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A GENERAL FORMULA FOR BEARING CAPACITY

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A General Formula for Bearing Capacity

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It is by now generally accepted, that the bearing capacity of a foundation depends, not only on the properties of the soil, but also on the dimensions, shape and depth of the foundation area, as well as on the inclination and eccentricity of the foundation load.

The most practical way of taking these effects into account consists in generalizing Terzaghi's bearing capacity formula by multiplying each of its terms with a shape-, a depth- and an inclination factor. The eccentricity is accounted for by making the calculation for the effective foundation area only.

Preliminary formulas of this type were proposed by the author in 1955 for the two special cases of $\varphi = 0$ (clay) and $c = 0$ (sand). In the present paper a general formula is indicated, and the different factors are studied, especially in regard to their dependence on the friction angle of the soil. As a result, approximate empirical formulas and corresponding diagrams are given for the bearing capacity-, inclination-, depth- and shape factors.

Finally, new definite rules are proposed for determining the effective and equivalent foundation areas as well as the contact pressure distribution.

1. Introduction.

In 1955 the author published a Danish paper [1], in which he proposed two simple, semi-empirical formulas for the calculation of the bearing capacity of foundations, on clay ($\varphi = 0°$) and on sand ($c = 0$) respectively. In the first case the formula was an extension of Skempton's [2], whereas in the latter case the formula was new but based, of course, on Terzaghi's original formula [3]. The formulas covered the general case of eccentric and inclined loads on foundation areas of any shape at any depth.

In his General Report on Foundations [4] to the London Conference in 1957 the author cited these two formulas but without giving any details.

In 1958 [5] the author extended his formulas to the case of simultaneous friction and cohesion, so that the long-term bearing capacity of clays could be calculated. In 1959 [6] and 1960 [7] he found it necessary to revise the expression for the inclination factor in the case of $\varphi = 0°$.

Lately — and partly due to a request from Professor E. Schultze in Aachen — further investigations have been made by the author and The Danish Geotechnical Institute, especially concerning the inclination factors. As a result, the original formulas have now been revised as described in the following.

2. The general formula.

Terzaghi [3] proposed as the first the following simple formula for the bearing capacity Q of a centrally and vertically loaded strip foundation on a horizontal earth surface:

$$Q : B = \frac{1}{2}\,\bar{\gamma}\,B\,N_\gamma + \bar{q}\,N_q + c\,N_c \qquad (1)$$

B is the width of the foundation, $\bar{\gamma}$ the effective unit weight of the soil and c its cohesion. $\bar{q}$ is the effective unit load on the surface outside the foundation, whereas the N's are functions of the friction angle φ of the soil.

In the special case of $\varphi = 0°$ (clay) an exact calculation has first been made by Prandtl [8], giving $N^\circ_\gamma = 0$, $N^\circ_q = 1$ and $N^\circ_c = \pi + 2 = 5.14 \sim 5$. Hence in this case:

$$Q^\circ : B = 5\,c + \bar{q} \qquad (2)$$

c is here the undrained shear strength of the clay, and Q° is the short-term bearing capacity.

Skempton [2] has extended this formula to cover the more general case of a foundation of finite length L ($\geqq B$), placed at a depth D below the surface:

$$Q^\circ : BL = 5\,c\,(1 + 0.2\,B : L)\,(1 + 0.2\,D : B) + \bar{q} \quad (3)$$

The ratio $D : B$ to be inserted in this formula is limited to 2.5, and $\bar{q}$ is now to be interpreted as the effective overburden pressure at foundation level.

Skempton has here multiplied the c-term in Terzaghi's formula with a "shape factor" and a "depth factor". The present author showed [1] that inclined foundation loads could be dealt with by multiplying also with an "inclination factor". Eccentric loads are dealt with by means of the so-called "effective foundation area" (see later).

Denoting shape factors by s, depth factors by d and inclination factors by i, Terzaghi's original formula (1)

can easily be generalized by multiplying each of its terms with a set of the above-mentioned factors:

$$Q : BL = \frac{1}{2}\bar{\gamma} B N_\gamma \, s_\gamma \, d_\gamma \, i_\gamma + \bar{q} N_q s_q d_q i_q + c N_c s_c d_c i_c \quad (4)$$

However, the q- and the c-factors are interrelated. If we have found a solution for the special case of ($\bar{\gamma} = 0$, $\bar{q} = 1$, $c = 0$), it can be shown [9], that the solution for the more general case of ($\bar{\gamma} = 0$, $\bar{q} \neq 0$, $c \neq 0$) is obtained by first multiplying all loads and stresses by ($\bar{q} + c \cot \varphi$) and then subtracting $c \cot \varphi$ from all normal loads and stresses (but not from the tangential ones). This means that instead of (4) we can write:

$$Q : BL = \frac{1}{2}\bar{\gamma} B N_\gamma \, s_\gamma d_\gamma \, i_\gamma + (\bar{q} + c \cot \varphi) \, N_q s_q d_q i_q - c \cot \varphi \quad (5)$$

Comparing (4) and (5) we find the following relation between the q- and the c-factors:

$$N_c s_c d_c i_c = (N_q s_q d_q i_q - 1) \cot \varphi \quad (6)$$

We can, of course, also use (6) to eliminate the q-factors from (4), if so desired:

$$Q : BL = \frac{1}{2}\bar{\gamma} B N_\gamma \, s_\gamma d_\gamma \, i_\gamma + (c + \bar{q} \tan \varphi) \, N_c s_c d_c i_c + \bar{q} \quad (7)$$

In principle, any of the three equations (4), (5) or (7) can be used as our general formula. However, (4) must be considered impractical, as it contains an unnecessarily great number of factors. Of the remaining two, (5) is evidently most practical for $c = 0$ (sand), whereas (7) is most convenient for $\varphi = 0°$ (clay in the undrained state).

From (6) it will be seen, that in the case of $\varphi = 0°$ we must have $N_q^{\circ} = s_q^{\circ} = d_q^{\circ} = i_q^{\circ} = 1$, which makes formula (5) unusable in this case. Consequently, if we want one single formula to cover all cases, this formula must be (7).

