

Data Mining:

Concepts and Techniques

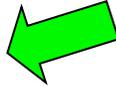
(3rd ed.)

— Chapter 6 —

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Chapter 5: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts 
- Frequent Itemset Mining Methods
- Which Patterns Are Interesting?—Pattern Evaluation Methods
- Summary

What Is Frequent Pattern Analysis?

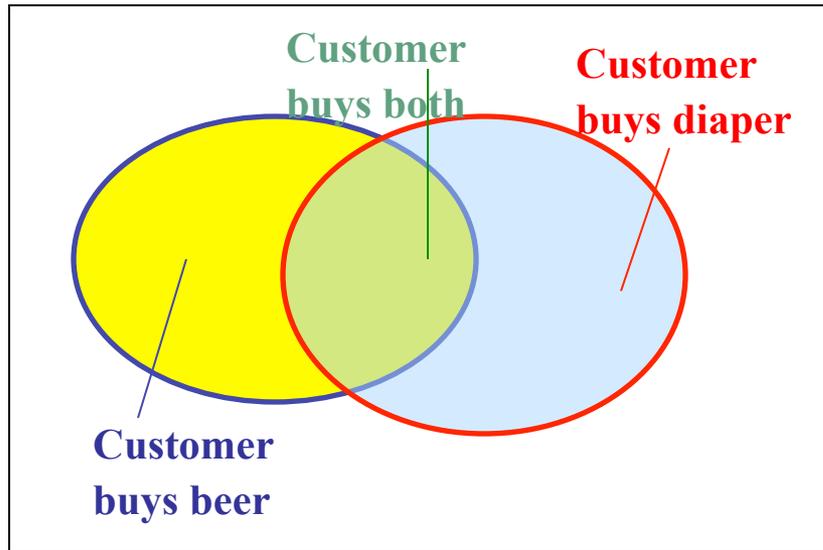
- **Frequent pattern**: a pattern (a set of items, subsequences, substructures, etc.) that occurs frequently in a data set
- First proposed by Agrawal, Imielinski, and Swami [AIS93] in the context of **frequent itemsets** and **association rule mining**
- Motivation: Finding inherent regularities in data
 - What products were often purchased together?— Beer and diapers?!
 - What are the subsequent purchases after buying a PC?
 - What kinds of DNA are sensitive to this new drug?
 - Can we automatically classify web documents?
- Applications
 - Basket data analysis, cross-marketing, catalog design, sale campaign analysis, Web log (click stream) analysis, and DNA sequence analysis.

Why Is Freq. Pattern Mining Important?

- Freq. pattern: An intrinsic and important property of datasets
- Foundation for many essential data mining tasks
 - Association, correlation, and causality analysis
 - Sequential, structural (e.g., sub-graph) patterns
 - Pattern analysis in spatiotemporal, multimedia, time-series, and stream data
 - Classification: discriminative, frequent pattern analysis
 - Cluster analysis: frequent pattern-based clustering
 - Data warehousing: iceberg cube and cube-gradient
 - Semantic data compression: fascicles
 - Broad applications

Basic Concepts: Frequent Patterns

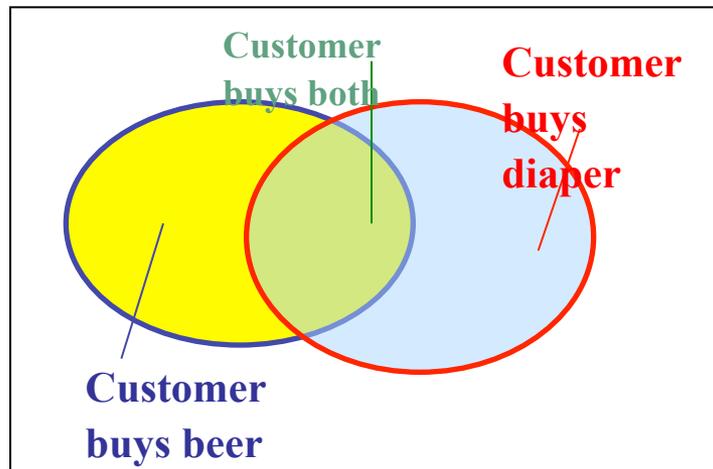
Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



- **itemset**: A set of one or more items
- **k-itemset** $X = \{x_1, \dots, x_k\}$
- **(absolute) support**, or, **support count** of X : Frequency or occurrence of an itemset X
- **(relative) support**, s , is the fraction of transactions that contains X (i.e., the **probability** that a transaction contains X)
- An itemset X is **frequent** if X 's support is no less than a **minsup** threshold

Basic Concepts: Association Rules

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



- Find all the rules $X \rightarrow Y$ with minimum support and confidence
 - **support**, s , **probability** that a transaction contains $X \cup Y$
 - **confidence**, c , **conditional probability** that a transaction having X also contains Y

Let $minsup = 50\%$, $minconf = 50\%$

Freq. Pat.: Beer:3, Nuts:3, Diaper:4, Eggs:3, {Beer, Diaper}:3

- Association rules: (many more!)
 - $Beer \rightarrow Diaper$ (60%, 100%)
 - $Diaper \rightarrow Beer$ (60%, 75%)

Closed Patterns and Max-Patterns

- A long pattern contains a combinatorial number of sub-patterns, e.g., $\{a_1, \dots, a_{100}\}$ contains $\binom{100}{1} + \binom{100}{2} + \dots + \binom{100}{100} = 2^{100} - 1 = 1.27 \times 10^{30}$ sub-patterns!
- Solution: Mine *closed patterns* and *max-patterns* instead
- An itemset X is **closed** if X is *frequent* and there exists *no super-pattern* $Y \supset X$, with the same support as X (proposed by Pasquier, et al. @ ICDT' 99)
- An itemset X is a **max-pattern** if X is frequent and there exists no frequent super-pattern $Y \supset X$ (proposed by Bayardo @ SIGMOD' 98)
- Closed pattern is a lossless compression of freq. patterns
 - Reducing the # of patterns and rules

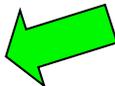
Closed Patterns and Max-Patterns

- Exercise. $DB = \{ \langle a_1, \dots, a_{100} \rangle, \langle a_1, \dots, a_{50} \rangle \}$
 - $Min_sup = 1$.
- What is the set of **closed itemset**?
 - $\langle a_1, \dots, a_{100} \rangle: 1$
 - $\langle a_1, \dots, a_{50} \rangle: 2$
- What is the set of **max-pattern**?
 - $\langle a_1, \dots, a_{100} \rangle: 1$
- What is the set of **all patterns**?
 - !!

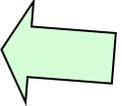
Computational Complexity of Frequent Itemset Mining

- How many itemsets are potentially to be generated in the worst case?
 - The number of frequent itemsets to be generated is sensitive to the minsup threshold
 - When minsup is low, there exist potentially an exponential number of frequent itemsets
 - The worst case: M^N where M : # distinct items, and N : max length of transactions
- The worst case complexity vs. the expected probability
 - Ex. Suppose Walmart has 10^4 kinds of products
 - The chance to pick up one product 10^{-4}
 - The chance to pick up a particular set of 10 products: $\sim 10^{-40}$
 - What is the chance this particular set of 10 products to be frequent 10^3 times in 10^9 transactions?

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Scalable Frequent Itemset Mining Methods

- Apriori: A Candidate Generation-and-Test Approach 
- Improving the Efficiency of Apriori
- FPGrowth: A Frequent Pattern-Growth Approach
- ECLAT: Frequent Pattern Mining with Vertical Data Format

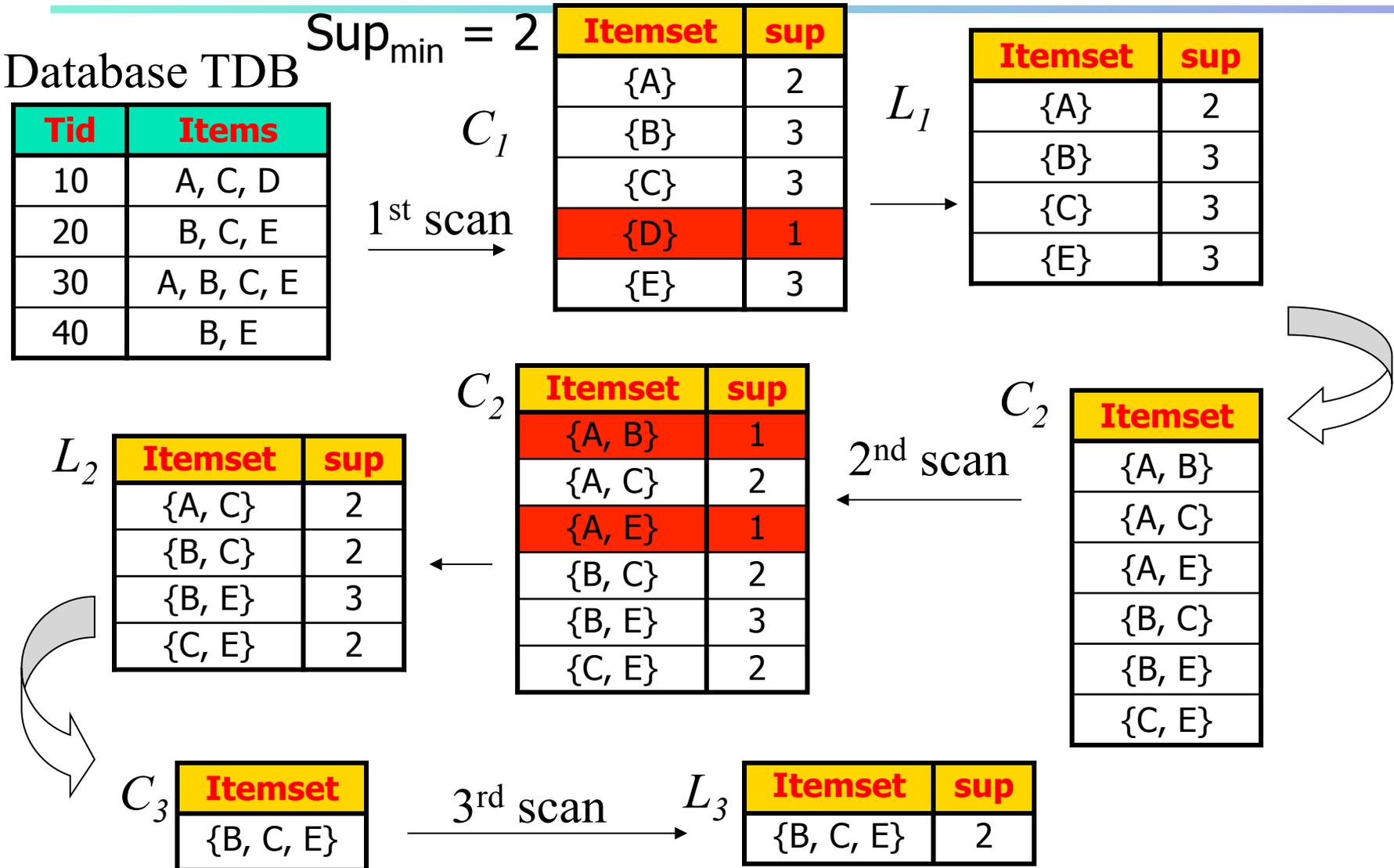
The Downward Closure Property and Scalable Mining Methods

