects exhibit a median risk premium of 20% and 6.7% of the expected value in the bid and the ask, respectively. Ambiguity adds a premium of 20 and 16.4 percentage points in the bid and the ask (see Table A2 in Appendix C.1.1). In sum, Hypothesis 1 is confirmed.

**Result 1** *Ambiguity in probabilities engenders wider spreads.*

**Implications of ambiguity for trades.** Subjects trade and earn less when the return distribution is ambiguous. Subjects trade risky prospects in 82% of all rounds. Trades fall by 14.8% (12 percentage points) when probabilities are ambiguous. The greatest reduction of 19.3% occurs when the probability is between 15% and 85% (see Table 3).

| Table 3: Percentage of trades across different ranges of probabilities |
|-----------------------------|-----------------------------|---------------|-----------------------------|
| \( \pi \)                  | [5% - 65%]                 | [15% - 85%]   | [35% - 95%]                 | Total obs. |
| Risk                       | 80.44                      | 79.55         | 79.55                       | 81.77      |
|                            | (1.5)                      | (1.6)         | (1.6)                       | (1.3)      |
| Amb.                       | 71.89                      | 64.17         | 73.88                       | 69.65      |
|                            | (3.9)                      | (4.2)         | (3.8)                       | (2.3)      |
| Diff.                      | 9.55**                     | 15.37***      | 5.67                        | 12.12***   |
|                            | (4.2)                      | (4.4)         | (4.1)                       | (2.6)      |
| N                          | 804                        | 804           | 804                         | 1340       |

*Note:* P-values of binomial test with CRSE: *: p-value<.1,**: p-value<.05,**:* p-value<.01.

The reduction in trading activity translates into significantly smaller profits. Subjects earn, on average, 41.98% (p=.0015, two-sample t-test) more in
risky rounds than in ambiguous rounds (See Table A1 in Appendix C.1.1).

4.2 Treatment L

This section describes first how information processing affects investment decisions. It then compares learning with ambiguous and unambiguous priors by fitting decision weights functions.

**Ambiguity effects with gradual information processing.** The general effects of ambiguity on the spread are robust to incoming information. In the aggregate, choices in treatment L are ambiguity-averse. Subjects choose wider spreads for ambiguous than for risky asset distributions, with increasing difference in the mean in the last 10 periods (see Table 4).

<table>
<thead>
<tr>
<th>Table 4: Median and mean spread with ambiguous and unambiguous priors in Treatment L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rounds</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Risk</td>
</tr>
<tr>
<td>Amb.</td>
</tr>
<tr>
<td>Diff.</td>
</tr>
</tbody>
</table>

*Note: One-sided median test and two-sample test in means: *: p-value<.1, **: p-value<.05, ***: p-value<.01. Standard errors in parentheses.*

Yet, subjects are not insensitive to information. Starting with a set of priors $\mu \in [.15,.85]$, full Bayesian inference reduces the interval of probabilities by 10 percentage points ($\Pi^*(s = \vartheta_I) = [.05,.65]$ or $\Pi^*(s = \vartheta_H) = [.35,.95]$), while MLU even eliminates ambiguity. The diminished ambiguity is expressed in subjects’ quotes. The ambiguous rounds in treatment L show more trading activity than the rounds with the same set of marginal probabilities $\pi \in [.15,.85]$ in NL: the average spread for ambiguous prospects is smaller by 29% (median (mean) spread of 28 (35.10) in NL vs. 20 (26.33) in
L, \( p=.01 \), median test). Trading activity is higher by 22% (64% in NL vs. 79% in L, \( p=0.011 \) binomial test with CRSE). Mean profits are 34% higher (6.29 ECU more on average, \( p=0.088 \), two-sample test).

However, controlling for the range in marginal and FBU probabilities, no difference in the aggregate distribution of spreads is observable (p-value=.92 in Kolmogorov-Smirnov test, see Figures A5a and A5b in the Appendix C.2). Comparing rounds where marginal probabilities (\( \pi \)) and FBU posteriors (\( \rho \)) lie in the same interval \([.05; .65]\) discloses a small difference in the spread: participants in NL choose a median spread of 20, whereas the median spread in L equals 15. This non-significant difference carries even less weight in the aggregate since the two treatment groups choose identical median spreads of 20 when both \( \pi \) and \( \rho(s = \vartheta_h) \in [.35; .95] \). Apparently, subjects do not perceive substantially less ambiguity when the same information is released gradually.

**Result 2** *Given the same range of marginal and FBU posterior probabilities, the aggregate distribution of spreads with ambiguous posterior beliefs does not differ from the one with ambiguous marginal beliefs.*

Therefore, data do not lend support to MLU theory. Yet, data is not completely consistent with FBU theory either: subjects react differently to ambiguity in final probabilities than to ambiguity in posteriors. Although spreads are, in the aggregate, constant, chosen bids and asks are more extreme after information. Participants in treatment NL choose a median bid and ask of 17.5 and 50 when \( \pi \in [.05; .65] \). Participants in treatment L, however, choose a median bid and ask of 10 and 40 for an FBU posterior \( \rho \in [.05; .65] \) (significant differences at 5% level each). Analogously, the median bid and ask is 40 and 70.5 in the rounds where \( \pi \in [.35; .95] \), but 50 and 81 in the rounds with a set of FBU posteriors \( \rho \in [.35; .95] \) (significant differences at 1% level each). To examine the extent to which these quotes are compatible with either updating rule, we next consider subjects’ probabilistic sophistication.
4.2.1 Analysis of quotes

Updating unambiguous priors. The probabilistic sophistication is analyzed with the decisions for risky prospects. First, the risky rounds in NL are used to establish a pattern between decisions and objective probabilities. Subjects should react in the same way to probabilities, regardless of probabilities being given or updated. Second, assuming that this pattern is stable - even if information is released gradually -, this pattern serves as benchmark to discuss the validity of Bayesian posterior probabilities.

