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Efficiency-Risk Tradeoffs in Dynamic Oligopoly Markets
– with application to electricity markets

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Abstract— **In this paper, we examine in an abstract framework, how a tradeoff between efficiency and risk arises in different dynamic oligopolistic market architectures. We consider a market in which there is a monopolistic resource provider and agents that enter and exit the market following a random process. Self-interested and fully rational agents dynamically update their resource consumption decisions over a finite time horizon, under the constraint that the total resource consumption requirements are met before each individual's deadline. We then compare the statistics of the stationary aggregate demand processes induced by the non-cooperative and cooperative load scheduling schemes. We show that although the non-cooperative load scheduling scheme leads to an efficiency loss - widely known as the "price of anarchy" - the stationary distribution of the corresponding aggregate demand process has a smaller tail. This tail, which corresponds to rare and undesirable demand spikes, is important in many applications of interest. On the other hand, when the agents can cooperate with each other in optimizing their total cost, a higher market efficiency is achieved at the cost of a higher probability of demand spikes. We thus posit that the origins of endogenous risk in such systems may lie in the market architecture, which is an inherent characteristic of the system.**

I. INTRODUCTION

Load scheduling, i.e., optimizing the demand for a resource over multiple periods to minimize the expected total cost of consumption, plays a crucial role in a wide array of applications, including dynamic demand response to realtime prices in electricity markets [1], load scheduling in cloud computing under QoS constraints [2], and multi-period rebalancing of multiple portfolio accounts in the presence of transaction costs [3]. In many cases where the price per unit resource in each period is determined by the instantaneous aggregate demand of finitely many agents, the problem falls into the category of dynamic oligopolistic competition.

In a multi-agent networked system, profit-seeking agents try to maximize their own utilities, perhaps by forming rational expectations over the behaviors of other agents, and responding to instantaneous changes in the environment. However, from a system operator's perspective, the impact of the aggregate behavior of rational agents, who aim at maximizing their own utility through dynamic interactions with each other and optimizing their responses to the changes in the environment, is nontrivial – agent interactions can lead to endogenous risk. For example, in electricity markets, aggregate demand spikes can incur additional costs to the resource providers or the power system as a whole. We shall focus on a measure of risk that quantifies such aggregate demand spikes, and examine how they may arise from the oligopolistic market architectures.

In many complex systems with interactive agents, for example, power networks, financial markets, social networks, and biological networks, the mechanisms that can possibly channel exogenous shocks into endogenous risks are still not well understood. Previous research efforts have explored various possible origins of endogenous risk, including heterogeneous beliefs [4], and failure of the agents to endogenize the feedback links [5]. In our work, we assume rational agents, who are fully aware of the pricing mechanism, know about other agents in the market, and form rational expectations. In this work, we provide an alternative explanation through a comparative study, and posit that endogenous risks can arise from the nature of the system architecture and dynamics, even at a rational expectation equilibrium (REE).

We build an abstract dynamic framework to model agent behavior in the form of load scheduling in response to realtime costs. We examine how different agent behaviors are induced by different oligopolistic market architectures. In particular, whether they make load scheduling decisions in a non-cooperative or a cooperative setting. The resulting aggregate demand process has various impacts on the market welfare, among which we shall focus on two metrics: market efficiency and the risk of aggregate demand spikes. We show that across different load scheduling strategies induced by various oligopolistic market architectures, there exists a tradeoff between efficiency and risk. More specifically, under the cooperative market architecture, the agents are more aggressive in absorbing exogenous uncertainties, and they can achieve higher market efficiency, i.e., lower cost. However, the tradeoff is a higher endogenous risk in terms of higher probability of aggregate demand spikes. The implication of our efficiency and risk analysis is that when the system architectures and operational policies are designed, system efficiency should not be the only goal that is pursued, endogenous risk and the associated tradeoffs should also be carefully considered.

An interesting example where we can apply the analytical framework to study the efficiency-risk tradeoffs is the dynamic demand response to realtime prices in electricity markets in the form of scheduling flexible loads. On the supply side, the intermittency of the renewable sources introduces exogenous supply shocks. On the demand side, large or perhaps small consumers may actively respond to the realtime market prices. A great portion of the consumer

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response will come from scheduling flexible loads. A specific example of electrical vehicle charging where our framework fits can be found in [6]. In electricity markets, exceedingly large price spikes introduce a level of volatility that can not only cause serious damage to both the service providers and the consumers, but also undermine viability of the power market as a whole. Assuming inelastic demand, the strategic bidding behavior of suppliers that can drive up prices is studied in [7]. However, we model the load scheduling behavior of both the consumers and distributed renewable generations. The resulting dynamic demand supply interaction can better model future smart grids.

This paper is structured as follows: in section II, we introduce the system model and formulate the problem; in section III, we focus on a specific case for which analytical solutions are obtained, the efficiency-risk tradeoffs are analyzed for this case; in section IV, we discuss the general case and study the tradeoff for a class of parameterized load scheduling strategies; in section V, we examine how the risk sensitivity of the agents affects the tradeoff. Due to page limit we omit the proofs. Please refer to [8] for more details.

