Bayesian game-theoretic modelling of uplink power determination in uniform, self-organising network

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A Bayesian game-theoretic model is applied to the problem of transmit power determination in the uplink of a self-organising CDMA cellular network with uniformly-distributed terminals. The existence of a Bayesian Nash equilibrium is shown and simulation results confirm only slight performance degradation compared to a fully centralised approach.

Introduction: It is well known that power control of the wireless channel, e.g. in the uplink of a traditional cellular network, is becoming increasingly critical owing to the emergence of diversified networks which require more precise allocation of resources [1]. In addition, increased research interest has been focused of late on self-organising networks in which nodes, in a decentralised fashion, allocate communication resources and commence transmission [2].

Non-cooperative game theory, as borrowed from economic theory [3], has emerged as an apt framework for the analysis of power determination and distributed self-organisation by offering a model of wireless nodes as 'selfish' with an inability to form binding contracts of behaviour [4]. The existence of Nash equilibria of various power control games, and in some instances their explicit determination, has been offered in the literature [5, 6].

However, inherent in virtually all previous efforts has been that all aspects of the system, notably path losses to the base station, are common knowledge across all participants. (A rare exception is [7].) In an effort to model more realistic self-organising systems, we herein add uncertainty to the path loss by assuming a uniform distribution of terminals. By means of the standard Harsanyi transformation of incomplete information (Bayesian) games [8], we investigate the existence of Bayesian Nash equilibrium power strategies and compare such equilibria to those obtained from a traditional centralised approach.

System model and assumptions: We consider the case of N nodes transmitting in the uplink of a CDMA network (assumed to be without the benefit of advanced multiuser detection techniques). The physical location of each node is drawn according to a two-dimensional uniform distribution over an ideal annulus of inner radius $R_{\min}$ and outer radius $R_{\max}$. We may identify $R_{\min}$ with the antenna far-field reference distance and $R_{\max}$ with the radius of the cell. Let R be the random variable representing the physical radius of any of the (indistinguishable) nodes to the base station. The probability density function (PDF) of R, $f_R(r)$, is then given by:

$$f_R(r) = \frac{d}{dr}\left[\frac{\pi r^2 - \pi R_{\min}^2}{\pi R_{\max}^2 - \pi R_{\min}^2}\right] = \frac{2r}{R_{\max}^2 - R_{\min}^2} \qquad (1)$$

for $R_{\min} \le r \le R_{\max}$ with $f_R(r) = 0$ elsewhere.

We assume a simple model of path loss that includes both distance loss and large-scale shadowing effects [9]:

$$L(R) = \bar{L}(R_{\min}) + 10n\log_{10} R - 10n\log_{10} R_{\min} + X_\sigma \qquad (2)$$

where L is the path loss in dB, $\bar{L}(R_{\min})$ is the (constant) reference distance path loss, n is the propagation exponent, and X_σ is a zero-mean normally-distributed shadowing term with standard deviation σ dB, truncated to $\pm 3\sigma$ to ensure a bounded path loss.

The independence of the distance and shadowing losses allows us to convolve the random variables in (2) by employing (1), yielding the following path loss PDF, $f_L(\ell)$:

$$f_L(\ell) = \begin{cases} \int_{a-C_0}^{\ell - C_0 + 3\sigma} g(x)\,dx, & \text{if } a - 3\sigma < \ell \le a + 3\sigma; \\ \int_{\ell - C_0 - 3\sigma}^{\ell - C_0 + 3\sigma} g(x)\,dx, & \text{if } a + 3\sigma < \ell \le b - 3\sigma; \\ \int_{\ell - C_0 - 3\sigma}^{b - C_0} g(x)\,dx, & \text{if } b - 3\sigma < \ell \le b + 3\sigma; \\ 0, & \text{otherwise} \end{cases} \qquad (3)$$

with

$$g(x) = \frac{10^{x/(5n)}}{(R_{\max}^2 - R_{\min}^2)}\,\frac{\ln 10}{10n\,\mathrm{erf}(3)}\,\frac{1}{\sqrt{2\pi\sigma^2}}\,e^{-(\ell - C_0 - x)^2/2\sigma^2}$$

where $C_0 \triangleq \bar{L}(R_{\min}) - 10n\log_{10} R_{\min}$, $a \triangleq 10n\log_{10} R_{\min} + C_0$, $b \triangleq 10n\log_{10} R_{\max} + C_0$, and $\mathrm{erf}(\cdot)$ is the standard error function. (The convolution in (3) is valid if $\sigma < 5n/3 \cdot \log_{10}(R_{\max}/R_{\min})$, which is a reasonable assumption.) Note that $\ell \in [a - 3\sigma,\ b + 3\sigma] \triangleq \mathscr{L}$, the path loss represents an atomless distribution, and the path loss space, when supplemented with the usual Euclidean distance metric, represents a complete, separable metric space.

Each node is assumed to transmit to the base station with a positive power (in watts) $p_i \in \{P_1, P_2, \ldots, P_M\} \triangleq \mathscr{P}$ for some M. Although the transmit power space is assumed to be finite primarily for analytical reasons, it realistically reflects the precision of current transceivers if $M \lesssim 10$.

