Fuzzy dynamic modelling and predictive control of a coagulation chemical dosing unit for water treatment plants

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Received 12 June 2014; accepted 11 August 2014

Abstract

In this paper, a fuzzy model predictive control (FMPC) strategy is proposed to regulate the output variables of a coagulation chemical dosing unit. A multiple-input, multiple-output (MIMO) process model in form of a linearised Takagi–Sugeno (T–S) fuzzy model is derived. The process model is obtained through subtractive clustering from the plant’s data set. The MIMO model is described by a set of coupled multiple-input, single-output models (MISO). In the controller design, the T–S fuzzy model is applied in combination with the nonlinear model predictive control (MPC) algorithm. The results show that the proposed controller has good set-point tracking when compared with nonlinear MPC and adequate disturbance rejection ability required for efficient coagulation control and process optimisation in water treatment operations.

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Keywords: Water treatment plants; Chemical dosing unit; Fuzzy dynamic model; Fuzzy model predictive control; Linearisation; Optimisation

1. Introduction

Coagulation process in water treatment operations involves removal of colloidal and suspended particles from raw waters through the addition of optimum quantity of relevant chemical reagents under a rapid mixing condition. Many water purification plants add coagulants and pH adjustment chemicals to raw waters in a rapid mixing tank or pipe to facilitate the coagulation process. The key issue with coagulation process is to control the quantity of chemical dosages that are added to the raw water influents taking into considerations variations in water quality and demands. When coagulation control is inadequate, the plant fails to satisfy water quality standards, expensive chemicals are wasted and less efficient filtration operation are performed due to unsettled flocs formation (Adgar et al., 2005; Valentin et al., 1999). In order to overcome these aforementioned problems, effective coagulation control is imperative for good portable water production and reduction in the total operational cost of the plant.

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Peer review under responsibility of Electronics Research Institute (ERI).

http://dx.doi.org/10.1016/j.jesit.2014.08.001
2314-7172/© 2014 Production and hosting by Elsevier B.V. on behalf of Electronics Research Institute (ERI).

Please cite this article in press as: Bello, O., et al., Fuzzy dynamic modelling and predictive control of a coagulation chemical dosing unit for water treatment plants. J. Electr. Syst. Inform. Technol. (2014), http://dx.doi.org/10.1016/j.jesit.2014.08.001
Coagulation control in water treatment plants is inherently difficult due to sudden changes in water quality and complex physicochemical reactions involved in the process. Many control strategies have been proposed in the literature to control the coagulant dosage system, but their works differ based on their process models, control objectives and methods. Evans et al. (1998) proposed a feedforward controller based on adaptive neuro-fuzzy network for Huntington water treatment plant in North West England. In Baxter et al. (2002), the integration of artificial neural network models with the supervisory control and data acquisition (SCADA) system through a number of process optimisation interfaces is presented in accordance with variations in influent water quality parameters.

In Fletcher et al. (2002), a feedforward coagulation control is developed using models based on nonlinear transformation of variables, multi-layer perceptron (MLP) and radial basis function (RBF) networks to improve the system in conjunction with a proportional controller. In another study reported by Adgar et al. (2005), the authors investigated the application of feedback control with decoupler on coagulation process in a twin pilot plant using streaming current detector and pH sensor to improve the existing manually flow-proportional control strategy. Analysis of the data collected during experiments on the pilot plant demonstrated that there is strong relationship and interaction between the streaming current detector and pH sensor measurements.

Model predictive control is widely used and accepted for effective and efficient process control in industries. The main objective of MPC strategy is to keep the output variables close to their reference trajectories taking into consideration the operating constraints (Morari and Lee, 1999). Linear model predictive control uses linear model to predict the system dynamics. It is effective for systems with only linear characteristics. However, nonlinear systems are poorly represented by linear models. Therefore, nonlinear model predictive control (NMPC) using nonlinear predictive model and optimisation methods are recommended for nonlinear systems and control applications.

The necessity to produce drinking water safe for public consumption has led to imposition of strict regulations on water treatment plants. It is therefore important to have in place an optimizing control with constraints handling capability to comply with the standards laid down by the regulatory bodies. The application of linear MPC has been studied on the coagulant dosage system for water treatment plants (Paz et al., 2009). The author used a linear model of the system for their study. It is focussed on a SISO model rather than solving nonlinear and multivariable control problem. The authors do not consider the effect of pH adjustment on the coagulation control and nonlinear behaviour of the system.