3. Bearing capacity factors.

As shown first by Prandtl [8], N_q and N_c can be calculated by considering the simple theoretical case of weightless earth ($\bar{\gamma} = 0$). The result is:

$$N_q = e^{\pi \tan \varphi} \tan^2 (45° + \frac{1}{2}\varphi) \quad (8)$$

$$N_c = (N_q - 1) \cot \varphi \quad (9)$$

Curves for N_q and N_c as functions of the friction angle φ are given in Fig. 1. The N-scale is logarithmic.

In principle, it should be possible to calculate N_γ by considering the special case of cohesionless, unloaded earth ($c = 0$, $\bar{q} = 0$). However, to the author's knowledge no one has as yet succeeded in indicating a corresponding figure of rupture which is both kinematically and statically possible.

Instead, several authors have used approximate rupture-figures of a type which will usually give too high values

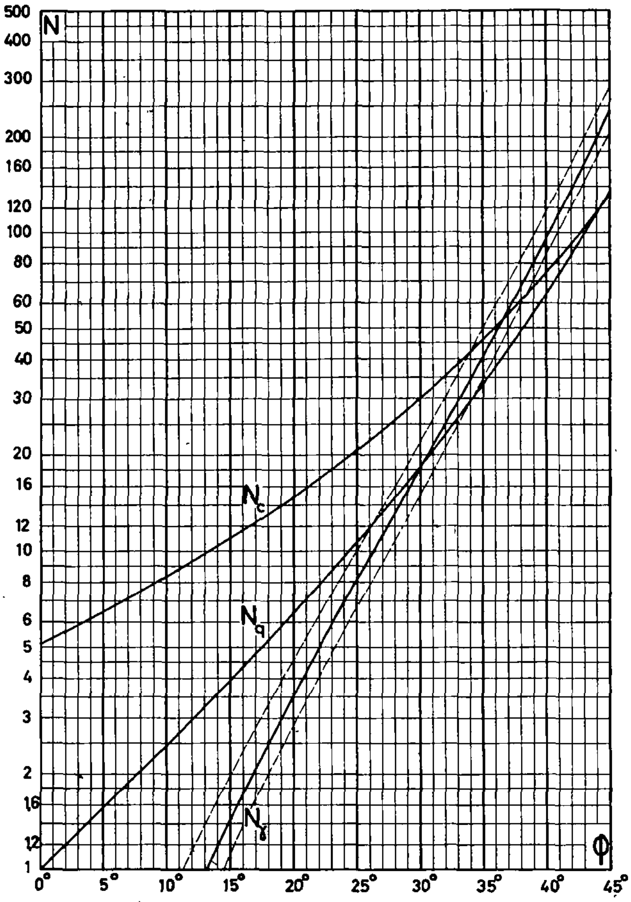

Fig. 1. Bearing capacity factors.

of N_γ. Meyerhof [14] has, f. inst., by means of such a procedure obtained N_γ-values corresponding to the upper dashed line in Fig. 1.

Lundgren and Mortensen [10] indicated in 1953 a figure of rupture, which is statically but not kinematically possible, and made the calculation for $\varphi = 30°$ (see Fig. 2). This procedure is known to give too low values of N_γ. Recently, D. Odgaard has at The Danish Geotechnical Institute made corresponding calculations for $\varphi = 20°$ and $40°$. The values found correspond to the lower dashed line in Fig. 1.

In 1955, when Lundgren and Mortensen's value for $\varphi = 30°$ only was known, the author of the present paper proposed as an empirical formula: $N_\gamma \sim N_q - 1$, this being exact for $\varphi = 0°$ and approximately correct for $\varphi = 30°$ (where $N_q = 18.4$, and $15 < N_\gamma < 22$).

However, as the correct curve for N_γ must lie between the two dashed lines in Fig. 1, a better approximation may f. inst. be obtained by means of the relation:

$$N_\gamma \sim 1.8 \, N_c \tan^2 \varphi = 1.8 \, (N_q - 1) \tan \varphi \quad (10)$$

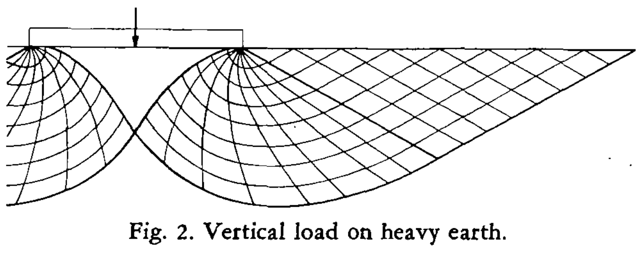
Fig. 2. Vertical load on heavy earth.

Table 1.

φ	N_γ	N_q	N_c
0.0°	0.0000	1.000	5.14
2.5°	0.0198	1.252	5.76
5.0°	0.0894	1.568	6.49
7.5°	0.229	1.966	7.34
10.0°	0.467	2.471	8.34
12.5°	0.844	3.11	9.54
15.0°	1.419	3.94	10.98
17.5°	2.275	5.01	12.71
20.0°	3.54	6.40	14.83
22.5°	5.39	8.23	17.45
25.0°	8.11	10.66	20.72
27.5°	12.12	13.94	24.85
30.0°	18.08	18.40	30.1
32.5°	27.04	24.58	37.0
35.0°	40.7	33.3	46.1
37.5°	61.9	45.8	58.4
40.0°	95.4	64.2	75.3
42.5°	149.9	91.9	99.2
45.0°	241.0	134.9	133.9

This relation is indicated by the full line marked N_γ in Fig. 1, and it is proposed to use this until more exact evidence becomes available.

By using an approximate figure of rupture similar to Lundgren and Mortensen's, Meyerhof found in 1955 [20] values close to the full line in Fig. 1.

The numerical values of N_γ, N_q and N_c — corresponding to the formulas (10), (8) and (9) — are given in Table 1.