- The **downward closure** property of frequent patterns
 - Any subset of a frequent itemset must be frequent
 - If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
 - i.e., every transaction having {beer, diaper, nuts} also contains {beer, diaper}
- Scalable mining methods: Three major approaches
 - Apriori (Agrawal & Srikant@VLDB' 94)
 - Freq. pattern growth (FPgrowth—Han, Pei & Yin @SIGMOD' 00)
 - Vertical data format approach (Charm—Zaki & Hsiao @SDM' 02)

Apriori: A Candidate Generation & Test Approach

- Apriori pruning principle: If there is **any** itemset which is infrequent, its superset should not be generated/tested!
(Agrawal & Srikant @VLDB' 94, Mannila, et al. @ KDD' 94)
- Method:
 - Initially, scan DB once to get frequent 1-itemset
 - **Generate** length $(k+1)$ **candidate** itemsets from length k **frequent** itemsets
 - **Test** the candidates against DB
 - Terminate when no frequent or candidate set can be generated

The Apriori Algorithm—An Example



The Apriori Algorithm (Pseudo-Code)

C_k : Candidate itemset of size k

L_k : frequent itemset of size k

$L_1 = \{\text{frequent items}\};$

for ($k = 1; L_k \neq \emptyset; k++$) **do begin**

$C_{k+1} =$ candidates generated from L_k ;

for each transaction t in database **do**

increment the count of all candidates in C_{k+1} that
are contained in t

$L_{k+1} =$ candidates in C_{k+1} with `min_support`

end

return $\cup_k L_k$;

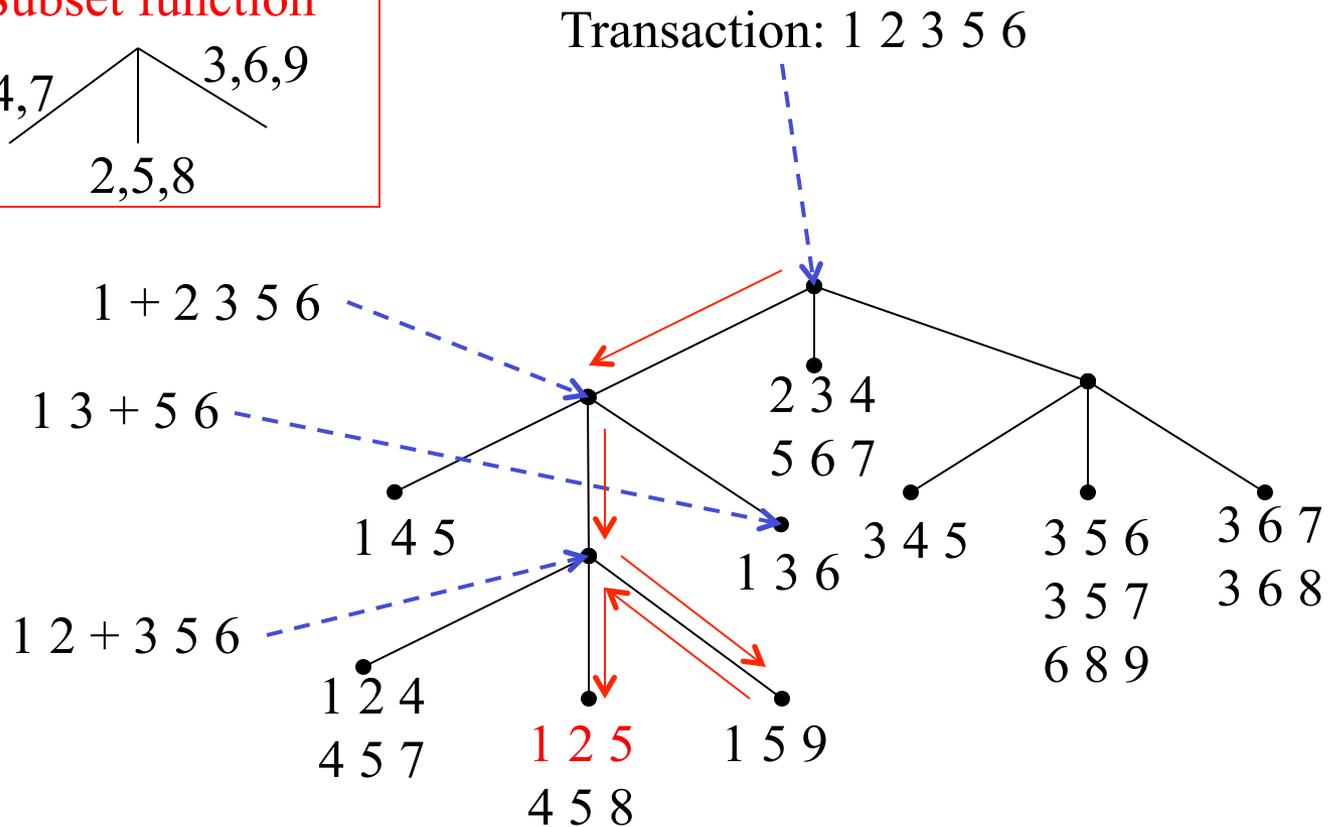
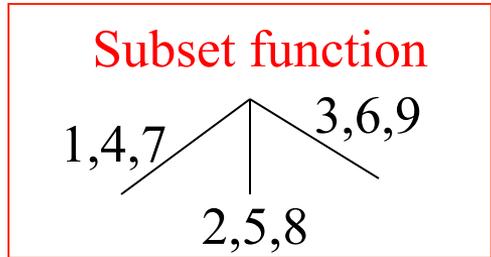
Implementation of Apriori

- How to generate candidates?
 - Step 1: self-joining L_k
 - Step 2: pruning
- Example of Candidate-generation
 - $L_3 = \{abc, abd, acd, ace, bcd\}$
 - Self-joining: $L_3 * L_3$
 - $abcd$ from abc and abd
 - $acde$ from acd and ace
 - Pruning:
 - $acde$ is removed because ade is not in L_3
 - $C_4 = \{abcd\}$

How to Count Supports of Candidates?

- Why counting supports of candidates a problem?
 - The total number of candidates can be very huge
 - One transaction may contain many candidates
- Method:
 - Candidate itemsets are stored in a *hash-tree*
 - *Leaf node* of hash-tree contains a list of itemsets and counts
 - *Interior node* contains a hash table
 - *Subset function*: finds all the candidates contained in a transaction

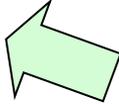
Counting Supports of Candidates Using Hash Tree



Candidate Generation: An SQL Implementation

- SQL Implementation of candidate generation
 - Suppose the items in L_{k-1} are listed in an order
 - Step 1: self-joining L_{k-1}
 - insert into C_k
 - select **$p.item_1, p.item_2, \dots, p.item_{k-1}, q.item_{k-1}$**
 - from **$L_{k-1} p, L_{k-1} q$**
 - where **$p.item_1 = q.item_1, \dots, p.item_{k-2} = q.item_{k-2}, p.item_{k-1} < q.item_{k-1}$**
 - Step 2: pruning
 - forall ***itemsets c in C_k*** do
 - forall ***(k-1)-subsets s of c*** do
 - if (*s is not in L_{k-1}) then delete c from C_k***
- Use object-relational extensions like UDFs, BLOBs, and Table functions for efficient implementation [See: S. Sarawagi, S. Thomas, and R. Agrawal. Integrating association rule mining with relational database systems: Alternatives and implications. SIGMOD' 98]

Scalable Frequent Itemset Mining Methods

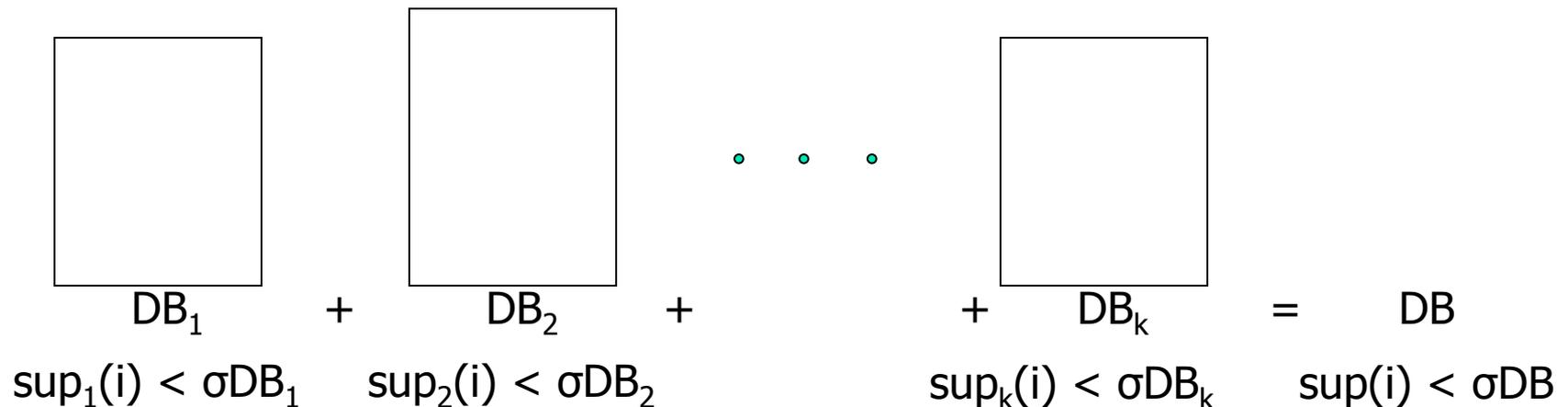
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Further Improvement of the Apriori Method

- Major computational challenges
 - Multiple scans of transaction database
 - Huge number of candidates
 - Tedious workload of support counting for candidates
- Improving Apriori: general ideas
 - Reduce passes of transaction database scans
 - Shrink number of candidates
 - Facilitate support counting of candidates

Partition: Scan Database Only Twice

- Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB
 - Scan 1: partition database and find local frequent patterns
 - Scan 2: consolidate global frequent patterns
- A. Savasere, E. Omiecinski and S. Navathe, *VLDB'95*



DHP: Reduce the Number of Candidates

- A k -itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
 - Candidates: a, b, c, d, e
 - Hash entries
 - {ab, ad, ae}
 - {bd, be, de}
 - ...
 - Frequent 1-itemset: a, b, d, e
 - ab is not a candidate 2-itemset if the sum of count of {ab, ad, ae} is below support threshold
- J. Park, M. Chen, and P. Yu. *An effective hash-based algorithm for mining association rules. SIGMOD'95*