In this paper, fuzzy model predictive control is considered and proposed to control the surface charge (SC) and pH values of the effluent stream from the coagulation chemical dosing unit of a water treatment plant. Generally, accurate modelling of nonlinear multivariable system using mechanistic modelling technique is challenging. However, with the application of intelligent methods such as fuzzy modelling, an easy and effective model could be developed. Fuzzy identification approach based on subtractive clustering is proposed to develop a fuzzy model of a chemical dosing unit from a set of data collected from a water treatment plant in South Africa. The linearised Takagi–Sugeno model is applied in the development of FMPC for the unit. Simulation tests are carried out to determine the suitability of the proposed control strategy for the coagulation control in water treatment operations.

The paper is organised as follows. Section 2 provides the description of the Rietvlei water treatment plant. The fuzzy modelling and identification technique and main concepts of fuzzy MPC are discussed in this section. The simulation setup, model validation, simulation results and analysis are discussed in Section 3. Section 4 draws some conclusions from the study.

2. Materials and methods

2.1. Description of Rietvlei water treatment plant

Rietvlei water treatment plant in the City of Tshwane, South Africa has a production capacity of about 40 million litres per day. The plant draws raw waters from Rietvlei dam. A pumping station and transport system are available to lift waters from the dam to the treatment plant located about 200 m away from it. Raw waters flow through the intake pipe into the chemical dosing unit. The volume of water abstracted is used to determine the quantity of chemical reagents required for the coagulation process.

Raw water inlet valve is placed after the flowmeters to control the quantity of water flowing into the plant. Fig. 1 illustrates the coagulation chemical dosing unit of Rietvlei water treatment plant. It contains a concrete mixing tank with inlet and outlet channels. There are two-pairs of metering pumps that are used for dosing coagulation chemicals into
the mixing tank. The two pumps on one side are active while the other two on the second side are on standby/redundant position. One of the active dosing pumps feeds sudfloc 3835, a blend of epichlorohydrin/dimethylamine (polyamine) and aluminium chlorohydrate (NSF International, 2013) and the other pump feeds ferric chloride solution into the mixing tank. Calcium hydroxide (hydrated lime) in slurry form is also added to the mixing tank using a diaphragm pump to stabilise the water and adjust its pH value between 8.1 and 8.3.

The chemically treated waters flow out slowly and evenly through a series of baffled or flocculation channels, to grow the flocs. The waters from the baffled channels flow into the Dissolved Air Floatation/Filtration (DAFF) unit. The unit removes all the flocs that have just been formed in the flocculation channels. The flocculated water is passed over the supersaturated water. This leads into a flocculation process where the flocs rises as a result of the water bubbles attached to them. The floating particles form a scummy brown layer or froth on the surface of the filter bed and separation takes place through the filter beds. The DAFF filters are cleaned regularly by manual backwashing operation. The backwash water is kept in settling tanks for couple of hours. Afterwards, the settled sludge is pumped into a sewerage system and the relatively clean water is pumped back into the dam. The filtered waters from the DAFF flow into Granular Activated Carbon (GAC) filtration unit to eliminate foul odour, taste and colour caused by the dissolved organic matter. The waters from GAC are conveyed into the chlorination chamber. Here, the chlorine gas is added to the waters for disinfection before they are pumped to the storage reservoirs and distributed to the final consumers (City of Tshwane, 2013; Bello et al., 2014a,b).

2.2. Data collection and analysis

Historical data was collected from the plant for a period of two-year (2011–2012). A total of 690 data samples were successfully obtained from the daily operating records of the plant. The collected data were the flow rate of sudfloc 3835 solution ($q_a$), flow rate of ferric chloride solution ($q_b$), flow rate of hydrated lime ($q_c$) and the pH value of the effluent stream from the coagulation chemical dosing unit of the plant. The other variable of interest is the surface charge or streaming current of the treated water leaving the chemical dosing unit. It is an important variable for the implementation of multivariable or feedback control strategy for coagulation process in a water treatment plant (Adgar et al., 2005; Bello et al., 2014a). This is not measured presently at plant but is computed using (1). Figs. 2 and 3 show the normalised data set used for modelling the coagulation chemical dosing unit.

$$\sigma = \left[ \frac{2}{\pi} n \varepsilon k T \right]^{1/2} \sinh 1.15(pH_0 - PH)$$  (1)
where $\sigma$ is the surface charge, $\kappa$ the Boltzmann constant, $T$ temperature, $\epsilon$ relative dielectric permittivity and $pH_0$ is pH at point of zero charge and $n$ ionic strength.

2.3. Fuzzy modelling

The fuzzy modelling accounts for the hidden imprecision in data and perform accurate input–output mapping using the fuzzy logic and rules (Jang, 1993). This process involves fuzzification of the input variables through membership function, a curve that maps the input values to membership grades within the interval of 0 and 1. Fuzzy conditional statements are the building blocks of the fuzzy inference system. Fuzzy conditional statements are useful to describe the imprecise manners of human reasoning necessary to make decisions in uncertain and imprecise environments or conditions. There are two common approaches to fuzzy conditional statements namely, Mamdani and Takagi–Sugeno.

