Low-Complexity Decoding of Repeat-Accumulate Codes over Quasi-Static Fading Channels

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Abstract—We consider iterative decoding of repeat-accumulate (RA) codes over frequency-flat, quasi-static fading channels. A soft-input, soft-output decoder is proposed for the inner convolutional decoding, which fuses the decoding approach of the soft-output Viterbi algorithm and the estimation approach of the maximum-likelihood sequence detector. The decoder deploys trellis search algorithm based on the generalized likelihood ratio test, whereby the channel state information is acquired implicitly using both the pilot and data signals during the decoding process. Through simulations, we show that the RA decoding with the proposed decoder has much better error performance than standard RA decoding with pilot-symbol-assisted channel estimation, while having approximately the same computational complexity. Compared with the conventional scheme of iterative channel estimation and decoding, the proposed decoder has much simpler structure and requires significantly less computational power, although it incurs some loss in error performance.

Index Terms — iterative decoding, repeat-accumulate codes, quasi-static fading channel, SOVA, ML sequence detection

I. INTRODUCTION

Repeat-accumulate (RA) codes, introduced in [1], are one of the simplest classes of “turbo-like” codes, which can provide capacity-approaching performance. Compared with turbo codes [2] and low-density parity-check codes [3], RA codes have the advantages of simple encoding structure and low decoding complexity, which make them particularly useful for practical applications with limited computational power. Recently, much research attention has been drawn to the application of RA codes over fading channels [4]–[9]. It has been shown that RA codes can have very good error performance via turbo decoding [10] or sum-product decoding [10], [11], especially over block-fading channels and slowly varying fading channels [4]–[9]. In these works, perfect knowledge of the channel state information (CSI) is assumed at the receiver. However, in practice, channel estimation is usually required for CSI acquisition and the performance of the iterative decoding of RA codes relies greatly on the estimation accuracy.

The CSI can be acquired via blind channel estimation [12]–[14], but this method usually incurs high computational complexity and also suffers from inherent estimation and decision ambiguities [14]. Thus, the pilot-symbol-assisted modulation (PSAM) technique [15] is used more often in the literature, whereby pilot symbols are inserted into the data sequence and channel estimates are obtained by filtering the received pilot signals. In the PSAM scheme, accurate channel estimation requires either frequent pilot insertion or high pilot-symbol energy, which can lead to a loss in bandwidth- and power-efficiency. To obtain reliable channel estimates without incurring a large pilot overhead, decision-aided channel estimation techniques [16]–[19] have been proposed, in which the data signals are utilized for channel estimation in addition to the pilot signals. Among them, the iterative channel estimation and decoding (ICED) technique [17]–[22] is most commonly used, and references [20]–[22] discuss, in particular, the ICED receivers with RA codes. These receivers usually have good error-rate performance. However, the main disadvantage lies in the increased complexity due to the decision feedback and the decision-aided channel estimation after each decoding iteration. This motivates us to investigate the possibility of low-complexity decoding with data-signal-aided channel estimation.

In this paper, we propose a simple turbo-based iterative decoding scheme for RA codes over frequency-flat, quasi-static fading channels, which utilizes the energy of the data signals to improve the channel estimation accuracy while requiring much lower computational power than the ICED. In particular, a soft-input, soft-output decoder is proposed for the inner convolutional decoding, which fuses the decoding approach of the soft-output Viterbi algorithm (SOVA) [23] and the estimation approach of the maximum-likelihood (ML) sequence detector [24]. This decoder does not require explicit CSI acquisition, and thus it is referred to as the SOVA-NCSI (convolutional) decoder. The SOVA-NCSI decoder deploys trellis search algorithm based on the generalized likelihood ratio test. Channel estimation is carried out implicitly using the received data signal sequence and the information of each survivor path during the trellis search. Similar to the conventional SOVA [23], the reliability metric of each message bit is obtained by comparing the reliabilities of the survivor selections. However, unlike the SOVA which requires the CSI of each data signal explicitly, the path metric in the SOVA-NCSI can be computed adaptively using only the received signal sequence.

Iterative decoding of RA codes using the proposed SOVA-
NCSI as the inner convolutional decoder and the standard repetition decoder as the outer decoder is referred to as the RA-SOVA-NCSI. Through computer simulations, we demonstrate that the RA-SOVA-NCSI has significantly better error performance than standard RA decoding using the SOVA or the BCJR algorithm with PSAM channel estimation. In terms of computational complexity, we show that the SOVA-NCSI is approximately the same as the conventional SOVA, both of which are much simpler than the BCJR algorithm [25], which has been commonly used in conventional RA decoders [20]–[22]. Compared with the ICED receiver, the proposed RA-SOVA-NCSI has much simpler structure and requires significantly less computational power, although it incurs some loss in error performance.

The rest of the paper is organized as follows. In section II, we briefly introduce the receiver structures that are commonly used in the literature, and compare them with the proposed RA-SOVA-NCSI. The SOVA-NCSI decoder is derived in section III, and the complexity analysis is provided in section IV. Simulation results are presented in section V. Section VI concludes the paper.