The underlying regression model assesses the extent to which the bid and the ask follow the asset’s expected value. Beliefs are estimated with nonlinear least squares in a seemingly unrelated regression with robust standard errors (NNLS-SUR):

\[
\begin{align*}
  b_i &= (1 - R_{P_b}) \cdot E[V|\tilde{\tau}] + \epsilon_{i,b} \\
  a_i &= (1 + R_{P_s}) \cdot E[V|\tilde{\tau}] + \epsilon_{i,a}
\end{align*}
\]

where \( E[V|\tilde{\tau}] = V_H \cdot \tilde{\tau} \).

It is therefore assumed that bids and asks both follow the subject’s expectation about the fundamental value, but potentially in a distorted way. Because subjects in treatment NL are more risk-averse in buying than in selling, the risk premium in selling \( R_{P_s} \) is allowed to differ from the risk premium in buying \( R_{P_b} \). The subject’s expectation is a function of his belief \( \tilde{\tau} \), which does not necessarily equal the objective probability. The mapping between objective probabilities and beliefs is represented with a weighted probability function proposed by Prelec (1998):

\[
\tilde{\tau}_i = e^{(-\beta(-\ln \tau)^\alpha)}
\]

The subject’s belief \( \tilde{\tau} \) is a weighted function of the objective probability \( \tau \). In treatment NL, \( \tau = \pi \), whereas in treatment L, the objective probability is assumed to be the Bayesian posterior \( \tau = \rho \). The coefficient \( \alpha \) regulates...
Figure 4: Estimated probability weighting function for unambiguous probabilities in NL & L.

The curvature of the function. The parameter $\beta$ determines the inflection point of the curve.

Table 5: Coefficient estimates for probability weighting function and risk premia

<table>
<thead>
<tr>
<th></th>
<th>NL</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.7971 (.0576)</td>
<td>0.7940 (.0424)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.6861 (.0612)</td>
<td>0.7411 (.0722)</td>
</tr>
<tr>
<td>$RP_s$</td>
<td>0.0110 (.0316)</td>
<td>0.0272 (.0326)</td>
</tr>
<tr>
<td>$RP_b$</td>
<td>0.2583 (.0366)</td>
<td>.2420 (.0280)</td>
</tr>
</tbody>
</table>

Note: Nonlinear least squares estimation with CRSE. Estimates are not significantly different.

The probability weighting function is in general inverse s-shaped, reflecting a general over-weighting of small and under-weighting of high probabilities. The functions do not differ between the two treatments. That is, the weighted priors. As I compare subjects’ reaction to objective probabilities, I use the definition of Bayesian updating that is closest to the objective probabilities.
subjects react to unambiguous given probabilities in the same way as to un-
ambiguous Bayesian posteriors. Assuming a stable relation between decisions
and probabilities, it cannot be excluded that subjects have Bayesian poste-
rior beliefs. Bayesian inference cannot be rejected.

*Updating ambiguous priors.* Analogous to the analysis of risky decisions,
I use the data in treatment NL to establish a pattern between decisions
and ambiguous priors. Assuming that the pattern does not change when
information is released gradually, this pattern is used to discuss the validity
of FBU and MLU posteriors.

The probability weighting function has single probability values as an
argument. Ambiguous distributions, however, are characterized by intervals
of probabilities. I approximate the estimate of a weighting function by using
the midpoint of the set of probabilities. In treatment NL, the set corresponds
to the ambiguous set of priors \([π_l, π_h]\). In treatment L, the set equals the set
of posteriors that varies with the updating rule. The midpoints of the set
of FBU posteriors are less extreme than the midpoints of the set of MLU
posteriors, which here is a singleton.

The solid line in Figures 5a and 5b depicts the relation between subjects’
estimated beliefs and ambiguous probabilities in NL. This inverse s-shape
relation serves as benchmark for the relation between estimated beliefs and
ambiguous posterior probabilities in L. The dashed line in Figure 5a rep-
resents the model fit with FBU posteriors. Estimated beliefs are s-shaped
in FBU posteriors, rather than inverse s-shaped. The discrepancy between
the benchmark (solid line) and the fit with FBU posteriors (dashed line)
points out that decision weights with FBU posteriors are too extreme. That
is, trading decisions are too extreme to be explained by the range of beliefs
under FBU.

The dashed line in Figure 5b depicts the model fit with MLU posteriors.
The weighting function is inverse s-shaped, but deviates from the benchmark
(solid line) as well. Given an MLU probability, estimated beliefs are not
sufficiently extreme to match the benchmark. Trading decisions are to close
to the belief of 50% to be explained by extreme MLU posteriors. Section C.2.1
in the Appendix displays the estimates of the NNLS-SUR with ambiguous probabilities and the results of a Lagrange-Multiplier test, which shows a significant difference between the benchmark model and the model fit under both FBU and MLU probabilities.

![Graphs showing estimated probability weighting functions in ambiguous rounds of NL & L.](image)

(a) Assuming FBU in L.  
(b) Assuming MLU in L.

**Figure 5:** Estimated probability weighting functions in ambiguous rounds of NL & L.

In a nutshell, quotes based on ambiguous posteriors are not extreme enough to be explained by MLU beliefs, but too extreme to be explained by FBU beliefs.