II. SYSTEM MODEL

A. Agent Arrival Process

We analyze a market model in which the agent arrival process is a discrete time random process. When an agent arrives, he activates a job that requires consuming a certain amount of the resource to complete. The agent has to finish the job within a finite window of time, and leave the market at his deadline. We define the number of periods that an agent stays in the market to be his *type*, denoted by $l \in \mathcal{L} = \{1, \dots, L\}$. We assume that agents of type l arrive according to a Bernoulli process $\{h_l(t) : t \in \mathbb{Z}\}$, with rate q_l. Upon arrival at the beginning of period t, an agent carries a job which requires consuming $d_l(t)$ units of the resource. We assume that the sequence $\{d_l(t) : t \in \mathbb{Z}\}$ is i.i.d., drawn from a general distribution $\mathcal{D}_l$ with mean $\mu_l = \mathbb{E}[\mathcal{D}_l]$, variance $\sigma_l^2 = \mathrm{Var}[\mathcal{D}_l]$, and support over the set of all real numbers $\mathbb{R}$. Let the L-dimensional column vectors $\mathbf{h}(t) = [h_l(t)] \in \{0, 1\}^L$, and $\mathbf{d}(t) = [d_l(t)] \in \mathbb{R}^L$ denote the vector forms of arrival events and the corresponding workloads. Let $U(t)$ denote the instantaneous aggregate demand for the resource from all agents in the market.

Remark 1: Note that we allow the load realizations as well as the instantaneous resource demand from the agents to become negative. This, models the situation where distributed agents can be both suppliers and consumers in the market. In financial market, the informed traders can be both buyers and sellers, and the uninformed traders have a role similar to the monopolistic provider [9]. In electricity markets, this corresponds to the scenario where consumers are equipped with distributed renewable generations or energy storage, and are able to sell energy back to the power grid. We also ran numerical simulations for the scenario where there is a lower bound on instantaneous resource demand / supply. In particular, when the lower bound equals zero, the agents are only consumers and cannot supply the resource to the market. In all of our the simulations, the efficiency-risk tradeoff is still observed.

B. Resource pricing

We assume that there is a monopolistic resource provider which always produces enough amount of the resource to meet the aggregate demand in each period. Moreover, we assume that the production costs borne by the provider is of quadratic form $\frac{1}{2}U(t)^2$, and the monopolist sets $p(t)$, the price per unit resource, to be the marginal cost of production in each period, thus $p(t) = U(t)$. Also, note that the quadratic cost function only models the production cost of the monopolistic resource provider, which we assume to have no intertemporal constraints. Overall, the aggregate demand is satisfied by the sum of the distributed stochastic supplies from the agents, and the resource produced by the monopolistic provider. The price is set by the provider and clears the market, i.e., at this price the overall production matches the aggregate consumption. In electricity markets, marginal cost pricing is a widely used mechanism [10]. Moreover, the monopolist corresponds to the conventional electricity generation which provides reliable electricity, as opposed to the distributed renewable generations, which are stochastic in the nature.

C. System State Evolution

At any period t, we group the agents by their departure times. For any $\tau \in \mathcal{L}$, there are at most $(L + 1 - \tau)$ agents who will stay in the market for τ periods (including t). They correpond to the type τ arrival at time t, the type $(\tau + 1)$ arrival at time $(t - 1)$, etc. An example for $L = 5$, $\tau = 3$, with 3 agents is shown in Fig. 1. For notational convenience,

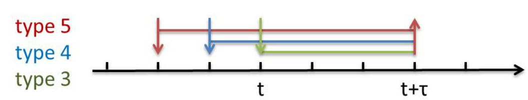

Fig. 1. For $L = 5$, at time t there are 3 agents that will stay in the market for 3 periods.

we index a type l agent who at time t will continue to stay in the market for τ periods by a tuple $(l, \tau)_t$, and list all possible agents in the ordered set:

$$\mathcal{C}=\{(1,1),(2,1),\cdots,(L,1),(2,2),\cdots,(L,2),\cdots,(L,L)\}$$

Let $D_c = L(L + 1)/2$ denote the cardinality of the ordered set $\mathcal{C}$. Let $u_{(l,\tau)}(t) \in \mathbb{R}$ denote the *instantaneous demand* for the resource from agent $(l, \tau)_t$, with the vector form:

$$\mathbf{u}(t) = [u_{(l,\tau)}(t) : (l, \tau) \in \mathcal{C}] \in \mathbb{R}^{D_c}$$

If at time t there is no agent $(l, \tau)_t$, i.e., $h_l(t + \tau - l) = 0$, we simply define $u_{(l,\tau)}(t) = 0$. The instantaneous aggregate demand is therefore:

$$U(t) = \mathbf{e}'\mathbf{u}(t)$$

where $\mathbf{e}$ is a D_c-dimensional column vector of all ones. Similarly, we define the *backlog state* to be a vector:

$$\mathbf{x}(t) = [x_{(l,\tau)}(t) : (l, \tau) \in \mathcal{C}] \in \mathbb{R}^{D_c},$$

with element $x_{(l,\tau)}(t)$ denoting agent $(l,\tau)_t$'s unsatisfied load at time t. We also define the *existence state* to be:

$$\mathbf{z}(t) = [z_{(l,\tau)}(t) : (l,\tau) \in \mathcal{C}] \in \{0,1\}^{D_c},$$

