A COMPARISON OF VARIOUS LOW-PASS FILTER ARCHITECTURES FOR SIGMA-DELTA DEMODULATORS

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ABSTRACT

A comparison of various low-pass filter architectures that are commonly used in Sigma-Delta (Σ–Δ) demodulators is presented in the paper. In this comparison the conventional demodulator filter architectures such as the optimal FIR filter, Sinc filter and non-linear Zoomer filter are considered. It is shown that the optimum FIR filter provides the most attractive properties among the conventional demodulator low-pass filters. As an alternative to the optimal FIR filter, optimal Laguerre IIR filter architecture is then proposed in the paper. A design scheme for the optimal IIR filter is presented via the use of orthonormal Laguerre functions. In comparison with the conventional FIR filter approach, this design offers several attractive features. The proposed IIR filter design problem is easy to solve and numerically robust. Furthermore, the optimal IIR filter is easy to implement because it needs only a small number of components and therefore well suited for VLSI implementations.

1. INTRODUCTION

Σ–Δ Modulators are simple (usually 1-bit) A/D converters that can be found in many practical applications and are well suited to VLSI implementation. The original single loop modulator consists of an integrator followed by a quantizer embedded in negative feedback. This simple modulator, however, produces undesirable low frequency tones that are difficult to filter at the demodulator output. In overcoming this difficulty, the use of higher order modulator schemes with multi-loop and multi-stage architectures has been proposed in the literature [1,2]. The quantization noise performances of these higher order modulator schemes are well known [3]. Based on the quantization noise characteristics, optimal FIR filters which can be used at the demodulator, have been proposed in the literature [3]. In general, optimal FIR filters require a large number of taps. Therefore, in VLSI implementation it is advantageous to use low order IIR filters, instead of long FIR filters. It is the intention of this paper to provide a methodology for the design of optimal Laguerre IIR filters for demodulators. The Laguerre filter circuit will be then compared with conventional low-pass filter architectures in terms of phase linearity and hardware implementation issues.

2. Σ–Δ DEMODULATOR FILTERS

Consider a Σ–Δ modulator driven by a d.c. input \( x(n) = \rho \). The power spectrum of the output, \( y(n) \), of the modulator is given by [2],

\[
S_{yy}(f) = \frac{(2\sin \pi f)^2m}{3} + \rho^2\delta(f)
\]

In equation (1), \( f \) denotes the frequency normalized by the sampling frequency and \( m \) is the order of the modulator. For a double loop modulator, \( m = 2 \). Here we consider the double loop modulator architecture but the proposed design techniques can be extended for higher order Σ–Δ modulators such as the multi-stage (MASH) architecture.

The second term of the right hand side of equation (1) is the signal term while the first term represents the quantization noise resulting from the modulator. The objective is to design a digital filter, \( H(z) \), such that the quantized quantization noise power at the output of the demodulator is minimized, i.e.,

\[
\Gamma = \min_{H} \int_{-0.5}^{0.5} |H(e^{j2\pi f})|^2 (2\sin \pi f)^{2m} df
\]

subject to \( H(e^{j2\pi f})|_{f=0} = 1 \) (2)

The linear constraint on the minimization problem of equation (2) is imposed in order to pass the d.c. signal unattenuated through the demodulator. In the rest of this section we shall provide a brief description of commonly used demodulator filters. For comparison of various filter architectures the following definition of filter bandwidth is used.

\[
f_B = \int_{-0.5}^{0.5} \frac{|H(e^{j2\pi f})|^2 df}{|H(e^{j2\pi f})|_{f=0}^2}
\]

Note that equation (3) provides the effective bandwidth of the filter. The use of the effective bandwidth in equation (3) is justified later in section 5. Using Parseval's relations the effective bandwidth can also be expressed as,

\[
f_B = \sum_{n=0}^{N-1} h^2(n) / \sum_{n=0}^{N-1} h(n)^2
\]

where \( h(n) \) and \( N \) are the impulse response and length of the digital filter, respectively.
2.1 Optimum FIR Filter

Based on equation (2), an optimal FIR filter that produces minimum filtered quantization noise power at the output of the demodulator can be derived \([2][3]\). For a double loop modulator, i.e. \(m=2\), coefficients of the \(N\)-tap optimum FIR filter are given by,

\[
e(n) = \frac{3(n+1)(n+2)(N-n)(N+1-n)}{N(N+1)(N+2)(N+3)(N+4)}
\]

The effective bandwidth of the FIR filter with coefficients defined in equation (5) can be obtained as, \(f_B = 1.42/N\). Furthermore, the objective function \(\Gamma\) for the FIR filter in equation (5) is given by, \(\Gamma = 720/N^5\).

Note that \(N\) is usually selected as the oversampling ratio (OSR) of the \(\Sigma - \Delta\) modulator.