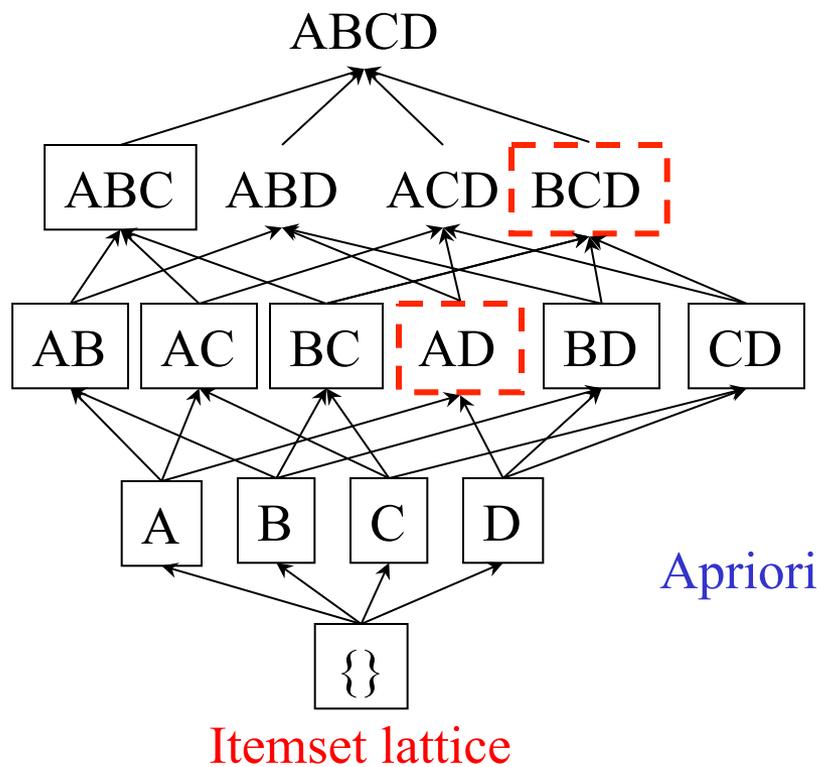
count	itemsets
35	{ab, ad, ae}
88	{bd, be, de}
·	·
·	·
·	·
102	{yz, qs, wt}

Hash Table

Sampling for Frequent Patterns

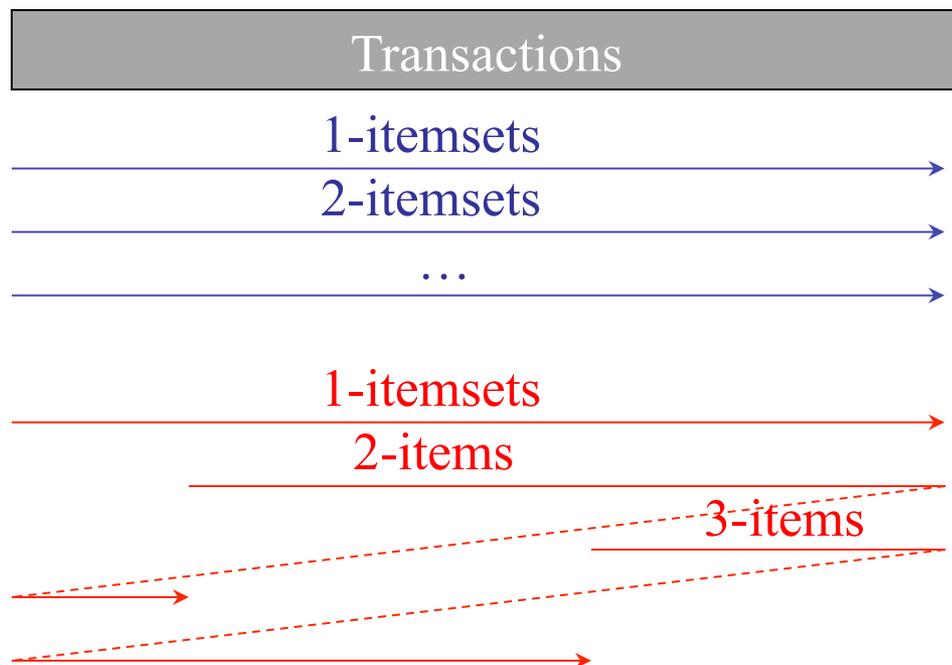
- Select a sample of original database, mine frequent patterns within sample using Apriori
- Scan database once to verify frequent itemsets found in sample, only *borders* of closure of frequent patterns are checked
 - Example: check *abcd* instead of *ab, ac, ..., etc.*
- Scan database again to find missed frequent patterns
- H. Toivonen. *Sampling large databases for association rules*. In *VLDB'96*

DIC: Reduce Number of Scans

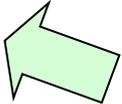


S. Brin R. Motwani, J. Ullman,
and S. Tsur. *Dynamic itemset
counting and implication rules for
market basket data. SIGMOD' 97*

- Once both A and D are determined frequent, the counting of AD begins
- Once all length-2 subsets of BCD are determined frequent, the counting of BCD begins



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Pattern-Growth Approach: Mining Frequent Patterns Without Candidate Generation

- Bottlenecks of the Apriori approach
 - Breadth-first (i.e., level-wise) search
 - Candidate generation and test
 - Often generates a huge number of candidates
- The FPGrowth Approach (J. Han, J. Pei, and Y. Yin, SIGMOD' 00)
 - Depth-first search
 - Avoid explicit candidate generation
- Major philosophy: Grow long patterns from short ones using local frequent items only
 - “abc” is a frequent pattern
 - Get all transactions having “abc”, i.e., project DB on abc: DB|abc
 - “d” is a local frequent item in DB|abc → abcd is a frequent pattern

Construct FP-tree from a Transaction Database

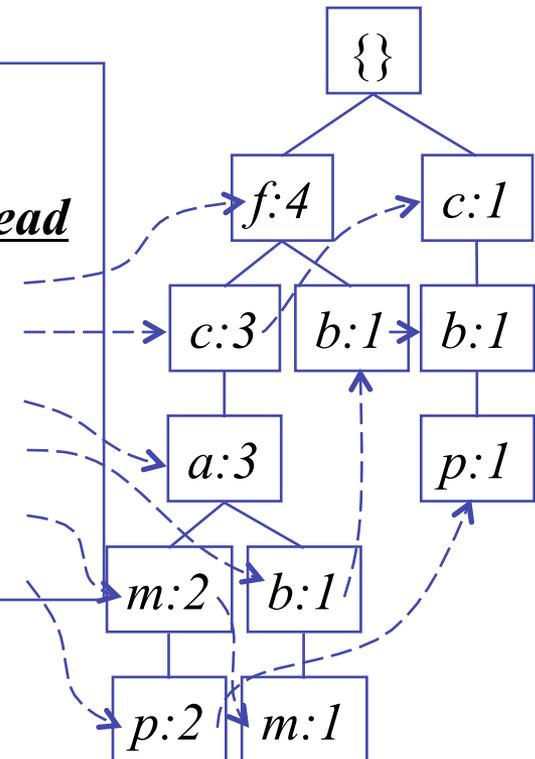
<i>TID</i>	<i>Items bought</i>	<i>(ordered) frequent items</i>
100	{f, a, c, d, g, i, m, p}	{f, c, a, m, p}
200	{a, b, c, f, l, m, o}	{f, c, a, b, m}
300	{b, f, h, j, o, w}	{f, b}
400	{b, c, k, s, p}	{c, b, p}
500	{a, f, c, e, l, p, m, n}	{f, c, a, m, p}

min_support = 3

1. Scan DB once, find frequent 1-itemset (single item pattern)
2. Sort frequent items in frequency descending order, f-list
3. Scan DB again, construct FP-tree

Header Table	
<u><i>Item frequency head</i></u>	
<i>f</i>	4
<i>c</i>	4
<i>a</i>	3
<i>b</i>	3
<i>m</i>	3
<i>p</i>	3

F-list = f-c-a-b-m-p

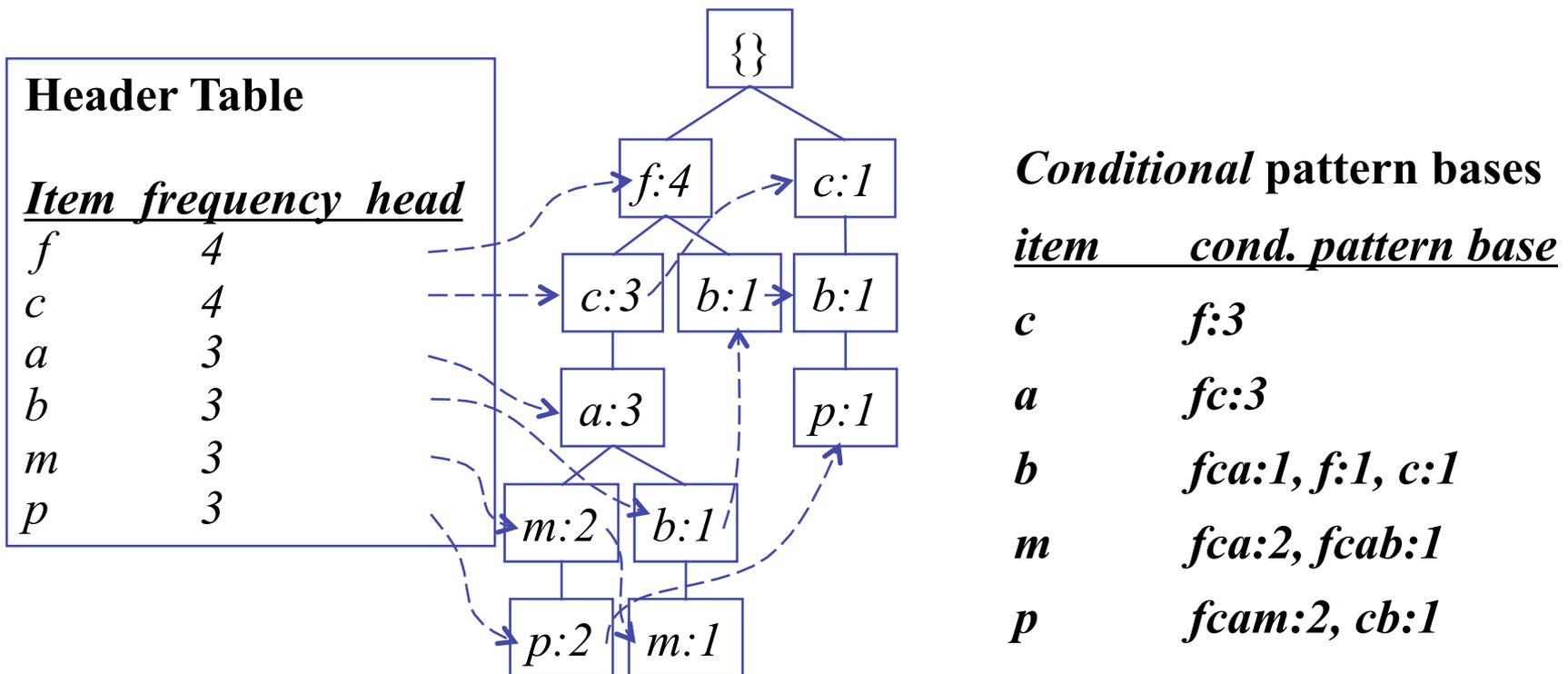


Partition Patterns and Databases

- Frequent patterns can be partitioned into subsets according to f-list
 - F-list = f-c-a-b-m-p
 - Patterns containing p
 - Patterns having m but no p
 - ...
 - Patterns having c but no a nor b, m, p
 - Pattern f
- Completeness and non-redundancy