### 4.2.2 Heterogeneous updating rules

Heterogeneity in updating behavior partly accounts for the bad fit of FBU and MLU models. To illustrate the heterogeneity in quotes, the midpoints of bid and ask pairs (henceforth mid-quotes) are depicted in Figure 6. The top two panels [6a & 6b] show the distribution of mid-quotes for the ambiguous probabilities $\pi \in [.05, .65]$ and $\pi \in [.35; .95]$, respectively. Without incoming information, mid-quotes are distributed symmetrically around the midpoint of the set of probabilities. The distributions differ clearly in the bottom two panels [6c & 6d] that show mid-quotes for the same intervals of FBU posteriors (i.e. $\rho \in [.05, .65]$ and $\rho \in [.35; .95]$). Mid-quotes are clustered at 3 mass points ($\{0 – 5; 20 – 25; 45 – 50\}, \{50 – 55; 70 – 75; 95 – 100\}$) suggesting 3 main updating methods.
Figure 6: Mid-quotes for $\pi$ or $\rho \in [.05, .65]$ (left) and $\pi$ or $\rho \in [.35, .95]$ (right). Treatment NL in top panels, L in bottom panels.
The cluster analysis in Appendix C.3 illustrates how trading decisions differ. In sum, a substantial share of quotes (25.75%) match highly ambiguity-averse trading behavior, that favors non-participation. These subjects center their bids and asks around the mid-prior 50 and choose wide spreads. Another substantial share (21.72%) is consistent with MLU: they choose extreme quotes and minimal spreads. The majority of trading decisions (42.17%) is consistent with less extreme Bayesian quotes. However, these Bayesian quotes do not reflect FBU posteriors. Under FBU, participants in treatment NL and L should consider the same support of probabilities and therefore make similar trading decisions. Bid-ask pairs in treatment L should resemble the ones in NL and should be similarly centered around the midpoints of the sets of probabilities, which are here \{35, 65\}. However, the bid-ask quotes based on incoming information encompass beliefs that are more extreme than the ones in treatment NL. Controlling for the range in marginal and FBU probabilities, 36.19% of bid-ask pairs in treatment NL encompass the value 50 versus 29.54% in treatment L (p-value=0.07 in binomial test). Table A6 in Appendix C.3 shows that the same cluster analysis in treatment NL provides ranges of bid-ask pairs that are less extreme, with observations that are distributed more evenly across the clusters.

Chosen quotes can be rationalized with the updating of the prior $\pi = .5$, the midpoint of the set of priors. Indeed, the Bayesian posterior $\rho(s, \pi = .5)$ fits the relation between trading decisions and probabilities (see Figure 7): the probability weighting functions with marginal and posterior probabilities do not differ, when posterior probabilities correspond to Bayesian updates of the midpoint of ambiguous priors\[^9\]

In a nutshell, generally subjects are ambiguity-averse, even after receiv-

\[^9\]Since the conditional probability for a correct signal is $q = .75$, the mass points around 25 and 75 suggest base-rate neglect as a possible explanation. However, base-rate neglect is unlikely to cause this pattern. Base-rate neglect should become apparent in decisions regarding both ambiguous and unambiguous return distributions. Yet, subjects - even those who fall in this specific cluster of Bayesian updaters - adjust their quotes to the prior in risky rounds. Figure A6 in Appendix C.2.2 shows how mid-quotes increase in the prior for the different signal values. Bids and asks are not heavily centered around 25 or 75.
learning information about ambiguous priors. Learning does not effect the spread, but induce different and more heterogeneous quotes. Furthermore, Bayesian updates of the mid-prior describe aggregate quotes better than FBU or MLU posteriors. These results point out the relevance of conditional smooth preferences. The next section outlines to what extent Bayesian smooth preferences explain data.

4.3 Conditional smooth preferences

This section shows that chosen quotes in the two treatments are consistent with second-order preferences over probabilities. In a first step, I show that ambiguity-averse, but midpoint-preserving recursive preferences generate a bid-ask spread around the midpoint of possible priors. In a second step, I show that quotes match Bayesian updates of recursive preferences.

Following the model of smooth preferences in Klibanoff et al. (2005), a strictly increasing and concave function $\phi(\cdot)$ is used to represent ambiguity-averse second-order preferences. The agent’s value function is assumed to take the double expectational form:

![Figure 7: Estimated probability weighting functions in ambiguous rounds of NL & L assuming BU of mid-prior in L.](image)
\[ \int_{\pi_l}^{\pi_h} \phi(\mathbb{E}_\pi U(\cdot))\psi(\pi)d\pi \] (3)

where \( \psi(\pi) \) represents the subjective probability over the set of priors \([\pi_l; \pi_h]\). The operator \( \mathbb{E}_\pi \) computes the expected value with respect to a specific Bernoulli distribution \( f(\pi) \) with success probability \( \pi \).

Like in standard expected utility models, attitudes towards risk are captured by the concavity of a von Neumann-Morgenstern utility function \( U(\cdot) \). In addition, attitudes towards ambiguity are captured separately by the function \( \phi(\cdot) \). Agents assign subjective second-order beliefs \( \psi(\pi) \) to some probability distribution \( \pi \). In their decision-making, they evaluate subjective expectations over expected utilities. Ambiguity aversion corresponds to a dislike of spreads around the mean expected utility and is reflected by the concavity of the function \( \phi(\cdot) \).