with element $z_{(l,\tau)}(t) = 1$ if and only if there is an arrival of type l agent at time $(t+\tau-l)$. Finally, system state at time t is defined to be $\mathbf{s}(t) = (\mathbf{x}(t), \mathbf{z}(t)) \in \mathcal{S}$, where $\mathcal{S} = \mathbb{R}^{D_c} \times \{0,1\}^{D_c}$ denotes the state space. We assume that system state is updated after the realization of $\mathbf{h}(t)$ and $\mathbf{d}(t)$ at the beginning of each period t, and the state information is publicly available to all agents in the market. Thus, the system state $\mathbf{s}(t) = (\mathbf{x}(t), \mathbf{z}(t))$ evolves as follows:

$$\mathbf{x}(t+1) = \mathbf{R}_1(\mathbf{x}(t) - \mathbf{u}(t)) + \mathbf{R}_2\mathbf{d}(t) \quad (1)$$

$$\mathbf{z}(t+1) = \mathbf{R}_1\mathbf{z}(t) + \mathbf{R}_2\mathbf{h}(t) \quad (2)$$

where $\mathbf{R}_1$ is a $D_c \times D_c$ matrix with non-zero elements:

$$\mathbf{R}_1\Big((k-1)(L+\frac{2-k}{2})+i+1,\ k(L+\frac{1-k}{2})+i\Big) = 1,$$
$$\text{for all } 1 \le i \le L-k \text{ and } 1 \le k \le L-1 .$$

Also, $\mathbf{R}_2$ is a $D_c \times L$ matrix with non-zero elements:

$$\mathbf{R}_2\Big((l-1)(L+\frac{2-l}{2})+1,\ l\Big) = 1, \text{ for all } 1 \le l \le L.$$

D. Non-cooperative Market Architecture

We define the *non-cooperative market architecture* to be a market setup in which there is no coordination among the strategic agents in scheduling their loads. With full information about the system model, an agent $(l,\tau)_t$ makes the decision of his instantaneous resource demand $u_{(l,\tau)}(t)$ based on his observation of system state $\mathbf{s}(t)$. We assume that the agents do not directly derive utility from consumption of the resource. Thus the only objective they have is to minimize their expected total cost, under the constraint that each agent's total consumption by his deadline must be equal to his workload. We define a strategy as a function:

$$\mathbf{u} : \mathcal{C} \times \mathcal{S} \to \mathbb{R}$$

which maps the index of an agent and the system state to his instantaneous demand for the resource. In particular, $u_{(l,\tau)}(\mathbf{s})$ gives the instantaneous demand of agent (l,τ) when he observes the system state $\mathbf{s}$. We shall focus on agent behaviors at steady state equilibrium. Due to the agent arrival random process and the decision making dynamics, the equilibrium concept we adopt here is Markov Perfect Equilibrium (MPE), in the sense of [11]. Moreover, we focus on symmetric equilibria where agents adopt the same strategy. We define the Markov Perfect Symmetric Equilibrium Strategy in our problem as follows:

Definition 1 (Markov Perfect Symmetric Equilibrium Strategy): A strategy $\mathbf{u}^{nc} = \{u^{nc}_{(l,\tau)}(\mathbf{s}) : (l,\tau) \in \mathcal{C}, \mathbf{s} \in \mathcal{S}\}$ is defined to be a Markov Perfect Symmetric Equilibrium Strategy, if the following fixed point equations are satisfied for all agents $(l,\tau) \in \mathcal{C}$ at any time t, for any system state $\mathbf{s}(t) \in \mathcal{S}$:

$$u^{nc}_{(l,\tau)}(\mathbf{s}(t)) = \arg\min_u \mathbb{E}\Big[up(t) + \sum_{i=1}^{\tau-1} u^{nc}_{(l,\tau-i)}(\mathbf{s}(t+i))p(t+i)\Big|\mathbf{s}(t)\Big] \quad (3)$$

$$\text{subject to: } \sum_{i=0}^{l-1} u^{nc}_{(l,l-i)}(\mathbf{s}(t+i)) = d_l(t), \quad \forall t, l,$$
$$p(t) = u + \sum_{(l',\tau') \neq (l,\tau)} u^{nc}_{(l',\tau')}(\mathbf{s}(t)),$$
$$p(t+i) = \sum_{(l',\tau')} u^{nc}_{(l',\tau')}(\mathbf{s}(t+i)), \quad \forall i \ge 1,$$

where $\mathbf{s}(t)$ evolves according to (1), (2).

Proposition 1 (Existence of equilibrium strategy): Assume that the price $p(t)$ is proportional to the instantaneous aggregate demand $U(t)$, and that agents of each type arrive in the market following independent Bernoulli processes with rates q_l, and workloads of each type independently and identically distributed according to distributions $\mathcal{D}_l$. There exists a Markov perfect symmetric equilibrium strategy $\mathbf{u}^s(\mathbf{s})$ which satisfies the fixed point equations in (3).