Find Patterns Having P From P-conditional Database

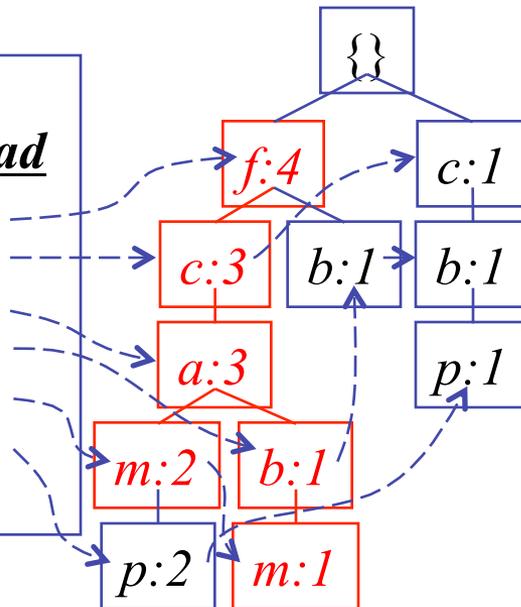
- Starting at the frequent item header table in the FP-tree
- Traverse the FP-tree by following the link of each frequent item p
- Accumulate all of *transformed prefix paths* of item p to form p 's conditional pattern base



From Conditional Pattern-bases to Conditional FP-trees

- For each pattern-base
 - Accumulate the count for each item in the base
 - Construct the FP-tree for the frequent items of the pattern base

Header Table	
<i>Item frequency head</i>	
<i>f</i>	4
<i>c</i>	4
<i>a</i>	3
<i>b</i>	3
<i>m</i>	3
<i>p</i>	3



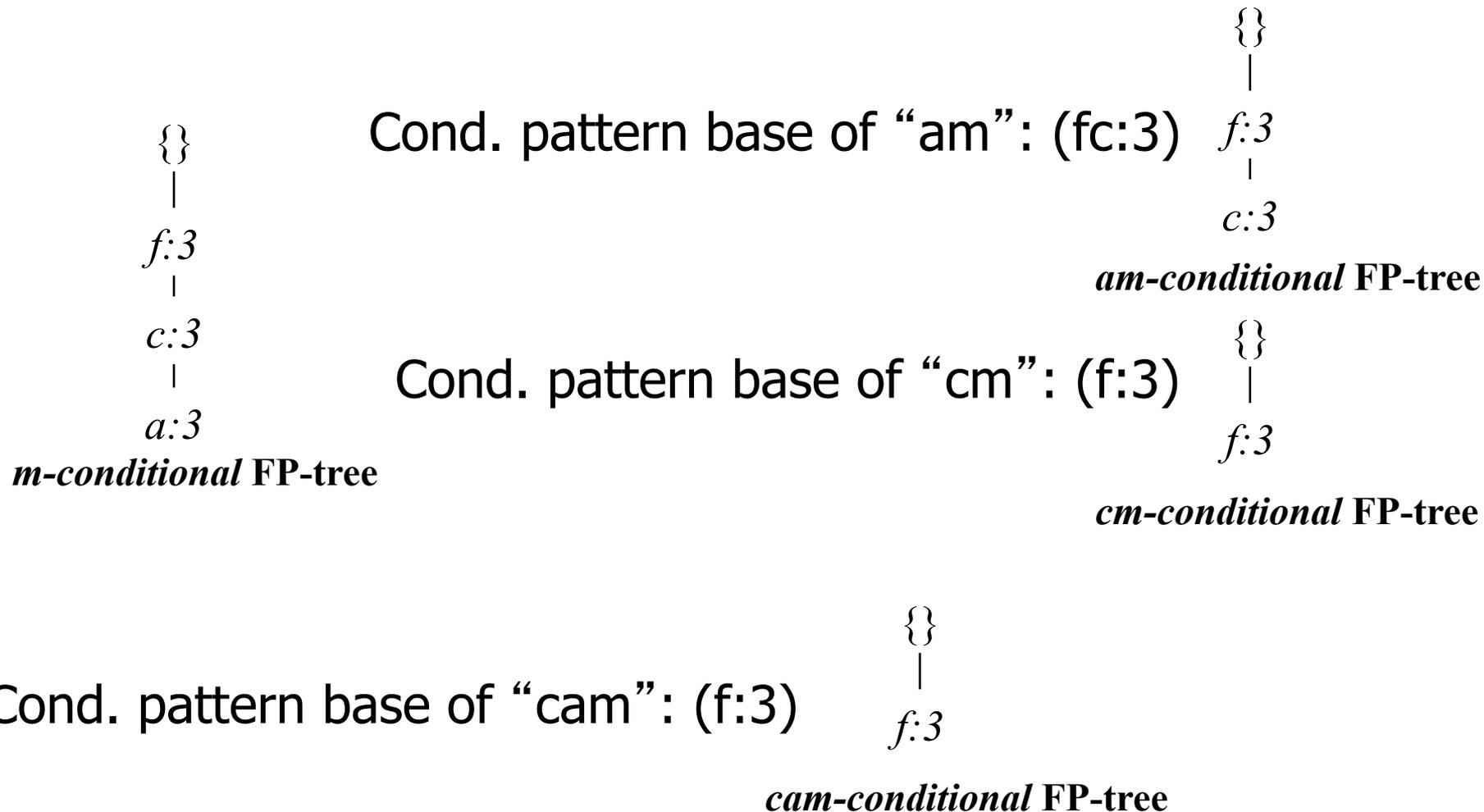
m-conditional pattern base:
fca:2, fcab:1



All frequent patterns relate to *m*,
m,
fm, cm, am,
fcm, fam, cam,
fcam

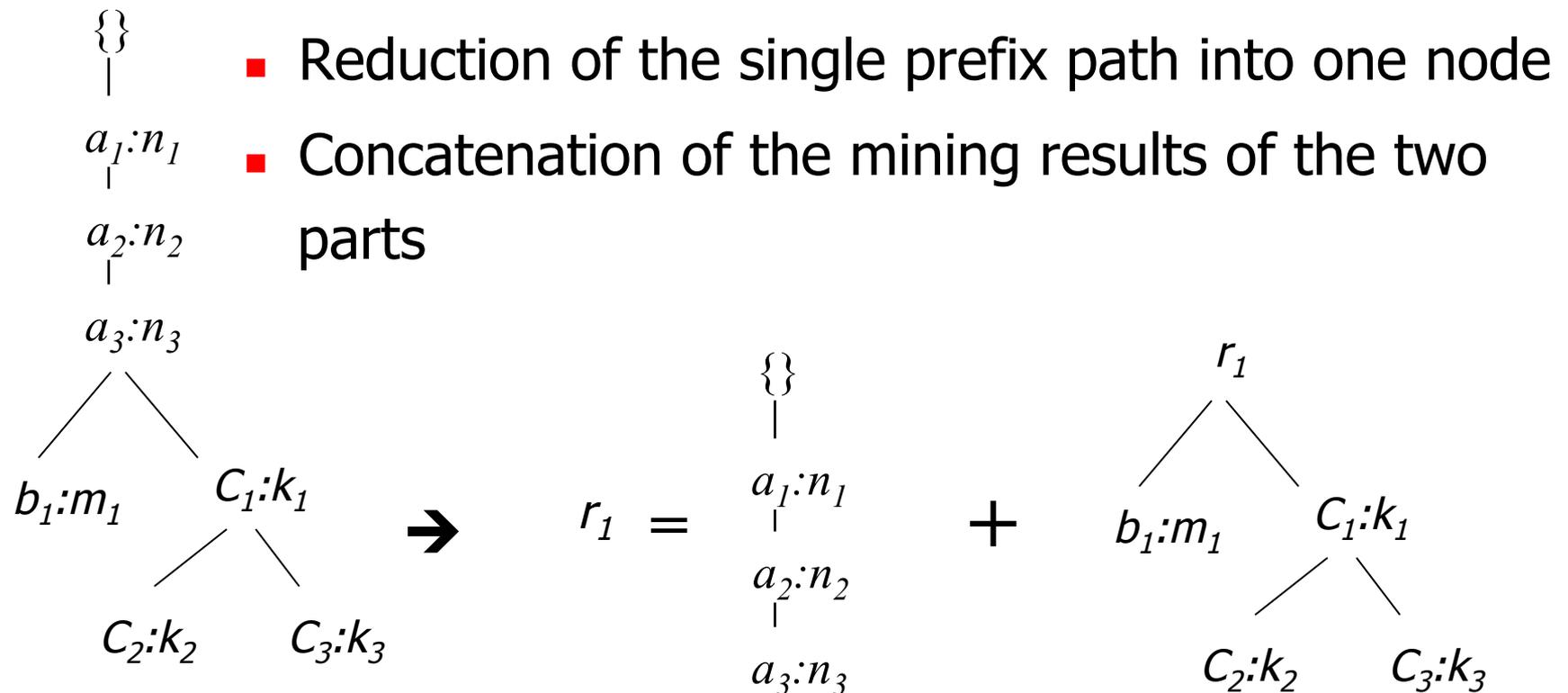
m-conditional FP-tree

Recursion: Mining Each Conditional FP-tree



A Special Case: Single Prefix Path in FP-tree

- Suppose a (conditional) FP-tree T has a shared single prefix-path P
- Mining can be decomposed into two parts



Benefits of the FP-tree Structure

- Completeness
 - Preserve complete information for frequent pattern mining
 - Never break a long pattern of any transaction
- Compactness
 - Reduce irrelevant info—infrequent items are gone
 - Items in frequency descending order: the more frequently occurring, the more likely to be shared
 - Never be larger than the original database (not count node-links and the *count* field)