2.2 Sinc\(^3\) (Comb) filter

An \(N\)-tap Sinc\(^3\) filter is defined by,

\[
h(n) = \frac{1}{N} \quad (n=0,1,...,N-1)
\]

An \(N\)-tap Sinc\(^3\) filter can be viewed as the cascaded connection of \(k\) \(N/k\)-tap Sinc\(^3\) filters. Its transfer function magnitude is therefore:

\[
|H(e^{j2\pi f/N})| = \left| \frac{k}{N} \frac{\sin(\pi N/k)}{\sin(\pi f)} \right|^k
\]

For a double loop modulator, a Sinc\(^3\) filter is used in the demodulator. The objective function \(\Gamma\) for the Sinc\(^3\) filter is given by \([3]\), \(\Gamma = 1458/N^5\). The effective bandwidth of Sinc\(^3\) filter can be obtained as \(f_B = 1.64/N\).

2.3 Zoomer filter

Zoomer filter is a nonlinear filter that takes an \(N\) number of output bits \(Q(i) (i = 0...N-1)\) from the modulator as its input and provides an estimate \(\bar{X}_n\) \((n=2...N)\) of the modulator input. For time-varying modulator input, the estimate is given by \([4]\),

\[
\bar{X}_n = \frac{Q(0)+W_n}{1/2} + \frac{3N-2n-5}{6}s
\]

where \(s\) is the estimated input slope and \(W_n\) is calculated as

\[
W_n = \sum_{i=1}^{n-1} (n-i+1)Q(i), \quad n \geq 2
\]

If the modulator input is constant, \(s = 0\).

A lower bound \(L\) or upper bound \(U\) for the final estimate \(\bar{X}\) is obtained depending on whether \(Q(n+1) = +1\) or \(-1\) \((n=1...N-1)\). That is: \(L\) is updated at each \(n\) if \(L > \bar{X}_n\) and \(Q(n+1) = +1\). Similarly, \(U\) is updated if \(U < \bar{X}_n\) and \(Q(n+1) = -1\). The final estimate \(\bar{X}\) is obtained as, \(\bar{X} = (L+U)/2\).

3. PERFORMANCES OF CONVENTIONAL FILTERS

Figures 1 and 2 show simulation results obtained using both constant and sinusoidal inputs to the modulator. MSE is the mean-square-error of the output filtered quantization noise. Results for three different constant inputs are shown in Figure 1 in range 0.1 to 0.7. It can be seen form Figure 1 that when the sample size \(N\) is changing from 16 to 256, the MSE of optimal FIR and Sinc\(^3\) filter decreases. But the MSE is independent of the input signal. The MSE resulting
from the Zoomer filter depends on the input signal but in general is smaller than the MSE of optimal FIR and Sinc3 filter. Figure 2 shows results for sinusoidal input signals. The signal amplitudes vary from 0.1 to 0.7, \(N(0SR)=128\), and the normalized frequencies change from \(1/128\) to \(1/8\).

It can be seen from Figures 1 and 2 that although Zoomer filter achieves better performances for constant input signals, optimal FIR filter is the most attractive for time-varying input.

4. OPTIMAL LAGUERRE IIR FILTERS

This section describes the design methodology of an optimal IIR filter, which produces minimum filtered quantization noise power at the output of the demodulator. For the constrained minimization problem presented in equation (2), it is assumed that \(H(z)\) can be expressed as the following truncated Laguerre series [5],

\[
H_L(z) = \sum_{k=0}^{M-1} \gamma_k \phi_k(z)
\]

(10)

where \(\gamma_k\) is the Laguerre series coefficients and \(\phi_k(z)\) is the frequency domain Laguerre function, given by,

\[
\phi_k(z) = \frac{z^k}{\Gamma(1-a) \Gamma(1+2a)} \left(1 - \frac{z}{a}ight)^k, \quad -1 < a < 1
\]

(11)

Let \(H_2\) denotes the space of functions, which are analytical and square integrable outside the unit circle. It is well known that the set of Laguerre functions forms an orthonormal basis in \(H_2\) space. Note that when \(a=0\), Laguerre filter defined in equation (10) becomes an \(M\)-tap FIR filter.