4. Inclination factors.

When using a general bearing capacity formula with inclination factors, Q is best defined as the vertical (normal) component of the bearing capacity. The foundation load has a vertical component V and a horizontal component H.

i_q and i_c can now be calculated by considering the theoretical case of weightless, cohesionless earth ($\bar{\gamma} = 0$, $c = 0$). This has been done e. g. by Schultze [11], Meyerhof [12] and the author [1]. The correct figure of rupture for such a case is shown in Fig. 3, and the results can be expressed as follows. For any given value of $H : V = \tan \delta$ the angle α is determined by the equation:

$$\tan\left(\alpha - \frac{1}{2}\varphi\right) = \frac{\sqrt{1 - (\tan\delta \cot\varphi)^2} - \tan\delta}{1 + \tan\delta : \sin\varphi} \quad (11)$$

Subsequently, i_q and i_c are found from the equations:

$$i_q = \frac{1 + \sin\varphi \sin(2\alpha - \varphi)}{1 + \sin\varphi} e^{-(0.5\pi + \varphi - 2\alpha)\tan\varphi} \quad (12)$$

$$i_c = i_q - \frac{1 - i_q}{N_c \tan\varphi} = i_q - \frac{1 - i_q}{N_q - 1} \quad (13)$$

Incidentally, equations analogous to (13) will — according to (6) — apply also to the depth factors d and the shape factors s.

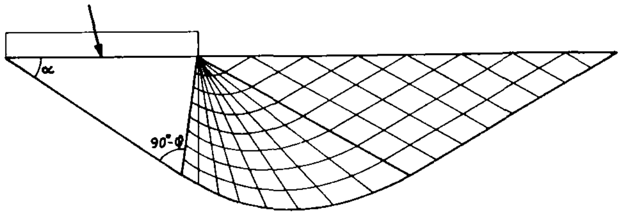

Fig. 3. Inclined load on weightless earth.

In the special case of $\varphi = 0°$ a similar calculation can be made, yielding the exact results:

$$\cos 2\alpha^\circ = \frac{H}{Bc} \quad (14)$$

$$i_c^\circ = \frac{1}{2} + \frac{2\alpha^\circ + \sin 2\alpha^\circ}{\pi + 2} \quad (15)$$

A somewhat simpler formula, giving approximately the same results, is the following, in which the effective foundation area A $(= B\,L)$ has been introduced instead of the width B of a strip foundation in order to make the formula more general:

$$i_c^\circ \sim \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{H}{A\,c}} \quad (16)$$

The still simpler formula proposed by the author in 1959 [6] : $i_c^\circ \sim 1 - H : 2Ac$ is actually too inaccurate, although on the safe side.

From the above formulas it will be seen that, in the case of $c = 0$, i_q is a function of $H : V$, whereas in the case of $\varphi = 0°$ i_c is a function of $H : A\,c$. By means of the principle used for developing equation (5) we can find that, if H_1 and V_1 correspond to the special case of ($\bar{\gamma} = 0$, $\bar{q} = 1$, $c = 0$), the values H and V corresponding to the more general case of ($\bar{\gamma} = 0$, $\bar{q} \neq 0$, $c \neq 0$) must be:

$$H = (\bar{q} + c \cot\varphi)\, H_1 \quad (17)$$

$$V = (\bar{q} + c \cot\varphi)\, V_1 - A\,c \cot\varphi \quad (18)$$

Hence, the ratio determining the inclination factors must actually be:

$$\frac{H_1}{V_1} = \frac{H}{V + A\,c \cot\varphi} = \frac{H}{A\,c + V\tan\varphi} \tan\varphi \quad (19)$$

The ratio $H : (A\,c + V \tan\varphi)$ might be considered especially suitable, since this ratio attains the value 1, when the (rough) foundation begins to slide horizontally along the surface. Using this ratio as reference Fig. 4 shows the exact values of i_c, as found from the formulas (11) — (15).

A very interesting result is found by plotting i_q as function of the ratio $H : (V + A\,c \cot\varphi)$. This is done in Fig. 5, and it will be seen, that it is then practically independent of φ. Actually the following approximate formula will usually suffice for practical calculations:

$$i_q \sim \left[1 - \frac{H}{V + A\,c \cot\varphi}\right]^2 \quad (20)$$

The simpler formula originally proposed by the author [1] for $\varphi = 30°$ and $c = 0$: $i_q \sim 1 - 1.5\, H : V$ is seen

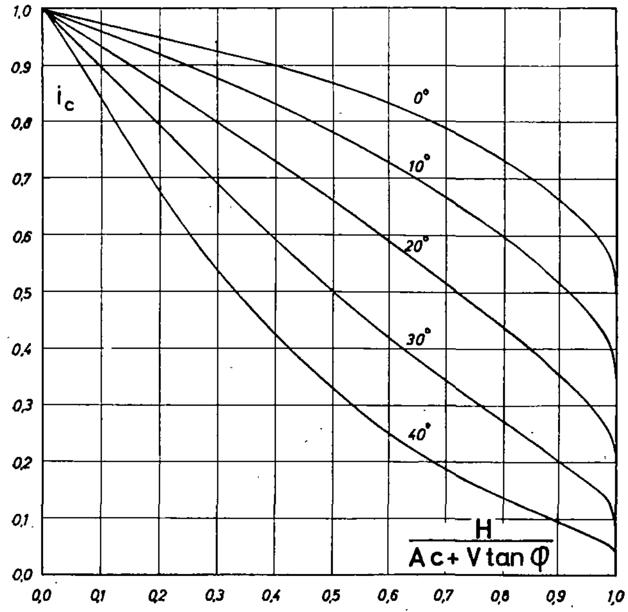

Fig. 4. Inclination factor for c-term.