The Frequent Pattern Growth Mining Method

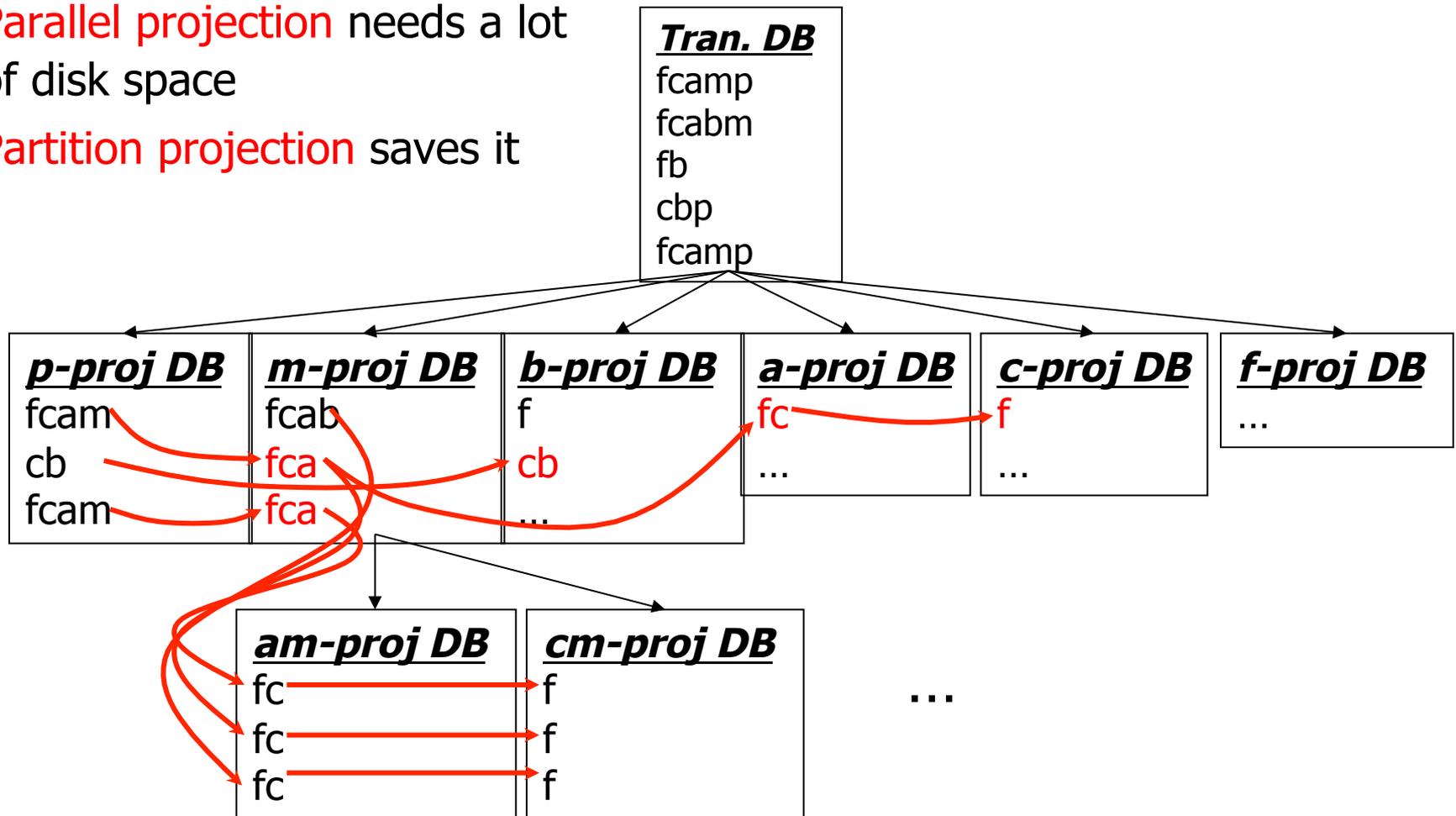
- Idea: Frequent pattern growth
 - Recursively grow frequent patterns by pattern and database partition
- Method
 - For each frequent item, construct its conditional pattern-base, and then its conditional FP-tree
 - Repeat the process on each newly created conditional FP-tree
 - Until the resulting FP-tree is empty, or it contains only one path—single path will generate all the combinations of its sub-paths, each of which is a frequent pattern

Scaling FP-growth by Database Projection

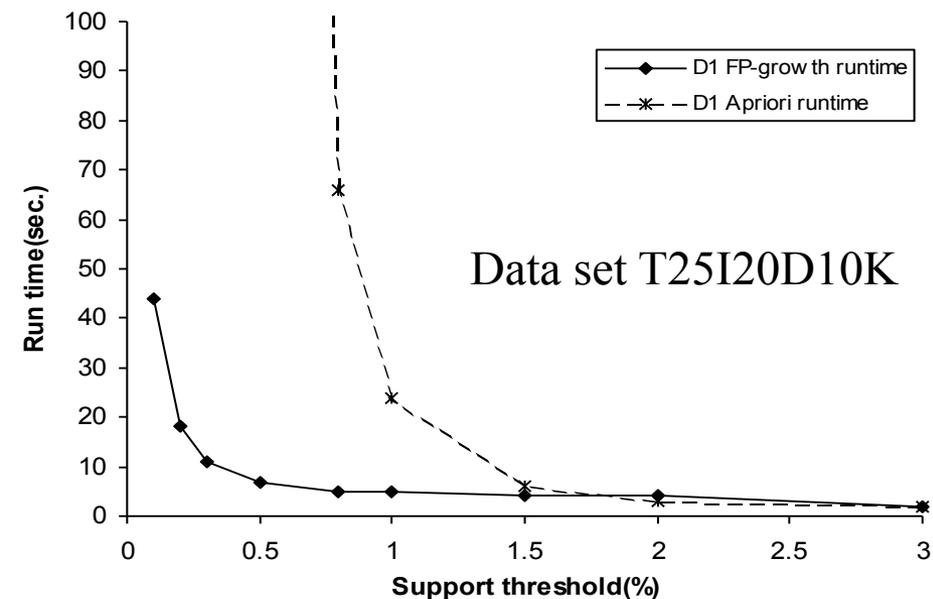
- What about if FP-tree cannot fit in memory?
 - DB projection
- First partition a database into a set of projected DBs
- Then construct and mine FP-tree for each projected DB
- **Parallel projection** vs. **partition projection** techniques
 - Parallel projection
 - Project the DB in parallel for each frequent item
 - Parallel projection is space costly
 - All the partitions can be processed in parallel
 - Partition projection
 - Partition the DB based on the ordered frequent items
 - Passing the unprocessed parts to the subsequent partitions

Partition-Based Projection

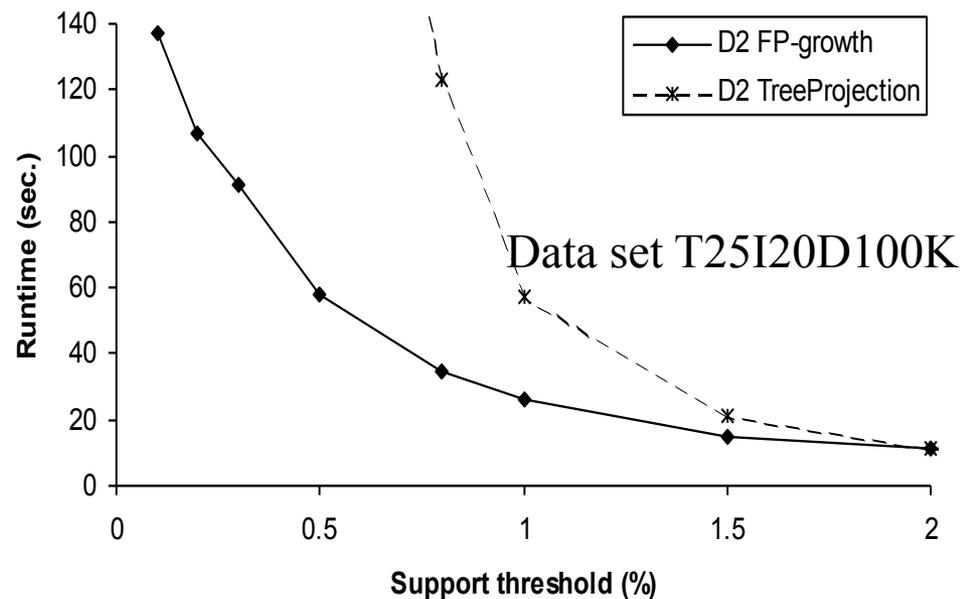
- **Parallel projection** needs a lot of disk space
- **Partition projection** saves it



Performance of FPGrowth in Large Datasets



FP-Growth vs. Apriori



FP-Growth vs. Tree-Projection

Advantages of the Pattern Growth Approach

- Divide-and-conquer:
 - Decompose both the mining task and DB according to the frequent patterns obtained so far
 - Lead to focused search of smaller databases
- Other factors
 - No candidate generation, no candidate test
 - Compressed database: FP-tree structure
 - No repeated scan of entire database
 - Basic ops: counting local freq items and building sub FP-tree, no pattern search and matching
- A good open-source implementation and refinement of FPGrowth
 - FPGrowth+ (Grahne and J. Zhu, FIMI'03)

Further Improvements of Mining Methods

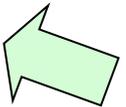
- AFOPT (Liu, et al. @ KDD' 03)
 - A “push-right” method for mining condensed frequent pattern (CFP) tree
- Carpenter (Pan, et al. @ KDD' 03)
 - Mine data sets with small rows but numerous columns
 - Construct a row-enumeration tree for efficient mining
- FPgrowth+ (Grahne and Zhu, FIMI' 03)
 - Efficiently Using Prefix-Trees in Mining Frequent Itemsets, Proc. ICDM'03 Int. Workshop on Frequent Itemset Mining Implementations (FIMI'03), Melbourne, FL, Nov. 2003
- TD-Close (Liu, et al, SDM' 06)

Extension of Pattern Growth Mining Methodology

- Mining closed frequent itemsets and max-patterns
 - CLOSET (DMKD' 00), FPclose, and FPMMax (Grahne & Zhu, Fimi' 03)
- Mining sequential patterns
 - PrefixSpan (ICDE' 01), CloSpan (SDM' 03), BIDE (ICDE' 04)
- Mining graph patterns
 - gSpan (ICDM' 02), CloseGraph (KDD' 03)
- Constraint-based mining of frequent patterns
 - Convertible constraints (ICDE' 01), gPrune (PAKDD' 03)
- Computing iceberg data cubes with complex measures
 - H-tree, H-cubing, and Star-cubing (SIGMOD' 01, VLDB' 03)
- Pattern-growth-based Clustering
 - MaPle (Pei, et al., ICDM' 03)
- Pattern-Growth-Based Classification
 - Mining frequent and discriminative patterns (Cheng, et al, ICDE' 07)

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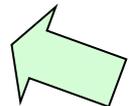


ECLAT: Mining by Exploring Vertical Data Format

- Vertical format: $t(AB) = \{T_{11}, T_{25}, \dots\}$
 - tid-list: list of trans.-ids containing an itemset
- Deriving frequent patterns based on vertical intersections
 - $t(X) = t(Y)$: X and Y always happen together
 - $t(X) \subset t(Y)$: transaction having X always has Y
- Using **diffset** to accelerate mining
 - Only keep track of differences of tids
 - $t(X) = \{T_1, T_2, T_3\}$, $t(XY) = \{T_1, T_3\}$
 - Diffset $(XY, X) = \{T_2\}$
- Eclat (Zaki et al. @KDD' 97)
- Mining Closed patterns using vertical format: CHARM (Zaki & Hsiao@SDM' 02)

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Mining Frequent Closed Patterns: CLOSET

- Flist: list of all frequent items in support ascending order
 - Flist: d-a-f-e-c
- Divide search space
 - Patterns having d
 - Patterns having d but no a, etc.
- Find frequent closed pattern recursively
 - Every transaction having d also has *cfa* → *cfad* is a frequent closed pattern
- J. Pei, J. Han & R. Mao. "CLOSET: An Efficient Algorithm for Mining Frequent Closed Itemsets", DMKD'00.