Mamdani fuzzy rules have both the antecedent (if-part) and consequent (then-part) expressed in terms of fuzzy sets. An example of Mamdani conditional statement is:

\[
\text{if coagulant flow rate is low, then surface charge is high}
\]
where surface charge and coagulant flow rate are linguistic variables, low and high are linguistic values of the membership functions.

On the other hand, Takagi–Sugeno fuzzy rule has fuzzy sets only in the antecedent part while the consequent part is expressed as a constant, linear or nonfuzzy equation of the input variables. An example of Takagi–Sugeno conditional statement is:

\[
\text{if lime flow rate is low, then } pH = k \times (\text{lime flow rate}) + c
\]

where low in the antecedent part is a linguistic label of the input membership functions and consequent part is denoted by an equation in terms of the input variable, lime flow rate. k and c are the equation coefficients of the consequent part. Fuzzy inference system which is the core part of fuzzy models generates result based on the following steps (Jang, 1993):

- Find the membership grade of each linguistic value on the antecedent part by comparing the input variables with the membership function;
- Determine the firing strength or weight of each fuzzy rule by aggregating the membership grades on the antecedent parts;
- Compute the qualified consequent of each fuzzy rule as a function of the firing strength;
- Aggregate the qualified consequent to generate a single valued output.
2.4. Multiple-input and multiple-output Takagi–Sugeno model

Consider a multivariable system with \( m \) inputs: \( u \in U \subset \mathbb{R}^m \) and \( \rho \) outputs: \( y \in Y \subset \mathbb{R}^\rho \). A set of coupled multiple-input and single-output fuzzy models of the input–output nonlinear autoregressive exogenous (NARX) type can be used to approximate the multivariable system (Mollov et al., 2004):

\[
y_i(k + 1) = F(\xi_i(k), u(k)), \quad i = 1, 2, \ldots, \rho
\]

where \( u(k) \in \mathbb{R}^m \) represents the current inputs and \( \xi_i \in \mathbb{R}^\rho \) contains both the current and past outputs, and past inputs:

\[
\xi_i(k) = [y_1(k), \ldots, y_\rho(k), u_1(k - 1), \ldots, u_m(k - 1)]^T
\]

with

\[
y_i(k) = [y_1(k), y_i(k - 1), \ldots, y_i(k - n_{y,i})], \quad i = 1, \ldots, \rho
\]

\[
u_j(k - 1) = [u_j(k - 1), u_j(k - 2), \ldots, u_j(k - n_{u,j})], \quad j = 1, \ldots, m
\]

where \( n_{y,i} \) and \( n_{u,j} \) denote the number of past for \( i \)th output and the \( j \)th input, respectively.

The Takagi–Sugeno fuzzy models have rules in form of:

\[
\mathcal{R}_{li}: \text{If } \xi_{li}(k) \text{ is } \Omega_{li,1} \text{ and } \ldots \text{ and } \xi_{li}(k) \text{ is } \Omega_{li,\rho} \text{ and } u_l(k) \text{ is } \Omega_{li,\rho+1} \text{ and } \ldots \text{ and } u_m(k) \text{ is } \Omega_{li,\rho+m} \text{ then } y_l(k + 1) = \zeta_{li}\xi_{li}(k) + \eta_{li}u(k) + \varphi_{li}, \quad i = 1, 2, \ldots, K
\]

where \( \Omega_{li} \) are the antecedent fuzzy sets of the \( i \)th rule, \( \zeta_{li} \) and \( \eta_{li} \) are vectors containing the consequent parameters, and \( \varphi_{li} \) is the offset. \( K \) is the number of rules for the \( l \)th output. The model output is obtained using the weighted average defuzzification method, given as (Babuska, 1998):

\[
y_l(k + 1) = \frac{\sum_{i=1}^{K_i} \lambda_{li}(\xi_{li}(k)) + \eta_{li}u(k) + \varphi_{li}}{\sum_{i=1}^{K} \lambda_{li}}
\]

where \( \lambda_{li} \) represents the degree of fulfilment for the \( i \)th rule. It is expressed as:

\[
\lambda_{li}(\xi_{li}, u) = \prod_{k=1}^{\rho} \mu_{\Omega_{li,k}}(\xi_{li,k}) \prod_{j=1}^{m} \mu_{\Omega_{li,j}}(u_{ij})
\]

2.5. Fuzzy model identification

The fuzzy model can be estimated from the input and output data using an appropriate model identification algorithm. The algorithm for fuzzy identification may include the following steps (Fan and Wang, 2004):

1. Fuzzy clustering is applied to determine the antecedents of the fuzzy model rules;
2. The appropriate cluster radius is determined;
3. The consequent parts of the fuzzy rules are determined by least squares parameter estimation technique and;
4. Compatible fuzzy sets are detected and merged using similarity-driven rule based algorithm.