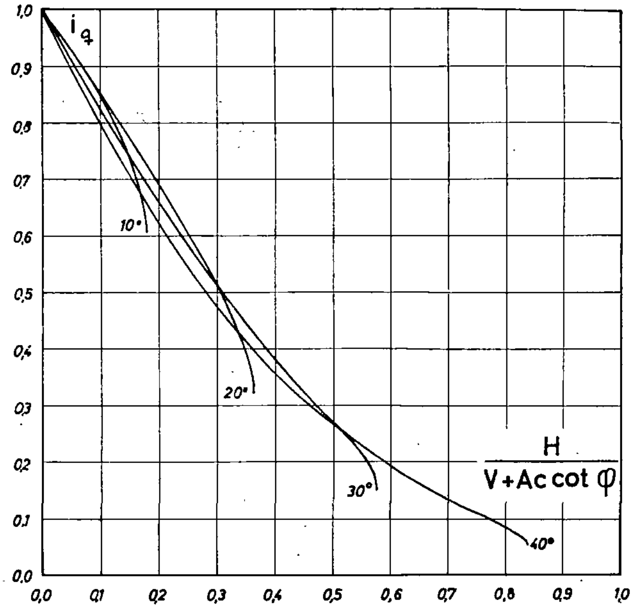

Fig. 5. Inclination factor for q-term.

to be a good approximation for this friction angle, but (20) is equally good and has the advantage of being valid for the other friction angles, too.

The value of i_c is found from (13), except in the case of $\varphi = 0°$, where (16) must be used.

When we want to calculate i_γ, we meet the same difficulty as in the calculation of N_γ, namely that the correct rupture-figure is not yet known. We can, however, make an approximate calculation by using circular rupture-lines. The calculations are made by means of the author's "equilibrium method" [9, 5, 7] and tables published by The Danish Geotechnical Institute [13]. The results of such calculations are indicated by the full curves in Fig. 6, where the ratio $H : (A\,c + V \tan\varphi)$ is used as reference.

Another, but much more complicated way is to use a more probable rupture-figure such as the one shown in Fig. 7 (first proposed by the author). In the plastic zone the results obtained by the method of Lundgren and Mortensen [10] are used. Such calculations have been made by D. Odgaard for $\varphi = 20°$, 30° and 40°, and the results are indicated by the dashed curves in Fig. 6.

As will be seen, the deviations between the results found by these two widely different methods are surprisingly small. Plotting the results found by the first method — which are the lowest — as functions of the ratio $H : (V + A\,c \cot\varphi)$, we get the picture shown in Fig. 8. It will be seen, that also i_γ is then practically independent of φ and can — with sufficient accuracy — be calculated from the following approximate formula:

$$i_\gamma \sim i_q^2 \sim \left[1 - \frac{H}{V + A\,c \cot\varphi}\right]^4 \qquad (21)$$

Incidentally, the relationship $i_\gamma \sim i_q^2$ was already proposed by the author in 1955 [1].

5. Depth factors.

In practice, foundation level is always placed at a depth D below surface level. This influences the bearing capacity in two ways. First, $\bar{q}$ must now be interpreted as the effective overburden pressure at foundation level and, second, depth factors d must be introduced.

As regards d_γ, is should be calculated assuming $c = 0$ and $\bar{q} = 0$. But this means that the earth above foundation level should be considered cohesionless, weightless and unloaded. It will then evidently contribute nothing to the bearing capacity, so that we must have:

$$d_\gamma = 1 \qquad (22)$$

In order to calculate d_q we must consider the special case of ($\gamma = 0$, $\bar{q} = 1$, $c = 0$). For $D > 0$ the correct rupture-figure is not known yet, however, although Bent Hansen has made some valuable suggestions. Meyerhof [14] has made approximate calculations assuming an inclined earth surface triangularly loaded, but his results are a little difficult to interprete, because he combines the γ- and q-terms.

A quite good approximation for reasonably small depths can be obtained by using circular rupture-lines, in combination with the author's equilibrium method. The forces acting on the vertical earth face are here assumed to correspond to Kötter's equation for a vertical rupture-line ($K = \cos^2\varphi$), but for comparatively small depths they do not influence the results very much.

Such calculations have been made for $\varphi = 0°$, 10°, 20°, 30° and 40°. The result is that, for values of $D : B < 1$,

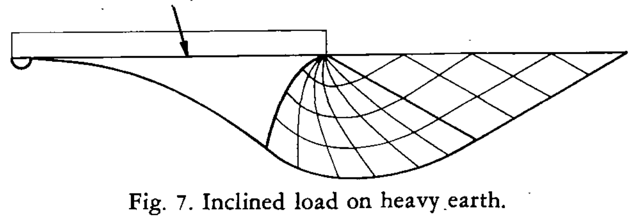

Fig. 7. Inclined load on heavy earth.

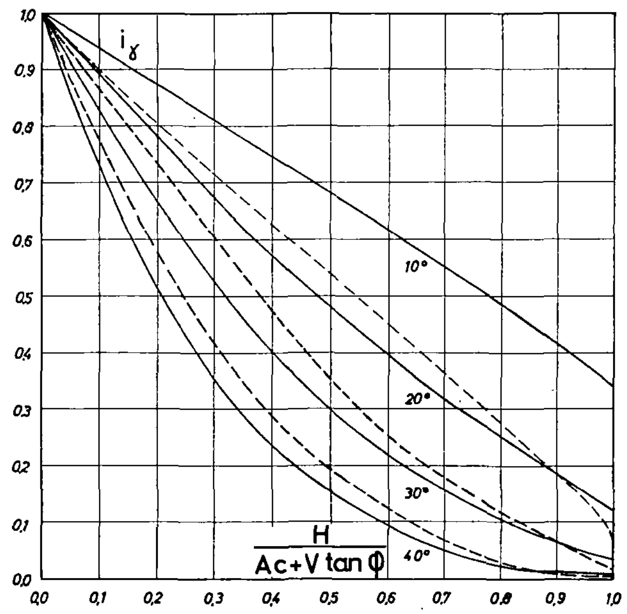

Fig. 6. Inclination factor for γ-term.