E. Cooperative Market Architecture

As an efficiency benchmark, we consider the cooperative market architecture, under which the agents can coordinate their actions to minimize their aggregate expected cost. The cooperative market architecture can model the scenario where the agents agree a priori upon a common strategy that minimizes their aggregate expected cost, and respond to the realtime market conditions according to the pre-specified strategy. Particularly, in future electricity markets, the cooperative scheme may correspond to a situation in which consumers with flexible loads pass all the relevant information to a load aggregator who schedules the consumptions on their behalves. We are interested in the system performance in the stationary equilibrium, and define the *optimal stationary cooperative strategy* under the cooperative market architecture as follows:

Definition 2 (Optimal Stationary Cooperative Strategy): A strategy $\mathbf{u}^c = \{\mathbf{u}^c(\mathbf{s}) : \mathbf{s} \in \mathcal{S}\}$, where $\mathbf{u}^c(\mathbf{s}) = \big[u^c_{(l,\tau)}(\mathbf{s}) : (l,\tau) \in \mathcal{C}\big]$, is defined to be an Optimal Stationary Cooperative Strategy if it solves the following fixed point equations for any system state $\mathbf{s}(t) \in \mathcal{S}$:

$$\mathbf{u}^c(\mathbf{s}(t)) = \arg \min_{\big[u_{(l,\tau)}:(l,\tau)\in\mathcal{C}\big]} \lim_{T\to\infty} \frac{1}{T-t} \mathbb{E}\Big[\Big(\sum_{(l,\tau)} u_{(l,\tau)}\Big)^2 + \sum_{t'=t+1}^{T} \Big(\sum_{(l,\tau)} u^c_{(l,\tau)}(\mathbf{s}(t'))\Big)^2 \Big|\mathbf{s}(t)\Big] \quad (4)$$

$$\text{subject to: } \sum_{i=0}^{l-1} u^c_{(l,l-i)}(\mathbf{s}(t+i)) = d_l(t), \quad \forall t, l,$$

where $\mathbf{s}(t)$ evolves according to (1), (2).
The above problem is a standard infinite horizon average cost MDP, and the associated Bellman equation can be solved via standard value iteration or policy iteration.

F. Welfare Metrics

Different oligopolistic market architectures induce different agent behaviors, which lead to different stationary distributions of the aggregate demand process $\{U(t) : t \in \mathbb{Z}\}$. We shall focus on two welfare metrics: **efficiency** and **risk**. More specifically, we define *efficiency* to be the expected sum of the resource provider's surplus and the agents' surplus:

$$W = \mathbb{E}[\underbrace{p(t)U(t) - \frac{1}{2}U(t)^2}_{W_p} + \underbrace{-p(t)U(t)}_{W_c}] = -\frac{1}{2}\mathbb{E}[U(t)^2] \quad (5)$$

Note that under the assumptions of quadratic production cost and marginal cost pricing, efficiency is decreasing in the second moment of the aggregate demand process. In (4), the optimal stationary cooperative strategy equivalently minimizes the aggregate demand variance, the strategy $\mathbf{u}^c(\cdot)$ thus achieves the highest efficiency in the sense of (5). We will denote this by W^c. Let W^{nc} denote the efficiency achieved by the equilibrium strategy $\mathbf{u}^{nc}(\cdot)$ under the non-cooperative market architecture. We have $W^{nc} \leq W^c$. The efficiency loss of the non-cooperative scheme is commonly known as the "price of anarchy" due to the strategic behavior of self-interested agents.

We define *risk* to be the tail probability of the stationary process of aggregate demand:

$$R = \Pr(U(t) > M) \quad (6)$$

for some positive constant M. As a result of marginal cost pricing, this notion of risk also captures the tendency for prices to spike drastically (above M). We also define *market robustness* to be:

$$B = 1 - R \quad (7)$$

Apart from market efficiency, risk, in terms of demand spikes, is also an important performance metric. Rational agents respond to endogenous realtime prices to minimize individual costs, without explicitly considering the externality they create for each other, and for the system as a whole. However, a system designer may have interests different from the agents, and be concerned about the risk, in particular the aggregate demand surges. Moreover, we are interested in pointing out the tradeoffs between efficiency and risk in a variety of market architectures. Even when the agents can coordinate their actions and are risk sensitive, so that large spikes are mitigated, the tradeoff still exists, as will be shown in section V.

In general, there are no closed form solutions for either of the above two formulations, and numerical solutions involve exponential complexity. In the following section, we will look into the case where the number of types $L = 2$, and the equilibrium strategy as well as the optimal cooperative strategy can be solved explicitly.

III. Dynamic Oligopoly Market with Two Types of Agents

A. Equilibrium Strategy and Optimal Cooperative Strategy

When $L = 2$, there are only two types of agents in the system: type 1 agents with uncontrollable loads that must be satisfied upon arrival, and type 2 agents who have the flexibility to split the consumption between two consecutive time periods. Under the assumption of Bernoulli arrival process, at any time t, there are at most 3 agents in the market, which are indexed as: $(1,1)_t$, $(2,1)_t$, and $(2,2)_t$. Among the three agents, only the type 2 agent $(2,2)_t$ that arrives in the current period needs to make a nontrivial load scheduling decision, while the other two agents have no choice but to fulfill their backlogs and leave the market by the end of period t. Considering the case of small L sheds light on understanding agent behaviors induced by different oligopolistic market architectures. For example, it can be used to model the interaction among a few load aggregators with considerable market power.