Game formulation and Nash equilibrium existence: The single-stage Bayesian game herein considered is formally defined as follows:

1. The game players are the N transmitting nodes.
2. The type space for each node is $\mathscr{L}$ with *a priori* PDF given by (3).
3. The action space for each node is $\mathscr{P}$.
4. The utility function for node i, formulated as a difference of throughput and power consumption terms (as in [6]), is a function of its realised path loss value $\ell_i \in \mathscr{L}$ and chosen transmit power $p_i \in \mathscr{P}$ as well as the path loss and transmit power of the other nodes (ℓ_{-i} and p_{-i}, respectively):

$$\begin{aligned} u_i(p_i, p_{-i}; \ell_i, \ell_{-i}) &= \kappa_1 R_b P_s(\gamma) - \kappa_2 p_i \\ &= \kappa_1 R_b P_s\left(10\log_{10}\frac{R_c}{R_b} + 10\log_{10} p_i - \ell_i \right. \\ &\quad \left. - 10\log_{10}\left[\sum_{j=1, j\ne i}^{N} \frac{p_j}{10^{\ell_j}/10}\right]\right) - \kappa_2 p_i \end{aligned} \qquad (4)$$

where κ_1 and κ_2 are the respective weights for the throughput and power consumption terms, R_b is the bit rate, R_c is the chip rate, thermal receiver noise is ignored, and P_s: $\mathbb{R} \to [0, 1]$ is a so-called 'success function', mapping the base station SIR γ (in dB) into the probability of successful reception. We choose here a sigmoidal function for $P_s(\cdot)$ that has been used previously [10] and has the form $P_s(\gamma) = [1 + \exp(-a_{sig}(b_{sig}))]^{-1}$.
5. The types of the N nodes are assumed independent, but not private, as $u_i(\cdot)$ is explicitly a function of ℓ_{-i}.

As is required by game theory, all exogenous aspects of the system (e.g. R_c, R_b etc.) are assumed to be common knowledge to all nodes.

The above structure of the game allows us to employ a seminal result of Milgrom [11, Theorem 3 Corollary] that at least one pure strategy ε-Bayes Nash equilibrium exists for this game for every $\varepsilon > 0$, but this equilibrium need not be unique. Stronger results, such as the existence of pure strategy equilibria, cannot be immediately applied as the players' types are not private.

Simulation and discussion: To compare the performance of the present strategy to other approaches, Bayesian Nash equilibria need to be explicitly determined, and we do so via simulation. We discretise $\mathscr{L}$ into K equally-spaced values and transform the PDF in (3) in the natural way. We consider $N = 2$ nodes and employ a simple exhaustive search that, although computationally inefficient, serves as adequate demonstration of principle. Although there is no *a priori* guarantee of existence of a symmetric pure strategy Bayes Nash equilibrium in this case, such equilibria have been identified for select cases. (Symmetric equilibria are desired, as the nodes are assumed to be indistinguishable.)

For comparison, we also examine the optimal result in which the base station has had reported to it the path losses of the two nodes. The transmit power strategy for each node is then calculated by the base station such that the total utility of the two nodes is maximised:

$$\begin{aligned} &\{p_{1,\max}(\ell_1, \ell_2), p_{2,\max}(\ell_1, \ell_2)\} \\ &\quad = \underset{p_1, p_2}{\arg\max}\left\{\sum_{i=1}^{2} u_i(p_i, p_{-i}; \ell_i, \ell_{-i})\right\} \end{aligned} \qquad (5)$$

Also calculated is a pessimistic single-user decision power strategy in which each node assumes that the neighbouring node maximises its transmit power.

Fig. 1 shows the expected utility of a node that employs the respective equilibrium/optimal power strategies. As can be seen, the reduction in utility from the present game-theoretic strategy is not precipitous from the optimal case (∼1200 utility units on average); only in the upper end of the path loss distribution, which occurs with diminishing probability, is a marked decrease in utility seen.

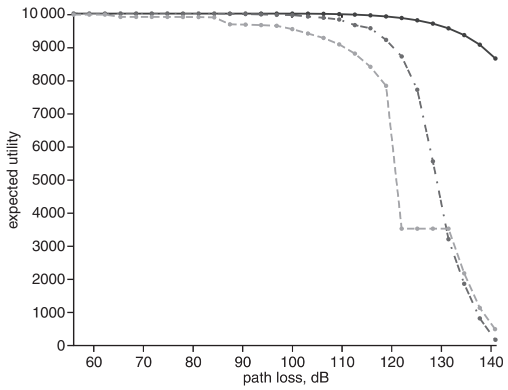

Fig. 1 *Equilibrium/optimal expected utilities against observed path loss*

Simulation parameters: $N=2$, $R_{\min}=100$ m, $R_{\max}=2000$ m, $R_b=10$ kbit/s, $R_c=1.2288$ Mc/s, $n=4$, $a_{sig}=10\ \text{dB}^{-1}$, $b_{sig}=10$ dB, $\sigma=6$ dB, $K=28$, $M=6$, $\kappa_1=1\ \text{kbit/s}^{-1}$, $\kappa_2=10^4\ \text{mW}^{-1}$, $P_1=0.1\ \text{mW}=-40$ dBW, $P_M=31.6228\ \text{mW}=-15$ dBW, $\mathscr{P}$ set to be evenly distributed on logarithmic scale, $freq=1$ GHz

—●— optimal strategy

· · ● · · multi-user (game-theoretic) strategy

- - ● - - single-user strategy

Conclusion: We have presented a game-theoretic analysis of transmit power determination in a self-organising network with path loss uncertainty and proved the existence of a Bayes Nash equilibrium. We note from the simulation results that the distributed solution presented here is only marginally less efficient than the optimal centralised solution, lending support to the self-organising paradigm. But, judging from the relative inefficiency of the single-user decision strategy, the path loss distribution must be taken into account via a proper game-theoretic model, as in the present approach, to be most effective.

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