The following analysis assumes that subjects have second-order beliefs, whose mean corresponds to the midpoint in the range of priors. This assumption is in line with the principle of insufficient reasons, under which agents assign equal probabilities to mutually exclusive events if they have no explicit reason to do differently.\(^{10}\)

I first show that ambiguity-averse second-order preferences generate a bid-ask spread. The optimal bid for going long is the certainty equivalent that satisfies:

\[ \int_{\pi_l}^{\pi_h} \phi(\mathbb{E}_\pi U(W_0 + V - b))\psi(\pi)d\pi = \phi(U(W_0)) \] (4)

Denote \( \int_{\pi_l}^{\pi_h} \psi(\pi)d\pi =: \mathbb{E}_\mu(\cdot) \). By Jensen’s inequality:

\[ \mathbb{E}_\psi \phi(\mathbb{E}_\pi U(W_0 + V - b)) < \phi(\mathbb{E}_\psi \mathbb{E}_\pi U(W_0 + V - b)) \] (5)

Under mean-preserving second-order beliefs, the subjective probability

\(^{10}\)Henceforth, the notion "mean-preserving" refers to "midpoint-preserving" in this context.
functions $ψ(π)$ satisfies $\int_{π_l}^{π_u} πψ(π)dπ = E_ψ(π) = \bar{π}$, where $\bar{π}$ represents the prior under risk. The RHS in Equation (5) equals then:

$$\phi(E_π U(W_0 + V - b)) = \phi(U(W_0)).$$

(6)

The optimal bid under risk makes the agent indifferent between buying the asset and keeping the endowment. It satisfies:

$$\phi(E_{\bar{π}} U(W_0 + V - b^R)) = \phi(U(W_0)).$$

(7)

From equations (4), (5) and (7) it follows that:

$$\phi(E_{\bar{π}} U(W_0 + V - b^R)) < \phi(E_π U(W_0 + V - b))$$

(8)

Because $\phi(\cdot)$ is strictly increasing, $U(\cdot)$ strictly concave, the optimal bid under ambiguity aversion is smaller than the optimal bid under risk, $b^{AA} < b^R$. Analogously, $a^{AA} > a^R$. Ambiguity-averse smooth preferences produce wider spreads than the spread under risk. With mean-preserving second-order beliefs, bid and ask quotes converge to the expected value under risk with decreasing ambiguity and risk aversion.

Incoming information alters the optimization problem at two points. First, expected utility is computed with posterior probabilities $ρ(s, µ)$ instead of given probabilities $π$. Second, the incoming information affects directly second-order beliefs $ψ(µ)$ by shifting more weights to more likely priors (Klibanoff et al. [2009]). With standard Bayesian updating:

$$ψ(s, µ) = \frac{ψ(µ)f(s, µ)}{\int_{µ_l}^{µ_h} ψ(\tilde{µ})f(s, \tilde{µ})d\tilde{µ}}$$

where

$$f(s, µ) = \begin{cases} qµ + (1 - q)(1 - µ) & \text{if } s = ϑ_h \\ (1 - q)µ + q(1 - µ) & \text{if } s = ϑ_l \end{cases}$$

(31)
The function \( f(s, \mu) \) is the probability of receiving signal \( s \) given a prior Bernoulli distribution with success probability \( \mu \). In particular, because \( \psi(s, \mu) \neq \psi(\mu) \):

\[
E_\psi(s = \vartheta_l, \mu) \phi\left(E_{\{s = \vartheta_l, \mu\}} U(\cdot)\right) < E_\psi(\mu) \phi\left(E_{\{s = \vartheta_l, \mu\}} U(\cdot)\right) \quad (9)
\]

\[
E_\psi(s = \vartheta_h, \mu) \phi\left(E_{\{s = \vartheta_h, \mu\}} U(\cdot)\right) > E_\psi(\mu) \phi\left(E_{\{s = \vartheta_h, \mu\}} U(\cdot)\right) \quad (10)
\]

Therefore, \( b_{CSP, \{s = \vartheta_l\}} < b_{SP, \{s = \vartheta_l\}} \); with conditional smooth preferences (CSP), second-order beliefs over priors that are updated upon the signal \( (s = \vartheta_l) \) induce a bid \( b_{CSP} \) that is lower than the optimal bid obtained with the same second-order beliefs over marginal probabilities. Analogously, \( b_{CSP, \{s = \vartheta_h\}} > b_{SP, \{s = \vartheta_h\}} \). Thus, conditional smooth preferences generate more extreme beliefs than marginal smooth preferences if traders have mean-preserving second-order beliefs. Consequently, gradual information release induces more extreme quotes compared to an environment where information is released all at once. Figures 8a and 8b display second-order beliefs with and without learning for the same support of probabilities. The dashed line depicts a uniform density over probabilities, which can be interpreted as subjects’ uniform second-order beliefs over marginal probabilities (applicable to treatment NL). The solid lines represent second-order beliefs over posteriors after Bayesian updating of uniform second-order beliefs over priors (applicable to treatment L). With smooth preferences, final expectations are more extreme if information is learned progressively.

In addition, it can be shown that under the assumption of mean-preserving spreads: \( b_{CSP} < b_R \). The risk-neutral agent quotes: \( b_{RN} = a_{RN} = E(V|s, \bar{\mu}) \).

With decreasing ambiguity and risk aversion: \( b^* \to E(V|s, \mu = E[\mu]) \). Analogously, \( a_{CSP} > a_R \) and \( a^* \to E(V|s, \mu = E[\mu]) \) with decreasing ambiguity and risk aversion.