2.5.1. Fuzzy clustering

In this study, subtractive clustering is applied to form clusters in the data and translate them into fuzzy rules. Subtractive clustering method is used to determine the number of rules and antecedent membership functions by considering the centre of each cluster as a fuzzy rule. In this approach each data point of a set of \( N \) data points \( x_1, x_2, \ldots, x_N \) in a \( \rho \)-dimensional space is regarded as the potential candidate for cluster centres. After normalisation and
scaling of data points in each direction, a density measure at data \( x_j \) is computed with reference to its location from other data points. The expression for the density measure of \( i \)th data point is:

\[
D_i = \sum_{j=1}^{N} \exp \left( -\left( \frac{2}{r_a} \right)^2 \cdot \| x_i - x_j \|^2 \right)
\]  

(8)

where \( r_a \) is a positive constant called cluster radius and \( \| \cdot \| \) denotes the Euclidean distance.

A data point is considered as a cluster centre when more data points are closer to it. Thus, the data point \( (x^+_i) \) with highest density measure \( D^+_i \) is considered as first cluster centre. With the exclusion of the first cluster centre, the density measure of all other data points is revised by using:

\[
D_i = D_i - D^+_i \cdot \mu(x^+_i)
\]  

(9)

\[
\mu(x^+_i) = \exp \left( \frac{\| x_i - x^+_i \|^2}{(rb/2)^2} \right)
\]  

(10)

where \( rb(r_b > r_a) \) is a positive constant that results in a measurable reduction in density measures of neighbourhood data points in order to avoid closely spaced cluster centres.

Using (9), the density measure of each point is obtained, the data point with the highest remaining density measure is assigned the next cluster centre, \( x^+_2 \) and all of the density measures for data points are revised again. The process is repeated and the density measures of the remaining data points after computation of \( k \)th cluster centre is revised by substituting the location \( (x^+_k) \) and density measure \( (D^+_k) \) of the \( k \)th cluster centre into (9). This process is stopped when adequate numbers of cluster centres have been generated. The cluster centres are the representations of the system to be modelled and exhibit certain similar characteristics. They are adopted as the centres for fuzzy rules’ antecedent membership functions that describe the system behaviour for \( j \)th variable of the input:

\[
\mu_{ij}(x_i) = \exp \left( \frac{x_i - x^+_j}{(r_a/2)^2} \right)
\]  

(11)

For every unique input vector a membership degree to each fuzzy set greater than zero (0) is computed, and therefore every rule in the rule base fires. This leads to the possibility of generating a couple of rules for describing the accurate relationship between input and output data (Lohani et al., 2006; Qun et al., 2006).

2.5.2. Cluster radius determination

Subtractive clustering requires that the cluster radius should be specified. Cluster radius shows the range of influence of a cluster when you consider the data space as a unit hypercube. Many smaller clusters are formed when a small cluster radius is specified in the data. This often leads to many rules. However, few rules are obtained when large cluster radius is specified yielding a few large clusters (Chiu, 1994). In determining the appropriate cluster radius \( r_a \), cluster validity analysis is carried out by running the clustering algorithm for several values of \( r_a \) starting form a small value to large value with different initialisations. The validity measure is calculated for each run, and the cluster radius which minimises the validity measure is selected as the appropriate cluster radius. In this study, prediction error is used as validity measure, expressed as:

\[
e = \frac{1}{N} \sum_{k=1}^{N} (y - \hat{y})
\]  

(12)

where \( y \) and \( \hat{y} \) are the true data and the predicted output respectively.
2.5.3. Consequent parameters estimation

The consequent parameters \((a_i, b_i)\) are estimated from the identification data set by least-squares techniques. The regressors \((\xi)\) and regressand \((y)\) of identification data and membership degrees of the fuzzy partition are arranged as follows (Babuska, 1998):

\[
\xi = \begin{bmatrix} 
\xi_1^T \\
\xi_2^T \\
\vdots \\
\xi_N^T 
\end{bmatrix}, \quad y = \begin{bmatrix} 
y_1 \\
y_2 \\
\vdots \\
y_N 
\end{bmatrix}, \quad W_i = \begin{bmatrix} 
\mu_{i1} & 0 & \ldots & 0 \\
0 & \mu_{i2} & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \mu_{iN} 
\end{bmatrix}
\]

(13)

The consequent parameters of the rule belonging to the \(i\)th cluster, \(a_i\) and \(b_i\) are concatenated into a single parameter vector, \(\theta_i\):

\[
\theta_i = [a_i^T \ b_i]^T
\]

(14)