Min_sup=2

TID	Items
10	a, c, d, e, f
20	a, b, e
30	c, e, f
40	a, c, d, f
50	c, e, f

CLOSET+: Mining Closed Itemsets by Pattern-Growth

- Itemset merging: if Y appears in every occurrence of X , then Y is merged with X
- Sub-itemset pruning: if $Y \supset X$, and $\text{sup}(X) = \text{sup}(Y)$, X and all of X 's descendants in the set enumeration tree can be pruned
- Hybrid tree projection
 - Bottom-up physical tree-projection
 - Top-down pseudo tree-projection
- Item skipping: if a local frequent item has the same support in several header tables at different levels, one can prune it from the header table at higher levels
- Efficient subset checking

MaxMiner: Mining Max-Patterns

- 1st scan: find frequent items

- A, B, C, D, E

- 2nd scan: find support for

- AB, AC, AD, AE, ABCDE

- BC, BD, BE, BCDE

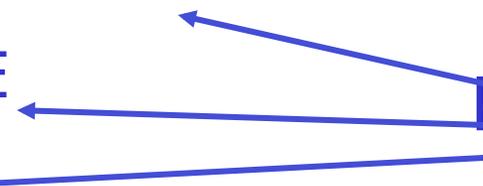
- CD, CE, CDE, DE

- Since BCDE is a max-pattern, no need to check BCD, BDE, CDE in later scan

- R. Bayardo. Efficiently mining long patterns from databases. *SIGMOD'98*

Tid	Items
10	A, B, C, D, E
20	B, C, D, E,
30	A, C, D, F

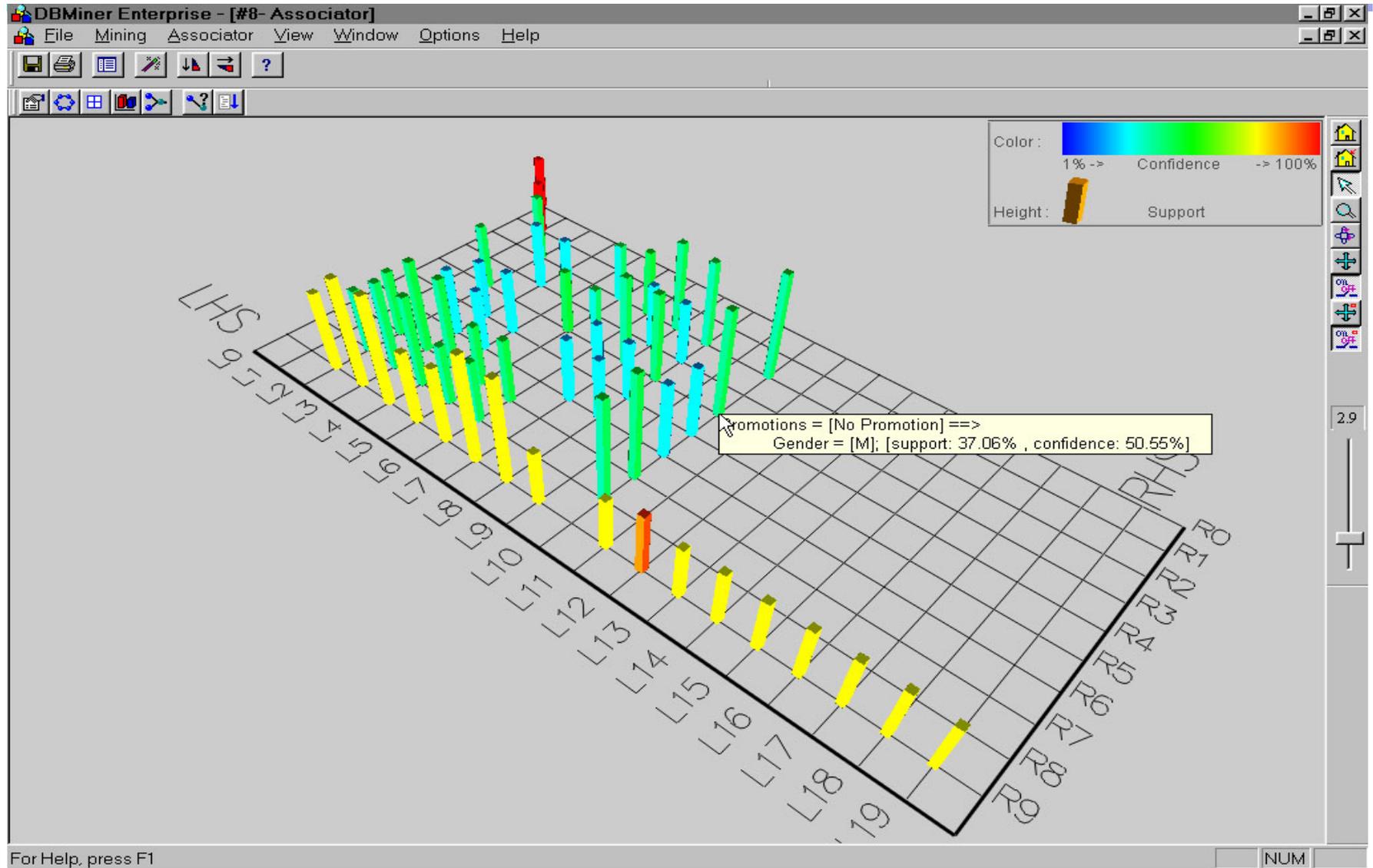
Potential max-patterns



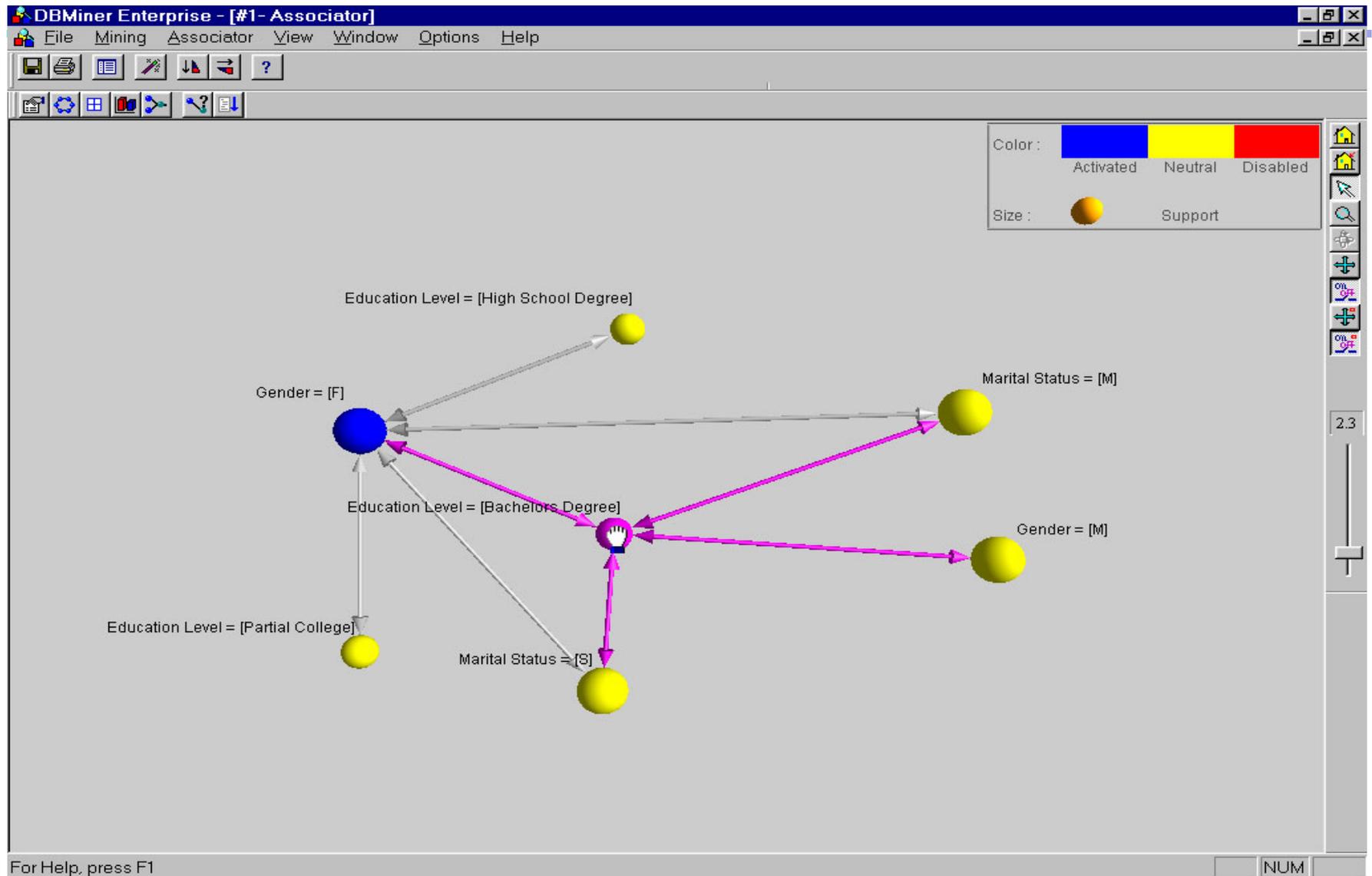
CHARM: Mining by Exploring Vertical Data Format