With the principle of insufficient reasons, for instance, the mean prior belief corresponds to \( E[\mu] = .5 \) for \( \mu \in [.15, .85] \). Bids and asks would be centered around \( E[V|s, \mu = .5] \), i.e. \( E[V|s = \vartheta_l, \mu = .5] = 25 \) after a low signal and \( E[V|s = \vartheta_h, \mu = .5] = 75 \) after a high signal. In this context, Bayesian updating of second-order preferences explains why quotes are more
(a) $\pi, \rho \in [.05, .65]$

(b) $\pi, \rho \in [.35, .95]$

Figure 8: Marginal and Bayesian second-order beliefs for a low (a) and a high (b) support of probabilities.

extreme in treatment L than in NL for the same support of probabilities.

5 Conclusion

The evidence of ambiguity aversion found so far in Ellsberg-type experiments extends to other frameworks. The experiment shows that, in cases where portfolio reallocation is limited, ambiguity impedes willingness to trade - with and without sequential information processing. These results confirm the intuition that investors appear to consider ambiguous assets as more risky (Sarin and Weber, 1993; Epstein and Wang, 1994).

A second main insight of the experiment is that ambiguity effects cannot be disentangled from the information condition. The same degree of ambiguity leads to different trading decisions, depending on how many pieces of information have been available so far. Despite the same willingness to trade, investors choose more extreme quotes when they receive information in pieces.

In addition, incoming information introduces more heterogeneity in trading behavior. A substantial share of agents is insensitive to additional information, another non-negligible share adopts extreme beliefs, and the majority of agents appears to update second-order preferences in a Bayesian way.

The heterogeneity in information processing advises caution on the general conclusion in Baillon et al. (2013) "... that pessimism is a stable trait of
decision makers, not affected much by information received." Indeed, the aggregate distribution of spreads remains stable when information is released sequentially. However, the heterogeneity in updating rules shows that the stability of ambiguity attitudes may not hold for everyone. This is in line with Bossaerts et al. (2010), who argue that heterogeneity has important implications for markets, which are, therefore, not best described by a representative agent. Heterogeneity in updating behavior, though, probably impacts markets differently than heterogeneity in ambiguity attitudes. For instance, heterogeneity in trades might be amplified if specific traders are more prone to use some specific updating rules than others. This raises the question of whether the updating rule is inherently influenced by ambiguity preferences. If highly ambiguity-averse subjects are insensitive to information and less ambiguity-averse subjects are instead more prone to apply MLU, gradual information flow may reinforce disparities in ambiguity preferences. In particular, asset pricing would be determined by extreme updaters, if those who update cautiously refrain from trading.

Other important questions remain to be clarified in future research. First, ambiguity effects possibly differ in markets. There is a difference between individual willingness to trade and its counterpart in markets, e.g. liquidity or market depth. The risk of adverse selection may incite investors to avoid ambiguous markets even more. Alternatively, trade may possibly be driven by one’s knowledge relative to other market participants (Zeckhauser 2006, cf. competence hypothesis in Heath and Tversky 1991). To be willing to trade, it might be sufficient to be not at informational disadvantage compared to other traders. Furthermore, the interaction between investors might eliminate any perception of ambiguity, especially if markets are dominated by aggressive traders. The findings in Sarin and Weber (1993), though, indicate that ambiguity effects are robust to market feedback. Yet, the extent to which information aggregation abates ambiguity effects is still not clear.

Second, the observed divergence in beliefs casts doubts on the hypothesis that trading volume falls with ambiguity. Even if ambiguity weakens individual willingness to trade, beliefs resulting from learning might be so divergent that different trading parties agree on speculative trade.
Third, updating behavior may vary with the type of information. Information in itself can be ambiguous. Extreme updating possibly disappears when the precision of signals is not known. The reactions to information matters in particular, if the decision to acquire information is endogenous. A correlation between willingness to pay for information under ambiguity and a subject’s ambiguity preferences might abate or reinforce ambiguity effects.

It is important to identify conditions under which ambiguity effects are self-enforcing. A faster resolution of ambiguity and a concomitant increase in liquidity benefit not only trading venues through higher profits, but also investors through lower transaction costs and, potentially, higher price efficiency. This study draws the attention to frequent information release as a mechanism to avoid or correct frictions in trades.
References


A Screen layout

Figures A1a and A1b depict examples of the composition in urn A when the prior is unambiguous and ambiguous, respectively. The grey balls in the ambiguous urn can be either red or blue.

![Risky prospect](image1.png) ![Ambiguous prospect](image2.png)

(a) Risky prospect  (b) Ambiguous prospect

**Figure A1:** Examples for visualization of probability distribution with urn A.

In treatment L, a second decision screen is shown to the subjects before they choose their quotes. In the upper left corner, the composition in urn A reminds the subjects of the prior distribution. If the asset takes the value 0 (i.e. the value ball is red), a second ball is drawn from the “urn N”. In 75% of all drawings, the subject will then observe a pink ball. The subject will see a green ball with 75% probability if the value ball is blue and the signal ball is drawn from “urn H”. The right side of the screen conveys the additional information by showing the color of the signal ball.
Figure A2: Example for an additional signal at the second stage.

B Mathematical appendix

B.1 Bid-ask spread generated by risk aversion

Risk-aversion introduces a spread between the bid and the ask.

Let \( b^{RN} = \mathbb{E}(V) \) be the optimal bid under risk neutrality. Assume risk-averse preferences are represented by a strictly concave utility function \( U(\cdot) \) with \( U'(\cdot) > 0 \) and \( U''(\cdot) < 0 \).