Using equation (10), the constrained minimization of equation (2) can now be written as,

\[
\Gamma = \min_{\gamma_k} \int_{-0.5}^{0.5} \left| H_L(e^{j2\pi f}) \right|^2 (2 \sin \pi f)^2 df
\]

subject to \(\sum_{k=0}^{M-1} \gamma_k = \sqrt{\frac{1-a}{1+a}}\)

(12)

It is easy to verify that equation (12) can be written as,

\[
\Gamma = \min_{\vec{\gamma}} \vec{\gamma}^T Q_m \vec{\gamma}
\]

(13)

subject to \(\sum_{k=0}^{M-1} \gamma_k = \sqrt{\frac{1-a}{1+a}}\)

where \(\vec{\gamma} = [\gamma_0, \gamma_1, \ldots, \gamma_{M-1}]^T\) and \(Q_m\) is a constant positive definite matrix whose \((i,k)\) element is given by,

\[
Q_m(i,k) = \int_{-0.5}^{0.5} (2 \sin \pi f)^2 \phi_i(e^{j2\pi f}) \phi_k(e^{j2\pi f}) df
\]

(14)

Note that it is necessary to specify the filter order, \(M\), in order to solve the above optimization problem. It is possible to obtain the necessary value of filter order by including a bandwidth constraint in the Laguerre filter design. That is for a given OSR the optimal IIR filter is designed such that it has a bandwidth equal to the optimal FIR filter. For the Laguerre IIR filter defined in equation (10) the bandwidth can be obtained as,

\[
f_{B,L} = \frac{1}{2\pi} \sqrt{\frac{1}{N}}.
\]

(15)

Equation (12) combined with equation (15) defines a quadratic programming problem with linear and quadratic equality constraints. This can be solved efficiently via a variable transformation technique followed by the use of standard software such as MATLAB and its Optimization Toolbox [6].

5. DESIGN EXAMPLE OF A LAGUERRE IIR FILTER

In the following, the performance of optimum Laguerre IIR filters is compared with the optimum FIR filters. The IIR filter performance can be improved by making the Laguerre filter pole approach the unit circle. However, for circuit stability, the pole \(a\) should be selected away from the unit circle. In the following IIR filter design the pole is selected as \(a=0.9\). As an example, consider the following Laguerre filter design for a double loop modulator \((m=2)\) with OSR, \(N=128\). From the constrained optimization we obtain, \(M=7\), with \(\vec{\gamma} = 10^{-5} \{0.73 2.77 5.01 5.92 5.01 2.77 0.73\}\).

Both the 128-tap optimum FIR filter and 7-tap Laguerre FIR filter produce the same objective function \(\Gamma\) given by
Γ = -77dB. This exemplifies the effectiveness of the Laguerre filter over the FIR filter. Figure 3 shows the transfer functions of the Optimal FIR and Laguerre filters. From Figure 3 it can be deduced that both filters have approximately the same 3dB bandwidth given by $f_{3-dB} = 1.08 \times 10^{-2}$. Using equations (4) we also get $f_{B-F,R} = 1.11 \times 10^{-2}$ and $f_{B,L} = 1.02 \times 10^{-2}$. The use of equation (4) to evaluate the filter bandwidth can be justified because $f_{B-\text{FIR}} = f_{B,L} = f_{3-dB}$.

Figure 4: FIR and Laguerre Filter Phase Transfer Functions

Figure 4 shows the phase transfer functions of the optimal IIR and optimal FIR filters. It can be seen that the phase of the optimal IIR filter is approximately linear in the frequency interval of interest (from 0 to 0.5/OSR).

6. VLSI IMPLEMENTATION ISSUES

The optimum FIR filter gives the lowest error for the filtered quantization noise. However, with equal coefficients, the $Sinc^k$ filter is easy to implement in comparison to the optimum FIR filter [7]. Because of the large range in coefficient values of the optimum FIR filter, strict precision requirements for multipliers are needed for its implementation. Both $Sinc^k$ filter and FIR filter require large tap implementation that is not attractive for VLSI.

Compared to the linear filters, the algorithmic complexity of Zoomer filter makes it difficult for VLSI implementation. Besides, the performance of Zoomer algorithm is inferior to the optimum FIR filter when the modulator inputs are time-varying.

Laguerre IIR filter is short and can be implemented as a tap delay line filter. The coefficient range is also small. They essentially have linear phase characteristics in the frequency range of interest. These features make the Laguerre filter architecture an attractive choice for the decoding filter of the $\Sigma - \Delta$ modulator.

7. CONCLUSIONS

A comparison of various $\Sigma - \Delta$ modulator low-pass filter architectures proposed in the literature, such as $Sinc^k$, optimum FIR and Zoomer filters, has been provided in the paper. This comparison has focused on the issues of non-linear phase characteristics and the ease of hardware implementation of the circuits. It has been demonstrated that the optimal FIR provides with most attractive properties except for having a very long impulse response. In overcoming this difficulty this paper has investigated the design of optimal Laguerre IIR filters. It has been shown that the corresponding constrained filter design problem can be converted into a quadratic programming problem with linear and quadratic equality constraints. Using a design example it is shown that, for the same demodulator filter bandwidth, the Laguerre filter achieves a much lower filter order than the optimal FIR filter. Therefore, in comparison to the conventional FIR filter approach, the use of Laguerre filter offers a more effective and more robust design. Because of this, the Laguerre filter architecture is well suited for VLSI implementations.

8. REFERENCES