Assuming that each cluster represents a local linear model of the system, the consequent parameter vectors \(\theta_i, \ i = 1, 2, \ldots, \rho\), can be estimated independently by the weighted least-squares method. The membership degrees \(\mu_{ik}\) of the fuzzy partition serve as the weights expressing the importance of the data pairs \((x_k, y_k)\) to the local model. If the columns of extended regressors \(X_e\) are linearly independent and \(\mu_{ik} > 0\) for \(1 \leq k \leq N\), then

\[
\theta_i = [\xi_i^T \ W_i \xi_e]^{-1} \xi_i^T \ W_i \ y
\]

(15)

is the least-squares solution of \(y = \xi \ e + \epsilon\), where the parameters \(a_i\) and \(b_i\) are expressed as:

\[
a_i = [\theta_1, \theta_2, \ldots, \theta_{\rho}], \quad b_i = \theta_{\rho+1}
\]

(16)

2.5.4. Rule based simplification

Fuzzy similarity measures are applied to initial fuzzy model obtained from identification data to eliminate or reduce the unnecessary or duplicated initial fuzzy rules and to obtain linguistics interpretation of the membership functions. The simplification algorithm combines similar fuzzy sets using the two thresholds: \(\eta \in (0, 1)\) for merging fuzzy sets that are similar to one another, and \(\eta \in (0, 1)\) for removing fuzzy sets similar to the universal set. The similarity between all fuzzy sets for each antecedent variable is examined in each iteration. The pair of membership functions with the highest similarity \(s \geq \eta\) are combined. The new fuzzy set replaces the ones merged when the rule base is updated. The algorithm continues to evaluate the similarities in the updated ruled base until there are no more fuzzy sets for which \(s \geq \eta\).

2.6. Linearisation of TS fuzzy model

The TS fuzzy model output in (6) can be expressed as:

\[
y_i(k+1) = \frac{\sum_{i=1}^{K_i} \lambda_{ii}(\xi_i(k), u(k)) (\xi_i(k) + \eta_i u(k) + \varphi_i)}{\sum_{i=1}^{K_i} \lambda_{ii}(\xi_i(k), u(k))}
\]

(17)

\[
y_i(k+1) = \sum_{i=1}^{K_i} \omega_{ii}(\xi_i(k), u(k), f_{ii}(\xi_i(k), u(k)))
\]

(18)

where \(\sum_{i=1}^{K_i} \omega_{ii}(\xi_i(k), u(k))\) is the normalised degree of fulfilment and \(f_{ii}(\xi_i(k), u(k)) = \xi_i \xi_i + \eta_i u(k) + \varphi_i\) is the linear model of the \(i\)th fuzzy rule.

Applying the Jacobian linearization to (17), the final expression becomes (Menees and Araujo, 1999; Johansen et al., 2000):

\[
\frac{\partial y_i(k+1)}{\partial \xi(k)} = \sum_{i=1}^{K_i} \left( \frac{\partial \omega_{ii}(\xi_i(k), u(k))}{\partial \xi(k)} \cdot f_{ii}(\xi_i(k)) + \omega_{ii}(\xi_i(k), u(k)) \cdot \frac{\partial f_{ii}(\xi_i(k))}{\partial \xi(k)} \right)
\]

(19)
2.7. Fuzzy model predictive control

The main idea for fuzzy MPC implemented in this study is to use the linearised fuzzy model to predict the future process output of the model predictive control (MPC) instead of models obtained from first principles or other modelling techniques (Roubos et al., 1999; Mollov et al., 2004). The MPC algorithm generally comprises of three main concepts:

1. The explicit application of a model to predict the future process output;
2. Computation of a sequence of future control inputs by minimising a specified performance index or objective cost;
3. The use of the only first control input in the sequence at the first instant. The horizons are moved to next sample period towards the future, and the optimisation is repeated. This is known as receding horizon strategy.

Fig. 4 shows the block diagram of the fuzzy model predictive controller. The future process output \( y = [y(k+1), \ldots, y(k+P)] \) are predicted over the prediction horizon \( P \) using the fuzzy model of the process. The process outputs depend on the process state and on the future control signals \( u = [u(k+1), \ldots, u(k+M)] \) within the control horizon, \( M \). The sequence of future control inputs \( u(k+1) \) is computed by minimising a given performance index. The performance index defines the process objective of minimising the error between output vector and the reference trajectory. The performance index expressed as a quadratic programming problem is:

\[
J(u) = \min_{\Delta u(k), \ldots, \Delta u(k+M)} S_y(k) + S_u(k) + S_{\Delta u}(k) 
\]

(20)

where

\[
S_y(k) = \sum_{i=1}^{P} \sum_{j=1}^{n_y} [w_j^y[r_j(k+i) - y_j(k+1)]]^2
\]