The optimal bid corresponds to the certainty equivalent that makes a risk-averse agent indifferent between the initial position \( W_0 \) and the investment in the long position. The optimal bid \( b^{RA} \) must therefore satisfy:

\[
\mathbb{E}_\pi U(W_0 + V - b) = U(W_0)
\]

The short-selling ask satisfies accordingly:

\[
\mathbb{E}_\pi U(W_0 - V + a) = U(W_0)
\]

By Jensen’s inequality:

\[
\mathbb{E}U(b^{RN}) = \mathbb{E}_\pi U(W_0 + V - b^{RN}) < U(\mathbb{E}_\pi(W_0 + V - E(V))) = U(W_0) = \mathbb{E}U(b^{RA})
\]
From \( U'(\cdot) > 0 \) and \( \mathbb{E}U(b^{RN}) < \mathbb{E}U(b^{RA}) \), it follows that \( b^{RA} < b^{RN} = \mathbb{E}(V) \). Analogously, \( a^{RA} > a^{RN} = \mathbb{E}(V) \).

C Results

C.1 Ambiguity

C.1.1 Descriptive statistics

Table [A1] shows the mean profits for risky and ambiguous prospects, across different ranges of probabilities.

Table A1: Mean profits across different ranges of probabilities

<table>
<thead>
<tr>
<th>Range of ( \pi )</th>
<th>[5% - 65%]</th>
<th>[15% - 85%]</th>
<th>[35% - 95%]</th>
<th>Total obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk</td>
<td>26.82</td>
<td>23.36</td>
<td>23.9</td>
<td>27.97</td>
</tr>
<tr>
<td></td>
<td>(1.82)</td>
<td>(1.88)</td>
<td>(1.85)</td>
<td>(1.52)</td>
</tr>
<tr>
<td>Amb.</td>
<td>24.92</td>
<td>18.57</td>
<td>15.63</td>
<td>19.70</td>
</tr>
<tr>
<td></td>
<td>(3.95)</td>
<td>(3.89)</td>
<td>(4.15)</td>
<td>(2.31)</td>
</tr>
<tr>
<td>Diff.</td>
<td>1.90</td>
<td>4.79</td>
<td>8.27**</td>
<td>8.27***</td>
</tr>
<tr>
<td></td>
<td>(4.35)</td>
<td>(4.32)</td>
<td>(4.54)</td>
<td>(2.78)</td>
</tr>
<tr>
<td>N</td>
<td>840</td>
<td>840</td>
<td>840</td>
<td>1340</td>
</tr>
</tbody>
</table>

*Note: *: p-value<.1, **: p-value<.05, ***: p-value<.01. The variable “Amb” represents the dummy variable for the ambiguous rounds.

Table [A2] shows the median values of bid and ask quotes as a fraction of the expected value. The premia in ambiguous rounds are computed with respect to the midpoint of the probability interval.
Table A2: Median values of quotes as a fraction of the expected value

<table>
<thead>
<tr>
<th></th>
<th>b (\mathbb{E}(\pi))</th>
<th>a (\mathbb{E}(\pi))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk</td>
<td>.8</td>
<td>1.0667</td>
</tr>
<tr>
<td>Amb.</td>
<td>.6</td>
<td>1.2308</td>
</tr>
<tr>
<td>Diff.</td>
<td>-0.20**</td>
<td>-0.1641***</td>
</tr>
</tbody>
</table>

*Note*: The variable “Amb.” represents the dummy variable for the ambiguous rounds. ***: p-value in median test <.01.

C.1.2 Regression estimates

Table A3 presents the results of the median polynomial regression. The estimates for risky prospects are plotted in the Figures 3a and 3b in Section 4.1.

Table A3: Median polynomial regression

<table>
<thead>
<tr>
<th>Dep. var.</th>
<th>Bid</th>
<th>Ask</th>
<th>Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior</td>
<td>0.3392***</td>
<td>1.1296***</td>
<td>0.5***</td>
</tr>
<tr>
<td></td>
<td>(.104)</td>
<td>(.104)</td>
<td>(.089)</td>
</tr>
<tr>
<td>Prior(^2)</td>
<td>0.0060***</td>
<td>-0.0018**</td>
<td>-0.005***</td>
</tr>
<tr>
<td></td>
<td>(.001)</td>
<td>(.001)</td>
<td>(.001)</td>
</tr>
<tr>
<td>Amb.</td>
<td>-5***</td>
<td>8***</td>
<td>10***</td>
</tr>
<tr>
<td></td>
<td>(1.899)</td>
<td>(2.398)</td>
<td>(3.014)</td>
</tr>
<tr>
<td>cons</td>
<td>3.1548**</td>
<td>4.3981**</td>
<td>-1.375</td>
</tr>
<tr>
<td></td>
<td>(1.284)</td>
<td>(2.062)</td>
<td>(.924)</td>
</tr>
<tr>
<td>N</td>
<td>1340</td>
<td>1340</td>
<td>1340</td>
</tr>
<tr>
<td>(R^2)</td>
<td>.3717</td>
<td>.3146</td>
<td>.0443</td>
</tr>
</tbody>
</table>

*Note*: Testing of coefficients with robust standard errors in parentheses: *: p-value<.1, **: p-value<.05, ***: p-value<.01. The variable “Amb.” represents the indicator variable for the ambiguous rounds.
C.1.3 Heterogeneity in ambiguity attitudes

This section examines differences in ambiguity preferences. The behavior in risky rounds is used to identify ambiguity-averse preferences. To control for learning effects, subjects are classified according to their behavior in the last 10 rounds, of which 3 are ambiguous.