Since the type 1 agents have no flexibility in scheduling the realized workload, we can incorporate the randomness of Bernoulli arrival of type 1 agents into the distribution $\mathcal{D}_1$ as a probability mass at 0. [1] Without loss of generality, we set $q_1 = 1$, and denote the arrival rate of type 2 agents by $q = q_2$. At period t, agents $(1,1)_t$ and $(2,1)_t$ must empty their backlogs, thus their instantaneous consumptions are determined as $u_{(1,1)}(\mathbf{s}(t)) = x_{(1,1)}(t)$, and $u_{(2,1)}(\mathbf{s}(t)) = x_{(2,1)}(t)$. We define the state of *aggregate uncontrollable load* as follows:

$$\underbrace{x(t)}_{\text{total uncontrollable load}} = \underbrace{d_1(t)}_{\text{from agent } (1,1)_t} + \underbrace{d_2(t-1) - u_{(2,2)}(t-1)}_{\text{from agent } (2,1)_t}.$$

Without ambiguity, we denote $u_{(2,2)}(t)$ by $u(t)$. When agent $(2,2)_t$ schedules his consumption $(u(t), d_2(t) - u(t))$, he does not distinguish between agents $(1,1)_t$ and $(2,1)_t$. To him the sufficient statistics of system state $\mathbf{s}(t) = (\mathbf{x}(t), \mathbf{z}(t))$ is just $(x(t), d_2(t))$, based on which he makes the decision. We define a *linear strategy* as a strategy $u(x, d_2)$ that is linear in both the state x and d_2, i.e.,

$$u(x, d_2) = -ax + bd_2 + g.$$

Finally, in each period t, the cost per unit consumption of the resource is given by $p(t) = U(t) = x(t) + u(t)$.

For the dynamic stochastic oligopolistic game under the non-cooperative market architecture, the equilibrium strategy $u^{nc}(x, d_2)$ is characterized by the solution to the following fixed point equation:

$$u^{nc}(x(t), d_2(t)) = \arg\min_u \Big\{ u(u + x(t)) + \mathbb{E}\Big[(d_2(t) - u) \Big(x(t{+}1) {+} h_2(t{+}1) u^{nc}(x(t{+}1), d_2(t{+}1))\Big) \Big| x(t), d_2(t)\Big]\Big\}, \quad (8)$$

where $x(t+1) = d_1(t+1) + (d_2(t) - u)$.

Proposition 2 (Existence and uniqueness of linear MPE): Under the non-cooperative market architecture, there exists a unique MPE with linear equilibrium strategy $u^{nc}(x, d_2)$ given by:

[1] However, this simplification does not apply to other types of agents with controllable loads, because even when the load realization D_l is zero for some type $l > 1$, the agent still has the freedom to produce and sell the resource in the market, as long as his net consumption is 0 by his deadline.

$$u^{nc}(x,d_2) = -\underbrace{\frac{1}{2(1+\sqrt{1-q/2})}}_{a^{nc}}x + \underbrace{\frac{1}{1+\frac{1}{\sqrt{1-q/2}}}}_{b^{nc}}d_2$$
$$\underbrace{+(\mu_1 + q\mu_2\frac{1}{1+\sqrt{1-q/2}})/(2(1+\sqrt{1-q/2}))}_{g^{nc}} \quad (9)$$

The coefficients a^{nc}, b^{nc}, and g^{nc} are determined by the parameters. In particular, a^{nc} is increasing, and b^{nc} is decreasing in the arrival rate q.

Next, we examine the cooperative scheme for the case $L = 2$. By Def. 2, under the cooperative market architecture the optimal stationary cooperative strategy can be obtained by solving the following Bellman equation with value function $V^c(x)$ and average cost per period λ^c:

$$\lambda^c + V^c(x) = (1-q)\Big(x^2 + \mathbb{E}_{d_1}\Big[V^c(d_1)\Big]\Big) + q\mathbb{E}_{d_1,d_2}\Big[\min_u \big\{(x+u)^2 + V^c(d_2-u+d_1)\big\}\Big] \quad (10)$$

Proposition 3: (*Existence of linear optimal stationary cooperative strategy*): Under the cooperative market architecture, there exists a linear optimal stationary cooperative load scheduling strategy $u^c(x,d_2)$ given by:

$$u^c(x,d_2) = -\underbrace{\frac{1}{1+\sqrt{1-q}}}_{a^c}x + \underbrace{\frac{1}{1+\frac{1}{\sqrt{1-q}}}}_{b^c}d_2$$
$$\underbrace{+\mu_1 + q\mu_2(\frac{1}{1+\sqrt{1-q}})/(1+\sqrt{1-q})}_{g^c} \quad (11)$$

which solves the Bellman equation in (10).

B. Welfare Impacts

We examine the welfare impacts of realtime pricing and agent behaviors under different market architectures. In particular, we look into the stationary distribution of the aggregate demand process $\{U(t) : t \in \mathbb{Z}\}$ induced by a generic linear strategy $u(x,d_2) = -ax+bd_2+g$, and evaluate the corresponding measures of efficiency and risk.