- Vertical format: $t(AB) = \{T_{11}, T_{25}, \dots\}$
 - tid-list: list of trans.-ids containing an itemset
- Deriving closed patterns based on vertical intersections
 - $t(X) = t(Y)$: X and Y always happen together
 - $t(X) \subset t(Y)$: transaction having X always has Y
- Using **diffset** to accelerate mining
 - Only keep track of differences of tids
 - $t(X) = \{T_1, T_2, T_3\}$, $t(XY) = \{T_1, T_3\}$
 - $\text{Diffset}(XY, X) = \{T_2\}$
- Eclat/MaxEclat (Zaki et al. @KDD' 97), VIPER(P. Shenoy et al.@SIGMOD' 00), CHARM (Zaki & Hsiao@SDM' 02)

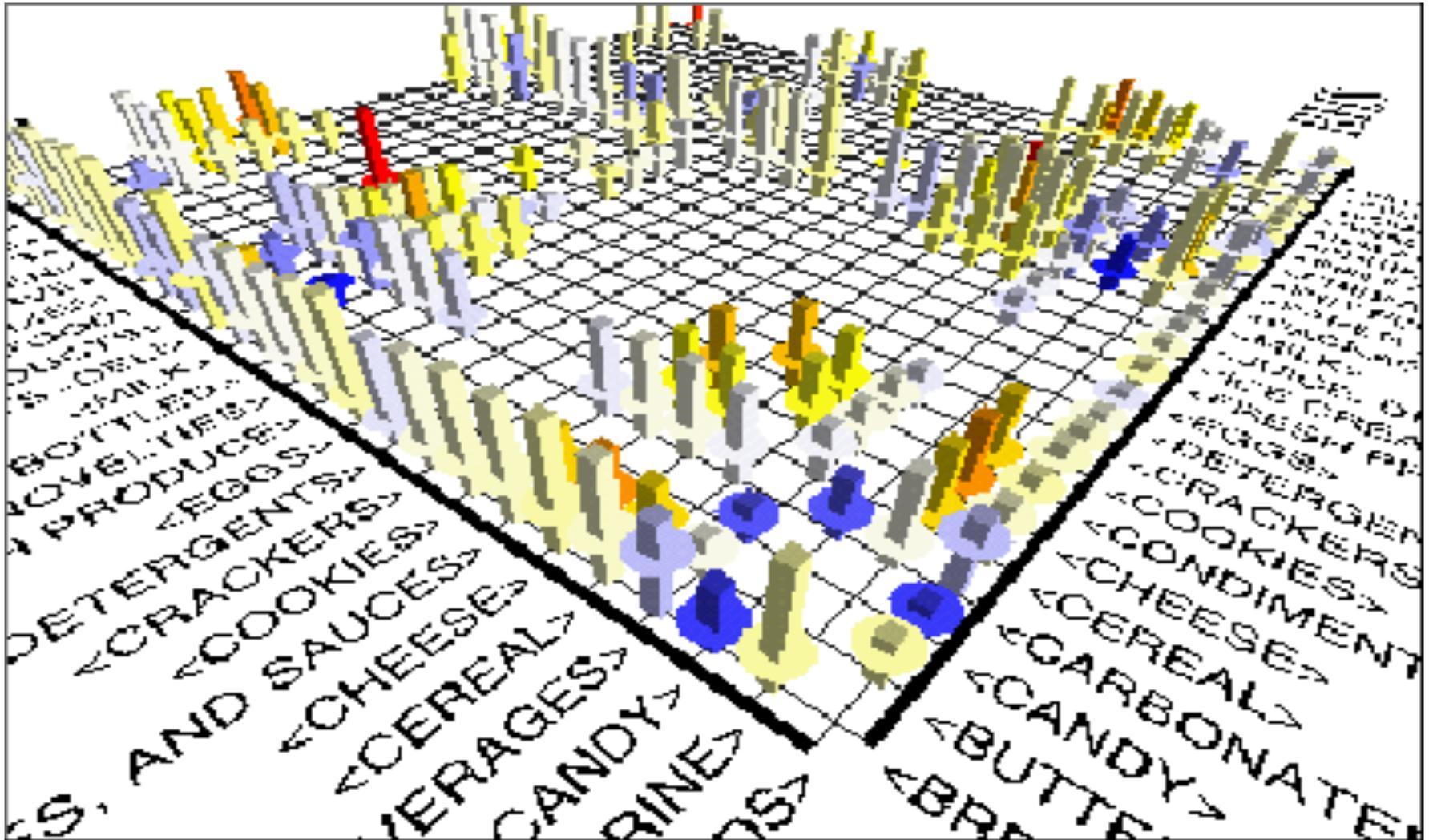
Visualization of Association Rules: Plane Graph



Visualization of Association Rules: Rule Graph

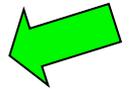


Visualization of Association Rules (SGI/MineSet 3.0)



Chapter 5: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Frequent Itemset Mining Methods
- Which Patterns Are Interesting?—Pattern Evaluation Methods
- Summary



Interestingness Measure: Correlations (Lift)

- *play basketball* \Rightarrow *eat cereal* [40%, 66.7%] is misleading
 - The overall % of students eating cereal is 75% > 66.7%.
- *play basketball* \Rightarrow *not eat cereal* [20%, 33.3%] is more accurate, although with lower support and confidence
- Measure of dependent/correlated events: **lift**

$$\text{lift} = \frac{P(A \cup B)}{P(A)P(B)}$$

$$\text{lift}(B, C) = \frac{2000 / 5000}{3000 / 5000 * 3750 / 5000} = 0.89$$

$$\text{lift}(B, \neg C) = \frac{1000 / 5000}{3000 / 5000 * 1250 / 5000} = 1.33$$

	Basketball	Not basketball	Sum (row)
Cereal	2000	1750	3750
Not cereal	1000	250	1250
Sum(col.)	3000	2000	5000

Are *lift* and χ^2 Good Measures of Correlation?

- “Buy walnuts \Rightarrow buy milk [1%, 80%]” is misleading if 85% of customers buy milk

- Support and confidence are not good to indicate correlations

- Over 20 interestingness measures have been proposed (see Tan, Kumar, Sritastava @KDD’ 02)

- Which are good ones?

symbol	measure	range	formula
ϕ	ϕ -coefficient	-1 ... 1	$\frac{P(A,B) - P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
Q	Yule’s Q	-1 ... 1	$\frac{P(A,B)P(\bar{A},\bar{B}) - P(A,\bar{B})P(\bar{A},B)}{P(A,B)P(\bar{A},\bar{B}) + P(A,\bar{B})P(\bar{A},B)}$
Y	Yule’s Y	-1 ... 1	$\frac{\sqrt{P(A,B)P(\bar{A},\bar{B})} - \sqrt{P(A,\bar{B})P(\bar{A},B)}}{\sqrt{P(A,B)P(\bar{A},\bar{B})} + \sqrt{P(A,\bar{B})P(\bar{A},B)}}$
k	Cohen’s	-1 ... 1	$\frac{P(A,B) + P(\bar{A},\bar{B}) - P(A)P(B) - P(\bar{A})P(\bar{B})}{1 - P(A)P(B) - P(\bar{A})P(\bar{B})}$
PS	Piatetsky-Shapiro’s	-0.25 ... 0.25	$P(A, B) - P(A)P(B)$
F	Certainty factor	-1 ... 1	$\max(\frac{P(B A) - P(B)}{1 - P(B)}, \frac{P(A B) - P(A)}{1 - P(A)})$
AV	added value	-0.5 ... 1	$\max(P(B A) - P(B), P(A B) - P(A))$
K	Klogsen’s Q	-0.33 ... 0.38	$\sqrt{P(A, B)} \max(P(B A) - P(B), P(A B) - P(A))$
g	Goodman-kruskal’s	0 ... 1	$\frac{\sum_j \max_k P(A_j, B_k) + \sum_k \max_j P(A_j, B_k) - \max_j P(A_j) - \max_k P(B_k)}{2 - \max_j P(A_j) - \max_k P(B_k)}$
M	Mutual Information	0 ... 1	$\frac{\sum_i \sum_j P(A_i, B_j) \log \frac{P(A_i, B_j)}{P(A_i)P(B_j)}}{\sum_i \sum_j P(A_i, B_j) \log \frac{P(A_i, B_j)}{P(A_i)P(B_j)}}$
J	J-Measure	0 ... 1	$\min(-\sum_i P(A_i) \log P(A_i) \log P(A_i), -\sum_i P(B_i) \log P(B_i) \log P(B_i))$ $\max(P(A, B) \log(\frac{P(B A)}{P(B)}) + P(\bar{A}\bar{B}) \log(\frac{P(\bar{B} \bar{A})}{P(\bar{B})}))$
G	Gini index	0 ... 1	$P(A, B) \log(\frac{P(A B)}{P(A)}) + P(\bar{A}\bar{B}) \log(\frac{P(\bar{A} \bar{B})}{P(\bar{A})})$ $\max(P(A)[P(B A)^2 + P(\bar{B} A)^2] + P(\bar{A})[P(B \bar{A})^2 + P(\bar{B} \bar{A})^2] - P(B)^2 - P(\bar{B})^2,$ $P(B)[P(A B)^2 + P(\bar{A} B)^2] + P(\bar{B})[P(A \bar{B})^2 + P(\bar{A} \bar{B})^2] - P(A)^2 - P(\bar{A})^2)$
s	support	0 ... 1	$P(A, B)$
c	confidence	0 ... 1	$\max(P(B A), P(A B))$
L	Laplace	0 ... 1	$\max(\frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2})$
IS	Cosine	0 ... 1	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
γ	coherence(Jaccard)	0 ... 1	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
α	all.confidence	0 ... 1	$\frac{P(A,B)}{\max(P(A), P(B))}$
o	odds ratio	0 ... ∞	$\frac{P(A,B)P(\bar{A},\bar{B})}{P(\bar{A},B)P(A,\bar{B})}$
V	Conviction	0.5 ... ∞	$\max(\frac{P(A)P(\bar{B})}{P(\bar{A}\bar{B})}, \frac{P(B)P(\bar{A})}{P(\bar{B}\bar{A})})$
λ	lift	0 ... ∞	$\frac{P(A,B)}{P(A)P(B)}$
S	Collective strength	0 ... ∞	$\frac{P(A,B) + P(\bar{A}\bar{B})}{P(A)P(B) + P(\bar{A})P(\bar{B})} \times \frac{1 - P(A)P(B) - P(\bar{A})P(\bar{B})}{1 - P(A,B) - P(\bar{A}\bar{B})}$
χ^2	χ^2	0 ... ∞	$\sum_i \frac{(P(A_i) - E_i)^2}{E_i}$

Null-Invariant Measures

Table 6: Properties of interestingness measures. Note that none of the measures satisfies all the properties.