In a first step, ambiguous rounds with a set of probabilities $[\pi_l, \pi_h]$ are compared to risky rounds whose probability equals the midpoint $\pi = \frac{\pi_l + \pi_h}{2}$ (henceforth mid-probability). A trading decision is defined as ambiguity-averse (in a tolerant sense) if, for ambiguous prospects, the subject chooses wider spreads than the spread $SP_{MP}^{MP}$ chosen when the unambiguous probability equals the mid-probability. In line with this definition, the variable $Y$ classifies decisions for ambiguous prospects in 3 categories, depending on whether the spread is smaller, equal, or wider than the individual maximum spread for risky prospects:

$$Y_{ij} = \begin{cases} 
1 & \text{if } SP_{(A,ij)} > SP_{(R,i)}^{MP} \\
0 & \text{if } SP_{(A,ij)} = SP_{(R,i)}^{MP} \\
-1 & \text{if } SP_{(A,ij)} < SP_{(R,i)}^{MP}
\end{cases}$$

$i = 1, ..., 67$ subjects, $j = 1, 2, 3$ ambiguous decisions

Subjects exhibit a larger spread in 43% of the ambiguous rounds (see
Figure A3a). Out of the 67 subjects, 12 chose a wider spread for all 3 ambiguous rounds versus 5 subjects who always chose a smaller spread (see Figure A3b). The majority of subjects is, on average, ambiguity-averse with $\sum_{j=1}^{3} Y_i > 0$.

A second step differentiates more thoroughly between ambiguity aversion and subjective expected utility (SEU). For instance, when $\pi \in [.05,.65]$, a risk-averse agent with subjective belief $\Pi^* = .45$ chooses a wider spread than for a risky prospect with $\pi = .35$, without being ambiguity-averse. For this purpose, rounds with $Y = 1$ are further distinguished into decisions that can and cannot be explained by SEU as well. A trading strategy is defined to be inconsistent with SEU when the spread for an ambiguous prospect with $\pi \in [\pi_l, \pi_h]$ is wider than any chosen spread for all risky prospects with $\pi \in [\pi_l, \pi_h]$. For instance, the subject’s decision in an ambiguous round with $\pi \in [.05,.65]$ is compared to all his decisions in risky rounds with $\pi \in \{.05,.15,.35,.50,.65\}$. In treatment NL, 52.33% of the rounds that are consistent with ambiguity-averse preferences reject SEU as a possible explanation.

In treatment L, significantly less ambiguous rounds can be classified as consistent with ambiguity-averse preferences (33% in L vs. 43% in NL, compare Figures A3a & A4). This is mainly due to some large spreads in risky rounds. In these rounds, contradicting signals endogenously create uncertainty: subjects choose wider spreads for risky prospects when the signal contradicts prior beliefs. Still, out of these ambiguous decisions with wider spreads, 52.31% display a spread wider than any chosen spread for unambiguous priors within [.15,.85], and exclude therefore SEU.

---

11Theoretically, subjects should choose a maximum spread at a prior at 50%, where the lottery exhibits the highest risk. Nevertheless, in treatment NL 27 subjects choose a maximum spread at a different probability. Therefore, spreads are compared to the maximum spread each subject has chosen for the risky prospects, regardless of whether it has been chosen at a probability of 50% or not.
C.2 Learning

Figure A5a depicts the distribution of chosen spreads in the ambiguous rounds of treatment NL with $\pi \in [.05, .65]$ or [.35; .95]. Figure A5b refers to the distribution of spreads in the ambiguous rounds of treatment L with $\rho \in [.05, .65]$ or [.35; .95]. In both figures, the vertical solid and dashed lines represent the median and mean spread, respectively. The distributions of spreads do not differ for the same range of marginal and posterior probabilities (p-value=.92 in Kolmogorov-Smirnov test).

Figure A5: Spreads for ambiguous prospects for the same theoretical dispersion in marginal (a) and Bayesian posterior (b) probabilities.
C.2.1 Results of NNLS-SUR

Table A4 shows the coefficient estimates of the NNLS-SUR model. Because, by design, there is less variation in the ambiguous probabilities, the estimation is more robust when assuming symmetric premia in the bid and the ask. The model estimates with marginal and Bayesian updates of the mid-prior (henceforth recursive Bayesian updating - RBU) do not differ (p-value of 1 for the ask equation and .2211 for the bid equation in the Lagrange-Multiplier test).

Table A4: Coefficient estimates for probability weighting function and risk premia

<table>
<thead>
<tr>
<th></th>
<th>NL mid-prior</th>
<th>L FBU***</th>
<th>MLU***</th>
<th>RBU ***</th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>0.9646 (.0370)</td>
<td>1.1532 (.0301)</td>
<td>0.9206 (.0440)</td>
<td>0.9824 (.0493)</td>
</tr>
<tr>
<td>α</td>
<td>0.6563 (.0754)</td>
<td>1.1754 (.0686)</td>
<td>0.2574 (.0288)</td>
<td>0.6658 (.0744)</td>
</tr>
<tr>
<td>RP</td>
<td>0.2982 (.0258)</td>
<td>0.2491 (.0301)</td>
<td>0.2491 (.0301)</td>
<td>0.2491 (.0301)</td>
</tr>
</tbody>
</table>

Note: Nonlinear least squares estimation with CRSE in a seemingly unrelated regression. ***: p-value<.01, refers to a significance difference between the model estimates in treatment NL and the ones with updated beliefs in Lagrange-Multiplier tests.

C.2.2 Heterogeneity in updating.