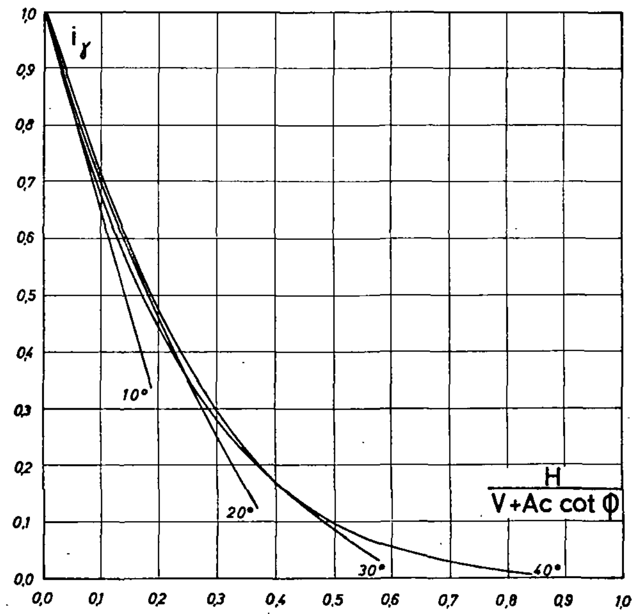

Fig. 8. Inclination factor for γ-term.

d_c can be expressed by the following approximate formula, valid for alle the investigated friction angles:

$$d_c \sim 1 + 0.35\frac{D}{B} \qquad (D < B) \tag{23}$$

This formula is, therefore, a good approximation for shallow foundations, but it cannot be used for caissons or piles.

For great values of $D:B$ it is evident that d_c must approach asymptotically a final value, which would be expected to be a function of the friction angle. According to Skempton [2] this final value should be about 1.5 for $\varphi = 0°$. From Meyerhof's Fig. 19 in [14] it can be deduced that the final value of d_q (which for sand is approximately equal to d_c) should be less that 2 for loose sand and more than 4 for dense sand.

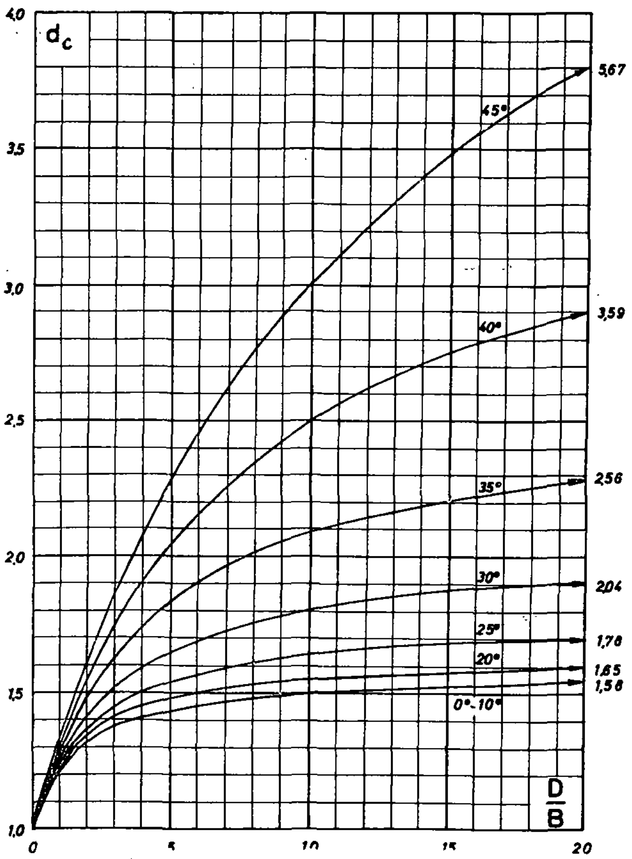

Fig. 9. Depth factor for c-term.

The simplest empirical formula, which fulfills all the above-mentioned requirements, is the following:

$$d_c \sim 1 + \frac{0.35}{B:D + 0.6:(1 + 7\tan^4\varphi)} \tag{24}$$

Fig. 9 shows the values of d_c according to this formula. The figures at the arrows indicate the final values of d_c (for $D = \infty$).

When d_c has been found, d_q can be calculated by means of (13), which is valid both for i, d and s:

$$d_q = d_c - \frac{d_c - 1}{N_c \tan\varphi + 1} = d_c - \frac{d_c - 1}{N_q} \tag{25}$$

For friction angles of 25° or more ($N_q > 11$) it will in practice be sufficiently correct to assume $d_q \sim d_c$. The error will then be less than 4 %. For $\varphi = 0°$ we have, of course, $d_q^\circ = 1$.

It is evident that, in calculating the depth factors d, D must be taken only as the depth of layers of equal or better strength than the layer immediately below foundation level. Softer layers above this level contribute with their effective weight to $\bar{q}$, but not to d.

When the foundation load is inclined, one of two things can be done. Either the depth factors are used, or a passive earth pressure on one side of the foundation (calculated as for a smooth wall) is taken into consideration by including it in H. It is, however, not allowed to do both of these things at the same time, as they are both caused by the shear strength of the soil above foundation level.

6. Shape factors.

If, instead of an infinitely long strip foundation of width B, we have a rectangular foundation area of width B and length L ($L \geqq B$), shape factors s must be introduced in the bearing capacity formulas.

Unfortunately, theoretical calculation is extremely difficult in this 3-dimensional case. However, a certain amount of empirical information can be derived from model and full scale tests.

According to Skempton [2], the following empirical formula should be used for $\varphi = 0°$:

$$s_c^{\circ} \sim 1 + 0.2\frac{B}{L} \tag{26}$$

From Meyerhof's Fig. 19 in [14] it appears that for a circular foundation s_q (which for sand is approximately equal to s_c) should not be much greater than 1.2 for loose sand, whereas it should exceed 2 for dense sand. According to his Fig. 20 the shape factor should vary, not only with the friction angle, but also with the depth ratio $D:B$. This, however, is due to Meyerhof's combination of the γ- and q-terms. Actually, quite similar results are obtained by using constant, but different shape factors for the γ- and q-terms.

The simplest empirical formula, which covers the abovementioned evidence, is the following:

$$s_c \sim 1 + (0.2 + \tan^6 \varphi)\frac{B}{L} \tag{27}$$

The upper part of Fig. 10 shows the values of s_c according to this formula.