Symbol	Measure	Range	P1	P2	P3	O1	O2	O3	O3'	O4
ϕ	ϕ -coefficient	$-1 \dots 0 \dots 1$	Yes	Yes	Yes	Yes	No	Yes	Yes	No
λ	Goodman-Kruskal's	$0 \dots 1$	Yes	No	No	Yes	No	No*	Yes	No
α	odds ratio	$0 \dots 1 \dots \infty$	Yes*	Yes	Yes	Yes	Yes	Yes*	Yes	No
Q	Yule's Q	$-1 \dots 0 \dots 1$	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
Y	Yule's Y	$-1 \dots 0 \dots 1$	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
κ	Cohen's	$-1 \dots 0 \dots 1$	Yes	Yes	Yes	Yes	No	No	Yes	No
M	Mutual Information	$0 \dots 1$	Yes	Yes	Yes	No**	No	No*	Yes	No
J	J-Measure	$0 \dots 1$	Yes	No	No	No**	No	No	No	No
G	Gini index	$0 \dots 1$	Yes	No	No	No**	No	No*	Yes	No
s	Support	$0 \dots 1$	No	Yes	No	Yes	No	No	No	No
c	Confidence	$0 \dots 1$	No	Yes	No	No**	No	No	No	Yes
L	Laplace	$0 \dots 1$	No	Yes	No	No**	No	No	No	No
V	Conviction	$0.5 \dots 1 \dots \infty$	No	Yes	No	No**	No	No	Yes	No
I	Interest	$0 \dots 1 \dots \infty$	Yes*	Yes	Yes	Yes	No	No	No	No
IS	Cosine	$0 \dots \sqrt{P(A, B)} \dots 1$	No	Yes	Yes	Yes	No	No	No	Yes
PS	Piatetsky-Shapiro's	$-0.25 \dots 0 \dots 0.25$	Yes	Yes	Yes	Yes	No	Yes	Yes	No
F	Certainty factor	$-1 \dots 0 \dots 1$	Yes	Yes	Yes	No**	No	No	Yes	No
AV	Added value	$-0.5 \dots 0 \dots 1$	Yes	Yes	Yes	No**	No	No	No	No
S	Collective strength	$0 \dots 1 \dots \infty$	No	Yes	Yes	Yes	No	Yes*	Yes	No
ζ	Jaccard	$0 \dots 1$	No	Yes	Yes	Yes	No	No	No	Yes
K	Klosgen's	$(\frac{2}{\sqrt{3}} - 1)^{1/2} [2 - \sqrt{3} - \frac{1}{\sqrt{3}}] \dots 0 \dots \frac{2}{3\sqrt{3}}$	Yes	Yes	Yes	No**	No	No	No	No

where: P1: $O(M) = 0$ if $\det(M) = 0$, i.e., whenever A and B are statistically independent.

P2: $O(M_2) > O(M_1)$ if $M_2 = M_1 + [k \ -k; \ -k \ k]$.

P3: $O(M_2) < O(M_1)$ if $M_2 = M_1 + [0 \ k; \ 0 \ -k]$ or $M_2 = M_1 + [0 \ 0; \ k \ -k]$.

O1: Property 1: Symmetry under variable permutation.

O2: Property 2: Row and Column scaling invariance.

O3: Property 3: Antisymmetry under row or column permutation.

O3': Property 4: Inversion invariance.

O4: Property 5: Null invariance.

Yes*: Yes if measure is normalized.

No*: Symmetry under row or column permutation.

No**: No unless the measure is symmetrized by taking $\max(M(A, B), M(B, A))$.

Comparison of Interestingness Measures

- Null-(transaction) invariance is crucial for correlation analysis
- Lift and χ^2 are not null-invariant
- 5 null-invariant measures

	Milk	No Milk	Sum (row)
Coffee	m, c	~m, c	c
No Coffee	m, ~c	~m, ~c	~c
Sum(col.)	m	~m	Σ

Measure	Definition	Range	Null-Invariant
$\chi^2(a, b)$	$\sum_{i,j=0,1} \frac{(e(a_i, b_j) - o(a_i, b_j))^2}{e(a_i, b_j)}$	$[0, \infty]$	No
$Lift(a, b)$	$\frac{P(ab)}{P(a)P(b)}$	$[0, \infty]$	No
$AllConf(a, b)$	$\frac{sup(ab)}{\max\{sup(a), sup(b)\}}$	$[0, 1]$	Yes
$Coherence(a, b)$	$\frac{sup(ab)}{sup(a) + sup(b) - sup(ab)}$	$[0, 1]$	Yes
$Cosine(a, b)$	$\frac{sup(ab)}{\sqrt{sup(a)sup(b)}}$	$[0, 1]$	Yes
$Kulc(a, b)$	$\frac{sup(ab)}{2} \left(\frac{1}{sup(a)} + \frac{1}{sup(b)} \right)$	$[0, 1]$	Yes
$MaxConf(a, b)$	$\max\left\{ \frac{sup(ab)}{sup(a)}, \frac{sup(ab)}{sup(b)} \right\}$	$[0, 1]$	Yes

Null-transactions w.r.t. m and c

Kulczynski measure (1927)

Table 3. Interestingness measure definitions.

Null-invariant

Data set	mc	$\bar{m}\bar{c}$	$m\bar{c}$	$\bar{m}c$	χ^2	Lift	AllConf	Coherence	Cosine	Kulc	MaxConf
D_1	10,000	1,000	1,000	100,000	90557	9.26	0.91	0.83	0.91	0.91	0.91
D_2	10,000	1,000	1,000	100	0	1	0.91	0.83	0.91	0.91	0.91
D_3	100	1,000	1,000	100,000	670	8.44	0.09	0.05	0.09	0.09	0.09
D_4	1,000	1,000	1,000	100,000	24740	25.75	0.5	0.33	0.5	0.5	0.5
D_5	1,000	100	10,000	100,000	8173	9.18	0.09	0.09	0.29	0.5	0.91
D_6	1,000	10	100,000	100,000	965	1.97	0.01	0.01	0.10	0.5	0.99

Table 2. Example data sets.

Subtle: They disagree

Analysis of DBLP Coauthor Relationships

Recent DB conferences, removing balanced associations, low sup, etc.

ID	Author <i>a</i>	Author <i>b</i>	<i>sup(ab)</i>	<i>sup(a)</i>	<i>sup(b)</i>	<i>Coherence</i>	<i>Cosine</i>	<i>Kulc</i>
1	Hans-Peter Kriegel	Martin Ester	28	146	54	0.163 (2)	0.315 (7)	0.355 (9)
2	Michael Carey	Miron Livny	26	104	58	0.191 (1)	0.335 (4)	0.349 (10)
3	Hans-Peter Kriegel	Joerg Sander	24	146	36	0.152 (3)	0.331 (5)	0.416 (8)
4	Christos Faloutsos	Spiros Papadimitriou	20	162	26	0.119 (7)	0.308 (10)	0.446 (7)
5	Hans-Peter Kriegel	Martin Pfeifle	18	146	18	0.123 (6)	0.351 (2)	0.562 (2)
6	Hector Garcia-Molina	Wilburt Labio	16	144	18	0.110 (9)	0.314 (8)	0.500 (4)
7	Divyakant Agrawal	Wang Hsiung	16	120	16	0.133 (5)	0.365 (1)	0.567 (1)
8	Elke Rundensteiner	Murali Mani	16	104	20	0.148 (4)	0.351 (3)	0.477 (6)
9	Divyakant Agrawal	Oliver Po	12	120	12	0.100 (10)	0.316 (6)	0.550 (3)
10	Gerhard Weikum	Martin Theobald	12	106	14	0.111 (8)	0.312 (9)	0.485 (5)

Table 5. Experiment on DBLP data set.

Advisor-advisee relation: Kulc: high, coherence: low, cosine: middle

- Tianyi Wu, Yuguo Chen and Jiawei Han, “[Association Mining in Large Databases: A Re-Examination of Its Measures](#)”, Proc. 2007 Int. Conf. Principles and Practice of Knowledge Discovery in Databases (PKDD'07), Sept. 2007

Which Null-Invariant Measure Is Better?

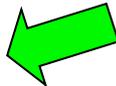
- IR (Imbalance Ratio): measure the imbalance of two itemsets A and B in rule implications

$$IR(A, B) = \frac{|sup(A) - sup(B)|}{sup(A) + sup(B) - sup(A \cup B)}$$

- Kulczynski and Imbalance Ratio (IR) together present a clear picture for all the three datasets D_4 through D_6
 - D_4 is balanced & neutral
 - D_5 is imbalanced & neutral
 - D_6 is very imbalanced & neutral

<i>Data</i>	<i>mc</i>	\overline{mc}	$m\overline{c}$	$\overline{m\overline{c}}$	<i>all_conf.</i>	<i>max_conf.</i>	<i>Kulc.</i>	<i>cosine</i>	IR
D_1	10,000	1,000	1,000	100,000	0.91	0.91	0.91	0.91	0.0
D_2	10,000	1,000	1,000	100	0.91	0.91	0.91	0.91	0.0
D_3	100	1,000	1,000	100,000	0.09	0.09	0.09	0.09	0.0
D_4	1,000	1,000	1,000	100,000	0.5	0.5	0.5	0.5	0.0
D_5	1,000	100	10,000	100,000	0.09	0.91	0.5	0.29	0.89
D_6	1,000	10	100,000	100,000	0.01	0.99	0.5	0.10	0.99

Chapter 5: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Frequent Itemset Mining Methods
- Which Patterns Are Interesting?—Pattern Evaluation Methods
- Summary 

Summary

- Basic concepts: association rules, support-confident framework, closed and max-patterns
- Scalable frequent pattern mining methods
 - Apriori (Candidate generation & test)
 - Projection-based (FPgrowth, CLOSET+, ...)
 - Vertical format approach (ECLAT, CHARM, ...)
- Which patterns are interesting?
 - Pattern evaluation methods

Ref: Basic Concepts of Frequent Pattern Mining

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- (**Max-pattern**) R. J. Bayardo. Efficiently mining long patterns from databases. SIGMOD'98
- (**Closed-pattern**) N. Pasquier, Y. Bastide, R. Taouil, and L. Lakhal. Discovering frequent closed itemsets for association rules. ICDT'99
- (**Sequential pattern**) R. Agrawal and R. Srikant. Mining sequential patterns. ICDE'95

Ref: Apriori and Its Improvements

- R. Agrawal and R. Srikant. Fast algorithms for mining association rules. VLDB'94
- H. Mannila, H. Toivonen, and A. I. Verkamo. Efficient algorithms for discovering association rules. KDD'94
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- S. Sarawagi, S. Thomas, and R. Agrawal. Integrating association rule mining with relational database systems: Alternatives and implications. SIGMOD'98

Ref: Depth-First, Projection-Based FP Mining

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Ref: Vertical Format and Row Enumeration Methods

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- F. Pan, G. Cong, A. K. H. Tung, J. Yang, and M. Zaki , CARPENTER: Finding Closed Patterns in Long Biological Datasets. KDD'03.
- H. Liu, J. Han, D. Xin, and Z. Shao, Mining Interesting Patterns from Very High Dimensional Data: A Top-Down Row Enumeration Approach, SDM'06.

Ref: Mining Correlations and Interesting Rules

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