1) Efficiency: As defined in (5), market efficiency is given by $W = -\mathbb{E}[U(t)^2]/2$. Assuming that all type 2 agents adopt the same linear strategy $u(x,d_2) = -ax + bd_2 + g$, market efficiency is given by:

$$W = -\lambda/2 \quad (12)$$

where $\lambda = \mathbb{E}[U(t)^2]$ is obtained by solving the following fixed point equation in $(\lambda, V(x))$:

$$\lambda + V(x) = (1-q)(x^2 + \mathbb{E}_{d_1}[V(d_1)]) + q\mathbb{E}_{d_1,d_2}\left[(x+u(x,d_2))^2 + V(d_2 - u(x,d_2)+d_1)\right]$$

Again, we conjecture and verify that the value function $V(x)$ is of quadratic form as $V(x) = Ax^2 + Bx$. The constants λ, A, B are determined by the parameters and can be solved explicitly. In particular, with the specific linear strategies $u^{nc}(\cdot,\cdot)$ and $u^c(\cdot,\cdot)$, we can calculate the efficiency metrics W^{nc} and W^c under different market architectures. In Fig. 2(a), we observe that for any arrival rate q, the non-cooperative load scheduling leads to a positive efficiency loss when compared to the cooperative scheme.

2) Risk: We move now to quantify the risk R defined in (6) for the $L = 2$ case. Note that given a linear strategy $u(x,d_2) = -ax + bd_2 + g$, the instantaneous aggregate demand is given by:

$$U(t) = (1 - ah_2(t))x(t) + h_2(t)(bd_2(t) + g)$$

Since the arrival process $\{h_2(t) : t \in \mathbb{Z}\}$ of type 2 agents is exogenous, we can instead focus on the stationary distribution $\mathcal{X}$ of the aggregate uncontrollable load process $\{x(t) : t \in \mathbb{Z}\}$. It can be shown that the probability distribution $\mathcal{X}$ is as follows:

$$\mathcal{X} = \mathcal{X}_k \text{ with probability } q^k(1-q), \quad (k = 0,1,\cdots)$$
$$\mathcal{X}_k = \sum_{i=1}^{k} a^{i-1}\Big(D_{1,i} + (1-b)D_{2,i}\Big) + a^k D_{1,k} - \frac{1-a^k}{1-a}g$$

where $\{D_{1,i} : i \in \mathbb{Z}^+\}$ and $\{D_{2,i} : i \in \mathbb{Z}^+\}$ are i.i.d. random sequences with distributions $\mathcal{D}_1$ and $\mathcal{D}_2$, respectively. It is easy to verify that the stationary distribution $\mathcal{X}^c$, induced by the linear optimal cooperative strategy $u^c(\cdot,\cdot)$, has a larger mean and a larger variance than that of $\mathcal{X}^{nc}$, induced by the non-cooperative equilibrium strategy $u^{nc}(\cdot,\cdot)$. In other words, the backlog state is more volatile under the cooperative market architecture.

Proposition 4 (Upperbound on the risk R): Suppose that the workload distribution $\mathcal{D}_i$ follows the Normal distribution $\mathcal{N}(\mu_i, \sigma_i^2)$ for $i = 1, 2$. Given a linear strategy $u(x,d_2) = -ax + bd_2 + g$, the risk can be upper bounded by the tail of the stationary distribution $\mathcal{X}$ as follows:

$$\Pr(U(t) > M) \le \Pr(\mathcal{X} \ge M) \le \frac{1}{\sqrt{2\pi}m}e^{-\frac{m^2}{2}}$$

where

$$m = (M - \frac{\mu_1 + (1-b)\mu_2 - g}{1-a})/(\frac{\sigma_1^2 + (1-b)^2\sigma_2^2}{1-a^2}).$$

The upper bound is decreasing in a and increasing in b. For a large enough M, the cooperative strategy $u^c(\cdot,\cdot)$ leads to a higher upper bound of risk than the non-cooperative strategy $u^{nc}(\cdot,\cdot)$. This is consistent with the following simulations results where the cooperative scheme indeed has a higher risk than the non-cooperative scheme.

Remark 2 (Interpretation of the coefficients): For a linear strategy $u(x,d_2) = -ax+bd_2+g$ adopted by type 2 agents, the coefficient a can be interpreted as the sensitivity to the state of uncontrollable loads $x(t)$. A larger a means that the strategy is more aggresive in absorbing the fluctuation of uncontrollable loads in the environment. Note that both a^{nc} and a^c are increasing in q. Intuitively, with a higher type 2 arrival rate q, each type 2 agent is more aggresive in responding to $x(t)$ at their first period, anticipating that during the second period with high probability another type 2 agent will arrive and respond to $x(t+1)$ in a similar aggresive way. Also note that $a^{nc} < a^c$ always holds. As a result of their strategic behavior, type 2 agents always respond less aggresively to uncontrollable load $x(t)$ under the non-cooperative market architecuture. Similarly, we can interpret the coefficient b as the sensitivity to the realizations of $d_2(t)$. We also make the observations that $b^{nc} > b^c$, and both b^{nc}, b^c are decreasing in q.

3) Numerical results: In the following, we will show by simulations the efficiency-risk tradeoffs. In particular, we compare the stationary distribution of the aggregate demand process induced by four different linear strategies. We have $u^c(\cdot,\cdot)$ as the cooperative scheme, and $u^{nc}(\cdot,\cdot)$ as the non-cooperative scheme. In addition, we define the "naive load scheduling" scheme to be $u^{naive}(x, d_2) = d_2/2$, and define the "no load scheduling" scheme to be $u^{no}(x, d_2) = d_2$.

Fig. 2(b) compares the measure of risk across the four strategies. We look at the 95% percentile of the stationary distribution of the aggregate demand process. A higher 95% percentile indicates a higher risk. We observe that as the arrival rate q increases, risk increases most rapidly with the optimal cooperative scheme.