When s_c has been found, s_q can be calculated by means of a formula analoguous to (25). However, for $\varphi \geqq 25°$ it will be sufficiently correct to assume $s_q \sim s_c$. For $\varphi = 0°$ we have, of course, $s_q^{\circ} = 1$.

According to Meyerhof's Fig. 20 his combined shape factor should be equal to 1 at $D:B \sim 0.25$ for all foundation shapes and all friction angles. This condition will be very nearly fulfilled, if we assume:

$$s_\gamma \sim 1 - \frac{1}{2}(0.2 + \tan^6 \varphi)\frac{B}{L} \tag{28}$$

The lower part of Fig. 10 shows the values of s_γ according to this formula. It will be noted that, for $\varphi \sim 37°$, we have $s_\gamma = 0.8$ for $B = L$. This agrees with Terzaghi's indication of $s_\gamma = 0.8$ for a square foundation area on sand [3].

7. Point resistance of piles.

The best check on the formulas indicated above for the depth and shape factors is obtained by using them to calculate the point resistance of piles.

For piles the γ-term can usually be neglected and the last term in equation (7) left out. Moreover, for most piles (other than sheet piles) we will have $B \sim L$. Calling the point area A_p, equation (7) gives us then the point resistance:

$$Q_p : A_p \sim (c + \bar{q} \tan \varphi)\, N_c\, s_c\, d_c \sim (c\, N_c + \bar{q}\, N_q)\, s_c\, d_c \tag{29}$$

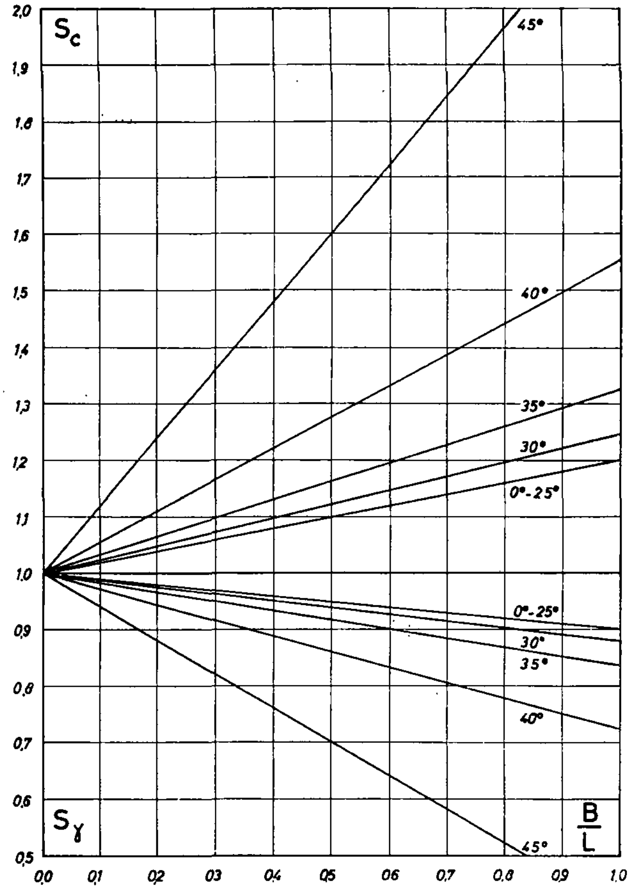

Fig. 10. Shape factors.

Fig. 11 shows the point resistance factor $s_c d_c$. The figures at the arrows indicate the final values (for $D = \infty$). As piles will usually have $D:B = 10-20$, we find for $\varphi = 0°$ a product $s_c^{\circ} d_c^{\circ} \sim 1.8$ which, as $N_c^{\circ} \sim 5$, gives the well-known point resistance factor of 9 for clay. This is, of course, due to the fact that we have employed Skempton's results [2] for $\varphi = 0°$.

For $\varphi = 35°$ we find the factor $s_q d_q \sim 3$ in accordance

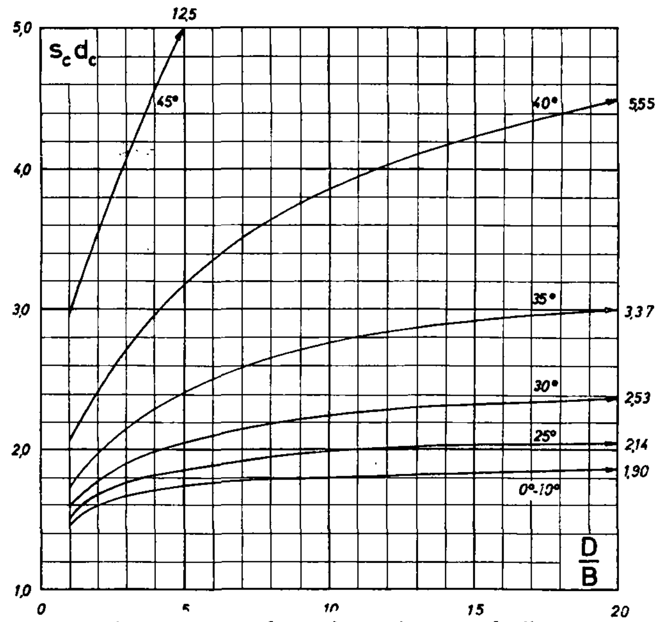

Fig. 11. Factor for point resistance of piles.

with the author's previous theory for the point resistance of piles in sand [15, 16]. According to Fig. 1 in [16] this should be approximately correct for $\varphi = 35°$, whereas for $\varphi = 40°$ the factor might be increased by about 50 %, and for $\varphi = 30°$ should be decreased somewhat. It will be seen that the results in Fig. 11 conform to these requirements.

8. Equivalent and effective foundation areas.

So far we have dealt only with rectangular foundation areas, centrally loaded. If, however, a centrally loaded foundation area has another shape, it must first be transformed into an "equivalent" rectangle in order to enable us to use the developed formulas. The position and side lengths (B and L) of this rectangle may be determined by the following conditions:

1) The centers of gravity should coincide.
2) The main axes should coincide.
3) The area should be the same ($= BL$).
4) The ratio of maximum to minimum plastic section modulus should be the same ($= L : B$)

If a foundation area (of any shape) is eccentrically loaded, we must first determine an "effective" foundation area, as proposed in 1953 by Meyerhof [12], who assumed its inner contour to be a straight line. We shall here determine it by means of the following conditions:

1) The effective area should be centrally loaded.
2) Its inner contour should be fixed by the principle of radial symmetry.