(21)

\[
S_u(k) = \sum_{i=1}^{M} \sum_{j=1}^{n_u} [w_j^u[u_j(k+i-1) - \bar{u}_j]]^2
\]

(22)

\[
S_{\Delta u}(k) = \sum_{i=1}^{M} \sum_{j=1}^{n_{\Delta u}} [w_j^{\Delta u}[\Delta u(k+i-1)]]^2
\]

(23)

subject to:

\[
u_{\text{min}} \leq u(k+i-1) \leq u_{\text{max}}, \quad \text{for } i = 1 \text{ to } M
\]

(24)

\[
\Delta u_{\text{min}} \leq \Delta u(k+i-1) - \Delta u(k+i-2) \leq \Delta u_{\text{max}}, \quad \text{for } i = 1 \text{ to } M
\]

(25)

\[
y_{\text{min}} \leq y(k+i) \leq y_{\text{max}}, \quad \text{for } i = 1 \text{ to } P
\]

(26)
where \( r(k) \) is the reference vector, \( y(k+j|k) \) is the j-step ahead predicted output given the present output measurements, \( w_j^o \) is the positive definite output error weighting matrix, \( w_{\Delta u}^j \) is the positive semi definite input weighting matrix. \( \pi_j \) is the nominal value of input \( j \). The weighting matrices, prediction horizon \( P \), and control horizon \( M \) are the tuning parameters which can be used to shape the closed-loop response of the system (Bemporad et al., 2013).

3. Simulation results and discussions

3.1. Simulation set-up

The data collected and analysed from the plant discussed in Section 2.2 were used to develop identification data for the fuzzy model of the chemical dosing unit at a sampling rate of 60 s. The data sets were divided into two parts: the first part (60%) was used to train the fuzzy model and second part (40%) was the checking data set to validate the model.

The FIS model for this study consists of two coupled fuzzy inference system (FIS) networks. The two FIS networks have similar structure. The first and second networks have the regressands: surface charge, \( SC(k) \); and pH values, \( pH(k) \) as output variables respectively. Using the heuristic approach, eight input variables or regressors to the first and second networks were selected. For the first network, the regressors are:

\[
\xi_1(k) = [SC(k-1), pH(k-1), qa(k-1), qa(k-2), qb(k-1), qb(k-2), qc(k-1), qc(k-2)]
\]

and for second networks, they are:

\[
\xi_2(k) = [pH(k-1), SC(k-1), qa(k-1), qa(k-2), qb(k-1), qb(k-2), qc(k-1), qc(k-2)]
\]

Fig. 5. Performance of the FIS network with the validation data set.
Table 1
Performance evaluation.

<table>
<thead>
<tr>
<th>Per. Index</th>
<th>SCchg</th>
<th>SCckg</th>
<th>pHchg</th>
<th>pHckg</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>0.0000066</td>
<td>0.0000068</td>
<td>0.7007</td>
<td>0.7011</td>
</tr>
<tr>
<td>ARV</td>
<td>0.9441</td>
<td>0.9451</td>
<td>0.9018</td>
<td>0.9025</td>
</tr>
</tbody>
</table>

The cluster centre for each network was determined by varying the cluster radius between 0.1 and 1 with steps of 0.1. The optimum cluster centres was obtained as 2 at a radius of 0.25 and 0.3 for the first network and second network respectively. Further, each input variable of FIS networks consisted of two Gaussian membership functions and the output variable were two linear membership functions. Each network had two fuzzy if-then rule and 82 parameters composed of 64 antecedent parameters and 18 consequent parameters. The rules of the first and second FIS networks are presented in Appendix A.

The consequent part of each network was assumed to be a second-order NARX structure. Its parameters were estimated using the least-squares method. The number of manipulated ($n_u$) and controlled variables ($n_c$) of each network are 3 and 1 respectively. Therefore, the order of NARX model was set as follows: number of past output terms used to predict the current output ($n_u$); 1 $\times$ 1 matrix, number of past input terms ($n_b$) and number of delay from input to output ($n_k$); 1 $\times$ 3 matrix respectively.

3.2. Model testing and validation

The first network was trained with $[\xi_1(k) - SC(k)]$ data set and the second network was trained with $[\xi_2(k) - pH(k)]$ data set. Each data set is made up of 414 data pairs. The performance of the model when checked with the validation data set is shown in Fig. 5. The quantitative measure of the model accuracy was performed using the root mean square error (RMSE) and average relative variance (ARV) for the training data and validation data. The expressions for the two performance indices (Pai et al., 2009; Jang, 1993) are:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2}$$

$$ARV = \left( \frac{RMSE}{s} \right)^2$$

where $\hat{y}_i$ is the output of model estimator, $y_i$ measured output, $\overline{y}_i$ mean of the measured output, ($N$) number of samples and $s$ standard deviation.