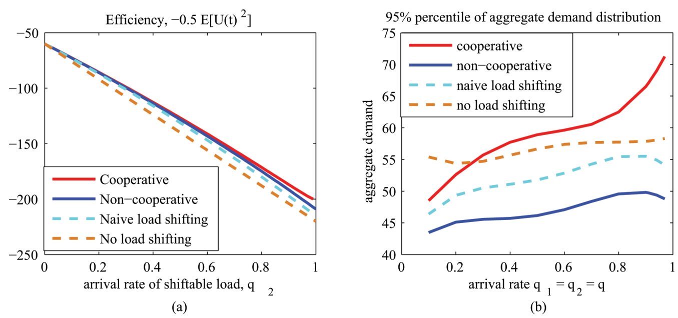

Fig. 2. Efficiency and risk under different load scheduling schemes.

Fig. 3 shows the sample paths of the aggregate demand process. We observe that at a shorter time scale, the cooperative scheme better smoothes the aggregate demand process, which is consistent with the lower aggregate demand variance. However, at a longer time scale, we can identify more demand spikes produced endogenously by the cooperative load scheduling scheme, corresponding to the higher risk.

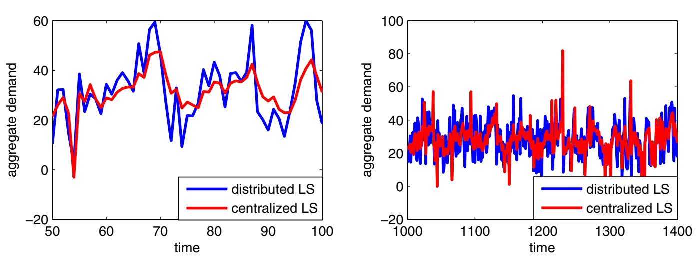

Fig. 3. Sample paths of the aggregate demand process. Left: short time scale. Right: long time scale.

IV. The General Case

In the $L = 2$ case we observed that most of the spikes take place during the absence of type 2 agents arrivals, which implies the connection between aggregate backlog fluctuation and the performance measure of risk: if the agents with controllable loads continue to arrive in each period, they can make use of the backlogs to schedule their consumptions, and the aggregate demand process will be smoothed; however, when those agents are absent and the backlog is accumulated to be high, an aggregate demand spike is produced endougenously. The tradeoff between efficiency and risk is thus closely related to the tradeoff of volatility between the aggregate demand process and the aggregate backlog process. We will show that for general L and for a class of boundedly rational load scheduling strategies, the volatility of the two processes cannot be made arbitrarily small simultaneously. Under the condition that $\mathbf{h}_l(t) = 1$ for each $l \in \mathcal{L}$ and for any t, aggregate demand volatility reflects how efficient the load scheduling strategy is, while aggregate backlog volatility reflects the risk of spikes. The system dynamics is as follows:

$$\mathbf{x}(t+1) = \mathbf{R}_1(\mathbf{x}(t) - \mathbf{u}(t)) + \mathbf{R}_2\mathbf{d}(t)$$

$$\mathbf{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} \mathbf{e}'\mathbf{u}(t) \\ \mathbf{e}'\mathbf{x}(t) \end{bmatrix}$$

where $y_2(t)$ and $y_2(t)$ are the two output measurements of aggregate demand and the aggregate backlog, respectively.

In general, load scheduling should be operated according to the following principle: with all other things being equal, an individual should demand more resource when he has a higher backlog state; and when other agents' backlog states are high, he forms the rational expectation that the instantaneous cost will be driven up, he thus should consume less to avoid the high cost. With this intuition, we restrict our focus to a class of *boundedly rational load scheduling strategies* defined as follows:

Definition 3 (Boundedly Rational Load Scheduling Strategy): A strategy $\mathbf{u}^{br}$ is defined to be a boundedly rational load scheduling strategy[2] if:

$$u^{br}_{(l,\tau)}(\mathbf{s}(t)) = \mathbf{F}'_{(l,\tau)}\mathbf{x}(t) \tag{13}$$

where $\{\mathbf{F}_{(l,\tau)} : (l,\tau) \in \mathcal{C}\}$ are D_c-dimensional column vectors defined as follows:

$$\mathbf{F}_{(l,1)} = \mathbf{e}_l, \quad \forall l \in \mathcal{L} \tag{14}$$

for $1 < \tau \leq L$:

$$\mathbf{F}_{(l,\tau),(l',\tau')} = \begin{cases} 1-\delta, & \text{if } (l',\tau') = (l,\tau) \\ -\frac{\delta}{D_c - 1}, & \text{if } (l',\tau') \neq (l,\tau) \end{cases} \tag{15}$$

where $0 < \delta < 0.5$; [3] $\mathbf{e}_l$ is a vector with the l-th element equal to 1, and all other elements equal to 0.

Note that the elements of the vector $\mathbf{F}_{(l,\tau)}$ are the coefficients of the linear strategy adopted by agent (l,τ). In (15), the vectors $\mathbf{F}_{(l,\tau)}$ for $\tau > 1$ are determined in equilibrium under specific oligopolistic market architectures, while the deadline constraints of load scheduling manifest as in (14). In particular, when the parameter δ is larger, individual load scheduling is more sensitive the other agents' backlog states and less sensitive to his own backlog state.