Figure A6 depicts mid-quotes for risky prospects and the mean regression estimates as a function of unambiguous priors. The dashed and solid lines correspond to mean estimates after subjects receive a high and a low signal, respectively. The average mid-quote increases in the prior, showing no evidence of base-rate-neglect.
C.3 Cluster analysis

To discern the different ranges of updated beliefs and their prevalence, bid-ask pairs for ambiguous prospects are clustered. The cluster analysis is performed in k-medians with 8 clusters, yielding the 8 different ranges for updated beliefs listed in Table A5.\footnote{The value of 8 clusters finds its justification in the theory, allowing the identification of 8 clusters in the upper triangular grid of bid-ask pairs: extreme beliefs upon both a low and a high signal (centered around the bid-ask points: (0,0); (100,100)), ambiguity-neutral Bayesian beliefs upon both a low and a high signal (the 45 line (5,5) to (95,95) ), ambiguity-averse Bayesian beliefs upon both a low and a high signal ((5,65), (35,95)), maximum ambiguity-aversion (0,100), ambiguity-neutral likelihood-insensitive beliefs (50,50). Robustness checks with more and less clusters do not yield a better comprehension of the data.}

In total, 21.72\% of the ambiguous decisions belong to the clusters 1 and 6 and are consistent with MLU. Quotes in these clusters are close to one extremum and exhibit, on average, the smallest spread of 1 ECU. The opposite behavior is described in clusters 7 and 8, that represent 25.75\% of the bid-ask pairs. These observations exhibit a substantial spread of more than 30 ECU. In approximately one third of these decisions, the spread is chosen wide enough to implement almost surely a no-trade outcome (cluster 8). In cluster 4, 10.35\% of the quotes disclose a small spread with bids and asks around 50\%, the midpoint of the set of priors. These quotes match the behav-

Figure A6: Mid-quotes for unambiguous assets and their mean-estimates for the two signals and the group of Bayesian updaters (clusters 3 and 5).
Table A5: Median bids, asks and spreads and corresponding statistics for 8 clusters in ambiguous rounds of treatment L

<table>
<thead>
<tr>
<th>Cluster</th>
<th>bid</th>
<th>ask</th>
<th>spread</th>
<th>% trade</th>
<th>% obs</th>
<th>consistent with</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>100</td>
<td>12.12</td>
<td>MLU</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>16</td>
<td>5</td>
<td>100</td>
<td>5.30</td>
<td>AN &amp; Bayesian</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>33.5</td>
<td>20</td>
<td>86.36</td>
<td>16.67</td>
<td>Bayesian</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>50</td>
<td>5</td>
<td>95.12</td>
<td>10.35</td>
<td>AN-LI/conseNLtism</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
<td>80</td>
<td>20</td>
<td>82.5</td>
<td>20.20</td>
<td>Bayesian</td>
</tr>
<tr>
<td>6</td>
<td>98.5</td>
<td>99</td>
<td>1</td>
<td>100</td>
<td>9.60</td>
<td>MLU</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>70</td>
<td>50</td>
<td>53.62</td>
<td>17.42</td>
<td>AA</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>99</td>
<td>98</td>
<td>15.15</td>
<td>8.33</td>
<td>AA - non-participants</td>
</tr>
</tbody>
</table>

Note: Cluster analysis in k-median.

Behavior of an ambiguity-neutral but likelihood-insensitive (AN-LI) investor who is rather unresponsive to incoming information. Under the assumption that subjects have second-order beliefs, whose mean equals the midpoint of the set of priors, over-emphasizing the mid-prior 50% concurs with conservatism. Conservatism predicts an over-weighting of the prior belief, but no increase in the spread. The remainder of the decisions amounts to 42.17% of bid-ask pairs in the clusters 2, 3 and 5. These quotes are consistent with Bayesian updating. The decisions in cluster 2 result in small spreads and, thus, do not show any evidence of ambiguity aversion. The majority of the bid-ask pairs, though, fall in clusters 3 or 5, that disclose a median spread of 20 ECU. Figure A7 summarizes the results of the cluster analysis. It depicts bid-ask pairs that are consistent with MLU, ambiguity-averse Bayesian, ambiguity-averse non-Bayesian and ambiguity-neutral beliefs in diamonds, squares, triangles and dots or crosses, respectively.

Table A6 lists the results of the same cluster analysis in treatment NL. The analysis yields less extreme clusters of beliefs. Furthermore, the observations are distributed more evenly across the eight clusters, yielding the more symmetric distributions of quotes reflected in Figures 6a and 6b.
**Figure A7:** Clusters of bid-ask pairs in ambiguous rounds of treatment L

**Table A6:** Median bids, asks and spreads and corresponding statistics for 8 clusters in ambiguous rounds of treatment NL

<table>
<thead>
<tr>
<th>Cluster</th>
<th>bid</th>
<th>ask</th>
<th>spread</th>
<th>% trade</th>
<th>% obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>10.5</td>
<td>1</td>
<td>100</td>
<td>8.96</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>25</td>
<td>2</td>
<td>95</td>
<td>7.46</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>37</td>
<td>1</td>
<td>97.06</td>
<td>12.69</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>50</td>
<td>35</td>
<td>69.70</td>
<td>12.31</td>
</tr>
<tr>
<td>5</td>
<td>49</td>
<td>60</td>
<td>12.5</td>
<td>82.61</td>
<td>17.46</td>
</tr>
<tr>
<td>6</td>
<td>70</td>
<td>80</td>
<td>5</td>
<td>90.70</td>
<td>16.04</td>
</tr>
<tr>
<td>7</td>
<td>35</td>
<td>90</td>
<td>52.5</td>
<td>34.38</td>
<td>11.94</td>
</tr>
<tr>
<td>8</td>
<td>4.5</td>
<td>85</td>
<td>72.5</td>
<td>19.44</td>
<td>13.43</td>
</tr>
</tbody>
</table>

*Note: Cluster analysis in k-medians.*