The results of the quantitative measures are compared and presented in Table 1. The closeness of the results of the two tests shows that the fuzzy model predicts or estimates the output variables satisfactorily.

Table 2
Parameters specification of the model predictive control schemes.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prediction horizon, $P$</td>
<td>15</td>
</tr>
<tr>
<td>Control horizon, $M$</td>
<td>5</td>
</tr>
<tr>
<td>Constraint: sulfloc 3835 flow rate</td>
<td>$0 &lt; q_c &lt; 2$</td>
</tr>
<tr>
<td>Constraint: ferric chloride flow rate</td>
<td>$0 &lt; q_c &lt; 2$</td>
</tr>
<tr>
<td>Constraint: hydrated lime flow rate</td>
<td>$0 &lt; q_c &lt; 2$</td>
</tr>
<tr>
<td>Weight: surface charge</td>
<td>1</td>
</tr>
<tr>
<td>Weight: pH</td>
<td>1</td>
</tr>
<tr>
<td>Rate weight for input variables</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Please cite this article in press as: Bello, O., et al., Fuzzy dynamic modelling and predictive control of a coagulation chemical dosing unit for water treatment plants. J. Electr. Syst. Inform. Technol. (2014), http://dx.doi.org/10.1016/j.jesit.2014.08.001
Fig. 6. Set-point tracking performances of FMPC and NMPC strategies.

Fig. 7. Disturbance rejection performance of the fuzzy MPC strategy.

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3.3. Controller configurations

Two different controller configurations were used to demonstrate the performances of the proposed control strategy:

- Nonlinear model predictive control (NMPC) using a nonlinear model of chemical dosing unit (Bello et al., 2014b) obtained by linearization at ($SC = -1.637 \times 10^{-3} \mu/mg$ and $pH = 7.2$); and
- Fuzzy MPC using the procedures described in Section 2.

The two configurations used the same control settings as shown in Table 2. However, the weights on the input variables are specified as zero and no constraint was placed on the output variables to allow them move freely.

3.4. Simulation analysis

The performance of the fuzzy MPC on the dosing unit was analysed after it was simulated for a simulation period of 24 h. Fig. 6 shows the performances of NMPC and FMPC compared to examine the set point tracking ability of the two controllers. In this simulation, it was assumed that the set points of the chemical dosing unit were changed every 6 h by the plant operator as a result of laboratory experimental procedures or tests. The set points of $SC$ and pH was changed from initial points from $-0.00035 \mu/eq/mg$, 7.4 to $-0.0007 \mu/eq/mg$, 8.5, then to $-0.0005 \mu/eq/mg$, 7.9, and finally to $-0.00055 \mu/eq/mg$, 8.1. The performances of the two controllers compared satisfactory with each other.
However, the performance metric based on integral squared error (ISE) of the two controllers is compared numerically using (Bemporad et al., 2013):

\[ J = \sum_{i=1}^{N} \left( \sum_{j=1}^{n_y} \left( w_j^y e_{u,ij} \right)^2 + \sum_{j=1}^{n_u} \left( w_j^u \Delta u_{ij} \right)^2 \right) \]  

(31)

where \( N \) is the number of controller sampling intervals in the scenario, \( e_{y,ij} \) is the deviation of output \( j \) from its set point (reference) at time step \( i \), \( e_{u,ij} \) is the deviation of manipulated variable \( j \) from its target position at time step \( i \), \( \Delta u_{ij} \) is the change in manipulated variable \( j \) at time step \( i \), \( w_j^y \), \( w_j^u \) and \( w_j^\Delta u \) are non-negative performance weights.

The performance metric, \( J \) for FMPC and NMPC are 21.25 and 26.42 respectively. The result indicates that FMPC performs better than NMPC.

The response of the proposed controller to disturbance rejection was examined. Randomly generated noise signals were added to the output variables accounting for the effect of the sensors on signal measurements. Input step signals were added to represent sudden disturbances to the operation of dosing unit. Fig. 7 illustrates the performance of the proposed FMPC to reject these input disturbances introduced to the dosing unit. Fig. 8 shows the control input signals to the chemical dosing unit. The simulation result shows that the FMPC has the ability to handle perturbations and sudden changes in the operational conditions of the chemical dosing unit.

4. Conclusion

In this paper, fuzzy model predictive control has been proposed and compared with the model predictive control specifically for applications in coagulation control in water treatment process. Fuzzy modelling and identification technique is applied to develop a dynamic process model of coagulation chemical dosing unit for a water treatment plant in South Africa. The linearised Takagi–Sugeno fuzzy model is obtained in form that could be used with the NMPC algorithm. The set point tracking capability of the proposed fuzzy MPC are examined and compared with NMPC. The simulation results and performance evaluation show that fuzzy MPC yields satisfactory performance over the NMPC. The results also demonstrate the merit of fuzzy MPC deployment for effective and practical water treatment process especially when there are frequent variations in its operating conditions.