Although a thus determined effective area may be kinematically impossible, it is often simpler to determine and leads to approximately the same results as Meyerhof's.

If necessary, the effective area is subsequently transformed into an equivalent rectangle. It is the width (B), length (L) and area ($A = BL$) of this effective, equivalent rectangle, which are to be used in the calculation of the bearing capacity.

Some typical examples are shown in Fig. 12. It will be realized that B and L can usually be estimated sufficiently correctly without actual calculation.

9. Doubly inclined foundation loads.

The effect of the horizontal force H on the bearing capacity is expressed by means of the inclination factors i, but these have been developed for the special case of a strip foundation with the force H acting perpendicularly to its length axis.

In the more general case of a finite, effective foundation area A, acted upon by a horizontal force H which may not be parallel with any of its main axes (Fig. 12), it is at present impossible to make any real calculations. It will, however, be on the safe side to use the previously indicated formulas for the inclination factors, provided that H is taken as the resultant horizontal force and A as the effective foundation area.

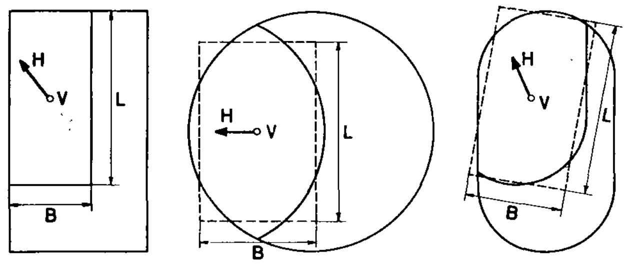

Fig. 12. Equivalent and effective foundation areas.

10. Distribution of contact pressure.

Assuming that failure actually takes place in the soil under the foundation, it is easy to show that — in the case of a centrally loaded strip foundation on the surface — the contact pressure corresponding to the $\bar{q}$- and c-terms in the bearing capacity formulas must be uniformly distributed. The same will probably be approximately correct also for foundation areas of other shapes and at finite depths. In the case of eccentric loads it will also be sufficiently correct to assume a uniform distribution over the effective foundation area.

As regards the distribution of the contact pressure corresponding to the γ-term this cannot be determined exactly, not even in the simplest case of a centrally and vertically loaded strip foundation on the surface. It is only known that in this case the pressure must be zero at the edges and attain a maximum value in the middle. Consequently, a parabolic or triangular distribution of the $\bar{\gamma}$-term is often assumed. It is evident, however, that in the case of eccentric or inclined loads the distribution will be altered. It will therefore not be realistic to assume f. inst. a triangular distribution of the $\bar{\gamma}$-term over the effective foundation width, as the author has proposed previously.

Another thing is that, in designing a foundation, a certain safety against failure is always introduced, so that in actual use the soil under the foundation is not in a state of (total) failure. This means that the actual pressure will be relatively more concentrated near the edges than according to the theory of plasticity.

All taken into consideration — and especially our admitted ignorance of the contact pressure distribution, both in the state of failure and in actual use — the author is now inclined to make the simplest assumption possible: a uniform distribution of the total contact pressure over the effective foundation area.

It is evident, however, that in the case of very extensive foundations, such as mats or rafts, a more detailed investigation of the contact pressure distribution will have to be made [17].

11. Safety factors.

It is, of course, entirely possible to employ the usual concept of a "total" safety factor F. In that case the calculated ultimate bearing capacity should simply be divided by F in order to give the "allowable" bearing

capacity. In Denmark $F = 2$ would be considered a suitable value.

The author prefers, however, the use of the so-called "partial" coefficients of safety [18, 19]. The principles shall be recapitulated here.

The foundation is designed for equilibrium in a "nominal" state of failure. Dead loads and water pressures are used unaltered, whereas the actual live loads p are multiplied by factors f_p. The corresponding nominal foundation load has the components V_n and H_n (to be used for calculating the inclination factors).

In calculating the nominal bearing capacity Q_n (vertical component) we do not use the actual shear strength parameters of the soil (c and φ) but nominal values defined by:

$$c_n = \frac{c}{f_c} \qquad \tan \varphi_n = \frac{\tan \varphi}{f_\varphi} \tag{30}$$

For calculating the short-term bearing capacity the "undrained" parameters $c = c_u$ and φ_u ($= 0°$ for fully saturated clay) must be used, whereas the long-term bearing capacity is calculated with the "effective" parameters $\bar{c}$ and $\bar{\varphi}$.

The foundation should in principle be given such dimensions that $Q_n = V_n$.

Finally, the foundation proper is designed for the nominal moments M_n etc. with nominal stresses $\sigma_n = \sigma_f : f_m$, where σ_f is the actual ultimate (or yield) strength of the material.

In Denmark, the following values of the different partial coefficients have been proposed for foundations [6]:

$$f_p = 1.5 \quad f_c = 1.75 \quad f_\varphi = 1.2$$
$$f_m = 1.4 \text{ (steel) or } 2.8 \text{ (concrete)}$$

The reason for putting f_φ much lower than f_c is, firstly, that φ usually will show much less variation than c and, secondly, that $f_\varphi = 1.2$ for sand corresponds approximately to a total safety of $F = 2$.

12. Calculation.

A design of a foundation according to the principles set forth in this paper will usually proceed as follows.

After having estimated the depth and main dimensions of the foundation, we calculate first the nominal foundation load (components V_n and H_n), using the partial coefficients of safety indicated in section 11.

The nominal load resultant intersects the underside of the foundation in a point which, per definition, is the center of the effective foundation area. By means of the principles indicated in section 8 we determine then the nominal, equivalent, effective foundation area (width B_n, length L_n and area $A_n = B_n L_n$).