[2] We have normalized $\mathbf{x}(t)$ and $\mathbf{u}(t)$ to eliminate the constant terms.

[3] The upperbound on δ is to ensure system stability for each $L \in \mathcal{L}$.

Theorem 1 (Efficiency-risk tradeoff for the general case): Assume that all agents adopt a boundedly rational load scheduling strategy $\mathbf{u}^{br}$ as defined in Def. 3. The L2 norm of the transfer function of the aggregate demand $Y_1(z)$ is decreasing in δ, and the L2 norm of the transfer function of the aggregate backlog $Y_2(z)$ is increasing in δ.

V. Risk sensitive agents

When compared to the risk neutral case, risk averse agents are more concerned about the possible price spikes, thus the aggregate demand spikes tend to be mitigated. We follow the Linear Exponential Quadratic Gaussian (LEQG) framework to study the risk sensitive optimal strategy under the cooperative market architecture. Similar to the problem formulation in [12], we define the risk sensitive objective function recursively as follows:

$$\begin{aligned} c_t(x) =& q\mathbb{E}_{d_2}\left[(x+u)^2 - \frac{2\beta}{\theta}\log \mathbb{E}_{d_1^+}[e^{-\frac{\theta}{2}c_{t+1}\left(d_2-u+d_1^+\right)}]\right] \\ &+ (1-q)\left[x^2 - \frac{2\beta}{\theta}\log \mathbb{E}_{d_1^+}[e^{-\frac{\theta}{2}c_{t+1}\left(d_1^+\right)}]\right] \end{aligned} \quad (16)$$

where we also assume the workload distributions $\{\mathcal{D}_i : i = 1, 2\}$ to be Gaussian. The risk sensitivity is captured by the parameter θ. When $\theta < 0$, the agents are risk averse, and when $\theta > 0$, the agents are risk loving. In our formulation, the discount factor β is chosen to be a small enough constant to ensure the existence of a solution for the range of θ we consider. The risk sensitive optimal coopearative strategy minimizes the objective function as follows:

$$u(t) = u^{c,\theta}(x(t), d_2(t)) = \arg\min_u c_t(x(t))$$

Theorem 2: For risk sensitivity $\theta \in \mathbb{R}$, there exists a lower bound $\underline{\beta}(\theta)$ and an upper bound $\overline{\beta}(\theta)$, such that for $\underline{\beta}(\theta) \le \beta \le \overline{\beta}(\theta)$, there exists a risk sensitive optimal cooperative load scheduling strategy of linear form as follows:

$$u^{c,\theta}(x, d_2) = -\underbrace{\frac{1}{1+r_3}}_{a^{c,\theta}} x + \underbrace{\frac{r_3}{1+r_3}}_{b^{c,\theta}} d_2 + \underbrace{\frac{r_3(\mu_1 + r_1/(2r_2))}{1+r_3}}_{g^{c,\theta}} \quad (17)$$

where $\{r_i : i = 1, 2, 3\}$ are given by:

$$r_1 = \frac{2\beta r_2(1-r_2)(\mu_1+\mu_2)}{1+\theta\sigma_1^2 r_2 - \beta(1-r_2)}, \quad r_3 = \frac{\beta r_2}{1+\theta\sigma_1^2 r_2},$$

$$r_2 = \frac{(1-\beta-(1-q)\theta\sigma_1^2)\left(\sqrt{1+\frac{4(1-q)(\beta+\theta\sigma_1^2)}{(1-\beta-(1-q)\theta\sigma_1^2)^2}}-1\right)}{2(\beta+\theta\sigma_1^2)}.$$

Note that the risk sensitive load scheduling strategy derived in (17) for $\theta \neq 0$ is different from the risk neutral optimal strategy in (11). Nevertheless, system performance measures of efficiency and robustness remain unchanged as in (12) and (7). Fig. 4 shows that when $\theta \le 0$ and as the magnitude of θ increases, the agents become more risk averse, and the market efficiency decreases while the robustness increases, and market efficiency achieves the maximum at $\theta = 0$. This is the case because the common goal of maximizing the risk neurtral objective coincides with optimizing the market efficiency metric defined in (12). Moreover, as the agents become risk loving for $\theta > 0$, their objective deviates from the market efficiency. Load scheduling produces more spikes at the aggregate level, which have large negative impacts that bring down the overall efficiency as well as increase endogenous risks.

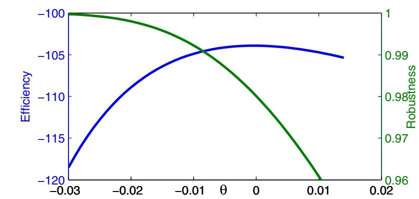

Fig. 4. Efficiency and robustness in risk sensitive load scheduling.

VI. Conclusion

In this paper, we proposed a framework to examine the welfare impacts of load scheduling under different market architectures. We took the approach of modeling agent behavior with dynamic oligopolistic games, and pointed out that different market architectures induce different agent behaviors, which lead to a tradeoff between efficiency and risk at the aggregate level. However, to what extent is the endogenous risk unavoidable and to what extent can it be controlled? Is there a fundamental tradeoff between efficiency and risk in more general systems with interactive agents? We leave the in-depth examination of these interesting questions to future works.

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