Acknowledgements

The authors acknowledged the support of Tshwane University of Technology, Pretoria for the work. We wish to thank Prof. AA Jimoh, the management and staff of Reitvlei water treatment plant, City of Tshwane (Water & Sanitation) for their contributions and cooperation in the course of the study.

Appendix A. Takagi–Sugeno rules for the coagulation chemical dosing unit

Rules for the first FIS network:

\( R_{11} : \) If \( SC(k - 1) \) is \( \Omega_{11,1} \) and \( pH(k - 1) \) is \( \Omega_{11,2} \) and \( q_d(k - 1) \) is \( \Omega_{11,3} \) and \( q_d(k - 2) \) is \( \Omega_{11,4} \) and \( q_b(k - 1) \) is \( \Omega_{11,5} \) and \( q_b(k - 2) \) is \( \Omega_{11,6} \) and \( q_c(k - 1) \) is \( \Omega_{11,7} \) and \( q_c(k - 2) \) is \( \Omega_{11,8} \) then \( SC(k) = -0.2239SC(k - 1) - 3.696 \times 10^{-7} pH(k - 1) - 1.807 \times 10^{-6} q_d(k - 1) + 2.042 \times 10^{-6} q_d(k - 2) + 5.34 \times 10^{-7} q_b(k - 1) + 1.99 \times 10^{-9} q_b(k - 2) + 5.762 \times 10^{-9} q_c(k - 1) + 2.059 \times 10^{-9} q_c(k - 2) + 3.21 \times 10^{-6} \)

\( R_{12} : \) If \( SC(k - 1) \) is \( \Omega_{12,1} \) and \( pH(k - 1) \) is \( \Omega_{12,2} \) and \( q_d(k - 1) \) is \( \Omega_{12,3} \) and \( q_d(k - 2) \) is \( \Omega_{12,4} \) and \( q_b(k - 1) \) is \( \Omega_{12,5} \) and \( q_b(k - 2) \) is \( \Omega_{12,6} \) and \( q_c(k - 1) \) is \( \Omega_{12,7} \) and \( q_c(k - 2) \) is \( \Omega_{12,8} \) then \( SC(k) = 0.4634SC(k - 1) - 2.061 \times 10^{-8} pH(k - 1) + 6.92 \times 10^{-6} q_d(k - 1) - 7.195 \times 10^{-6} q_d(k - 2) - 2.514 \times 10^{-6} q_b(k - 1) - 5.076 \times 10^{-6} q_b(k - 2) + 3.88 \times 10^{-8} q_c(k - 1) + 6.272 \times 10^{-9} q_c(k - 2) + 2.771 \times 10^{-7} \)
The rules of the second FIS network:

\[ R_{21} : \text{If } pH(k-1) \text{ is } \Omega_{21,1} \text{ and } SC(k-1) \text{ is } \Omega_{21,2} \text{ and } q_a(k-1) \text{ is } \Omega_{21,3} \text{ and } q_b(k-1) \text{ is } \Omega_{21,4} \text{ and } q_b(k-1) \text{ is } \Omega_{21,5} \text{ and } q_b(k-2) \text{ is } \Omega_{21,6} \text{ and } q_a(k-1) \text{ is } \Omega_{21,7} \text{ and } q_e(k-2) \text{ is } \Omega_{21,8} \text{ then } pH(k) = 0.4351pH(k-1) - 7.156 \cdot 10^5 SC(k-1) + 10.29q_a(k-1) + 0.4817q_a(k-2) - 23.21q_b(k-1) + 7.814q_b(k-2) - 0.05128q_e(k-1) + 0.0056q_e(k-2) + 4.682 \]

\[ R_{22} : \text{If } pH(k-1) \text{ is } \Omega_{22,1} \text{ and } SC(k-1) \text{ is } \Omega_{22,2} \text{ and } q_a(k-1) \text{ is } \Omega_{22,3} \text{ and } q_b(k-2) \text{ is } \Omega_{22,4} \text{ and } q_b(k-1) \text{ is } \Omega_{22,5} \text{ and } q_b(k-2) \text{ is } \Omega_{22,6} \text{ and } q_a(k-1) \text{ is } \Omega_{22,7} \text{ and } q_e(k-2) \text{ is } \Omega_{22,8} \text{ then } pH(k) = 0.08837pH(k-1) + 1.138 \cdot 10^6 SC(k-1) + 13.54q_a(k-1) + 12.48q_a(k-2) + 14.77q_b(k-1) + 6.019q_b(k-2) + 0.258q_e(k-1) - 0.00132q_e(k-2) + 7.29 \]

References


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