Next, the nominal shear strength parameters (c_n and φ_n) are calculated as indicated in section 11. They are used for determining the bearing capacity factors N_γ N_q and N_c (Fig. 1), as well as the inclination factors i_γ, i_q and i_c — by means of formulas (20), (21) and (13) — the depth factors d_γ ($= 1$) and d_c (Fig. 9) and the shape factors s_γ and s_c (Fig. 10). All the values of V, H, B, L and A to be used for this purpose should be the nominal ones.

The nominal bearing capacity is now found from equation (7) or — in the case of $c = 0$ (sand) — simpler from equation (5). It should then be checked whether $Q_n \sim V_n$; if not, some of the main dimensions must be changed and the calculation repeated until sufficiently good agreement is obtained.

The nominal moments etc. in the pertaining sections of the foundation can now be calculated, assuming a uniform distribution of the total nominal contact pressure over the effective foundation area. The design of these sections is, finally, made with nominal strengths of the building materials as indicated in section 11.

13. Example.

A TV. transmission tower of reinforced concrete should be founded on a circular foundation slab. The total weight of the tower is 3000 t (including the foundation slab and overlying soil), and the total wind force is calculated at 150 t, its resultant being located at a height of 35 m above foundation level.

The soil is a Danish glacial (moraine) clay with a unit weight of 2.2 t/m³. The (theoretical) ground water table is assumed to coincide with foundation level at $D = 2$ m under the surface. The clay has an undrained shear strength of $c_u = 18$ t/m² ($\varphi_u = 0°$) and its effective shear strength parameters are $\bar{c} = 3$ t/m² and $\bar{\varphi} = 35°$.

Using the system of partial coefficients we find:

$$V_n = 3000 \text{ t}$$
$$H_n = 150 \cdot 1.5 = 225 \text{ t}$$
$$\bar{q}_n = 2.2 \cdot 2.0 = 4.4 \text{ t/m}^2$$
$$c_n = 18 : 1.75 = 10{,}3 \text{ t/m}^2$$
$$\bar{c}_n = 3 : 1.75 = 1.7 \text{ t/m}^2$$
$$\tan \bar{\varphi}_n = (\tan 35°) : 1.2 = 0.700 : 1.2 = 0.582$$
$$\bar{\varphi}_n \sim 30°$$

The eccentricity of the foundation load is $225 \cdot 35 : 3000 = 2.63$ m. Estimating the diameter of the foundation at 12 m the effective and equivalent foundation areas will be as shown in the middle of Fig. 12. Here we find approximately $B = 5.5$ m and $L = 9.0$ m.

We shall now first calculate the short-term bearing capacity, corresponding to $\varphi = 0°$. In this case we have:

$$N^\circ_\gamma = 0 \qquad N^\circ_q = 1 \qquad N^\circ_c = 5.1$$

As $D : B = 2.0 : 5.5 = 0.36$ and $B : L = 5.5 : 9.0 = 0.61$, formulas (23) and (26) give for $\varphi = 0°$:

$$d^\circ_c = 1 + 0.35 \cdot 0.36 = 1.13$$
$$s^\circ_c = 1 + 0.2 \cdot 0.61 = 1.12$$

For the inclination factor formula (16) gives:

$$i_c^{\circ} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{225}{5.5 \cdot 9.0 \cdot 10.3}} = 0.87$$

Equation (7) now gives the nominal short-term bearing capacity:

$$Q_n : BL = 10.3 \cdot 5.1 \cdot 1.12 \cdot 1.13 \cdot 0.87 + 4.4 = 62 \text{ t/m}^2$$

$$Q_n = 62 \cdot 5.5 \cdot 9.0 = 3070 \text{ t} \sim V_n = 3000 \text{ t}$$

As far as the short-term stability is concerned, the diameter has apparently been estimated correctly. We investigate now the long-term bearing capacity, corresponding to $\varphi_n = 30°$. In this case we have (Fig. 1):

$$N_\gamma = 18 \qquad N_q = 18.5 \qquad N_c = 30$$

For $D : B = 0.36$ formula (23) gives as before $d_c = 1.13$, whereas Fig. 10 for $B : L = 0.61$ gives the following shape factors:

$$s_\gamma = 0.92 \qquad s_c = 1.15$$

For the inclination factors formulas (20), (21) and (13) give:

$$i_q = \left[1 - \frac{225}{3000 + 5.5 \cdot 9.0 \cdot 1.7 \cdot 1.73}\right]^2 = 0.86$$

$$i_\gamma = 0.86^2 = 0.74 \qquad i_c = 0.86 - \frac{1 - 0.86}{18.5 - 1} = 0.85$$

Equation (7) gives now the nominal long-term bearing capacity:

$$Q_n : BL = \frac{1}{2}(2.2 - 1) \cdot 5.5 \cdot 18 \cdot 0.92 \cdot 1 \cdot 0.74$$
$$+ (1.7 + 4.4 \cdot 0.58) \cdot 30 \cdot 1.15 \cdot 1.13 \cdot 0.85$$
$$+ 4.4 = 185 \text{ t/m}^2 > 62$$

The long-term stability is evidently ample, even when the full wind load is taken into account. This is, of course, not necessary in a long-term analysis.

14. Summary.

Terzaghi's simple formula for the bearing capacity of a foundation can be generalized by means of shape-, depth- and inclination factors. The simplest form of such a general formula is (7).

The exact values of the bearing capacity factors N_q and N_c are first calculated (8 and 9). For N_γ an upper and a lower limiting curve are indicated, and a simple empirical formula is given (10).

The inclination factors are now investigated, and it is shown that for i_q and i_γ two simple, empirical formulas can be indicated (20 and 21).

For the depth- and shape factors simple, empirical formulas are also developed (24, 27 and 28), corresponding to the available evidence (Meyerhof), and it is shown that they lead to plausible values of the point resistance factors for piles.

Eccentric loads are dealt with by means of the so-called effective foundation area which, per definition, is centrally loaded by the foundation load.

After a discussion of the contact pressure distribution and safety factors an example (a TV tower) is finally calculated by means of the new formula and the revised factors, using the so-called partial coefficients of safety.

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