II. SYSTEM MODEL AND TRANSCIEVER STRUCTURES

In this paper, we consider the simplest class of RA code, i.e., the $q$-regular non-systematic RA code which is formed by the serial concatenation of a rate-$\frac{1}{q}$ repetition code and a $\frac{1}{1+D}$ convolutional code, which is called the accumulator, with an interleaver between them. The transmitter model for the RA-coded transmission is shown in Fig. 1. The message sequence is first encoded by the RA encoder. One pilot symbol is inserted into the coded data stream after every $B−1$ data symbols and $B$ is usually referred to as the pilot symbol spacing. The composite sequence is modulated with binary phase-shift keying (BPSK) and transmitted through a frequency-flat, quasi-static fading channel.

Two types of receiver structures are considered, namely, the standard RA decoding and the ICED, which are depicted in Fig. 2. In the standard RA decoding, the CSI of each data signal is acquired using the received pilot signals through a moving average estimator. Assuming that the acquired estimates are equal to the true channel gains, standard iterative decoding is performed to recover the transmitted message. In the ICED scheme, the BCJR algorithm is used more often for the inner convolutional decoding, because it can produce soft decisions for the convolutionally encoded bits, in addition to the soft decisions of the input bits to the convolutional encoder. With the feedback loop, the tentative decisions from the RA decoder are used to carry out decision-aided channel estimation after each decoding iteration, and the refined channel estimates updates the reliability information at the decoder for further decoding iterations. It is clear that the receiver complexity of the ICED is higher than that of the standard decoding. Nevertheless, it is still widely considered because of good error performance [17]–[22]. In comparison, the receiver structure of the proposed RA-SOVA-NCSI, as shown in Fig. 3, is much simpler, and it does not even require any explicit channel estimator. We will show in section IV that the complexity of the SOVA-NCSI is much lower than that of the BCJR algorithm, and thus demonstrate that the RA-SOVA-NCSI requires significantly less computational power than the conventional ICED.

III. SOVA-NCSI DECODER

In this section, we propose the SOVA-NCSI decoder by considering a convolutional code with one input stream and $n$ output streams. Since the SOVA-NCSI performs trellis search decoding, a survivor has to be decided when two contending paths merge at one node. The survivor selection is carried out by comparing their reliability metrics, which are now derived as follows.

Consider a binary sequence $u = [u(1), u(2), \ldots, u(K)]$, which is encoded by a convolutional encoder of rate $\frac{1}{n}$. The BPSK-modulated code sequence is denoted by $\chi = [\chi(1), \chi(2), \ldots, \chi(k), \ldots, \chi(K)]$, where $\chi(k) = [x^{(1)}(k), x^{(2)}(k), \ldots, x^{(n)}(k)]$ and $x^{(i)}(k)$ is the modulated output from the $i$-th output branch of the convolutional encoder. The received signal for $x^{(i)}(k)$ can be expressed as

$$r^{(i)}(k) = h^{(i)}(k)\sqrt{E_s}x^{(i)}(k) + n^{(i)}(k),$$

where $E_s$ denotes the energy of each modulated signal, $h^{(i)}(k)$ is the channel gain and $n^{(i)}(k)$ is the complex AWGN with mean zero and variance $N_0$. We will use the notation $u_{[t',t]}$ to denote the subsequence of $u$ from the $t'$-th to the $t$-th term. At time instance $t$, the received signal sequence before $t$ is denoted by $r_{[1,t]} = [r(1), r(2), \ldots, r(t)]$, where
The reliability metric of the path $u_{1,t}$, the corresponding modulated code sequence is denoted by $x_{1,t}$.

To compute the ML probability density function (PDF) with respect to the channel gains $h_{1,t} = [h(1), h(2), \ldots h(t)]$, where $h(k) = [h^{(1)}(k), h^{(2)}(k), \ldots h^{(n)}(k)]$, i.e.,

$$\prod_{k=0}^{n} \max_{h(k)} p(u_{[a(k),b(k)]}, r_{[a(k),b(k)]} | h(k)),$$

where $a(k) = \max \{1, t - (k + 1)L + 1\}$ and $b(k) = t - k L$ represent the starting and ending positions of the $k$-th segment, respectively, and $h(k)$ is the constant channel gain for the segment. The reliability metric for the hypothesized path, which is defined as the logarithm of (2), can thus be expressed as

$$\lambda(u_{1,t}) = \sum_{k=0}^{n} \max_{h(k)} \log p(u_{[a(k),b(k)]}, r_{[a(k),b(k)]} | h(k)).$$

Because of the trellis decoding structure, and the partial sum of (3) from $k = 1$ to $n$ has already been computed at time instance $t - L$. Thus, it remains to compute the term for $k = 0$ only, i.e.,

$$\max_{h(0)} \log p(u_{[a(0),b(0)]}, r_{[a(0),b(0)]} | h(0)),$$

Note that since the metric computation in (3) and (4) only makes use of the received data signals, the trellis decoding using this metric will suffer from decision ambiguities, as shown in [14]. To resolve these ambiguities, the received pilot signals should also be incorporated into the metric computation. Let $r_p$ be the set consisting of the nearest $2W$ received pilot signals to $r(t)$ ($W$ preceding and $W$ succeeding, except at the beginning or at the end of the transmission). Assuming that they experience the same channel gain as $r_{[a(0),b(0)]}$, we get

$$\log p(u_{[a(0),b(0)]}, r_{[a(0),b(0)]}, r_p | h(0)) = \log p(u_{[a(0),b(0)]}, r_{[a(0),b(0)]} | h(0)) + \log p(r_p | h(0)).$$

Conditioned on the channel gain $h(0)$, the term $\log p(r_p | h(0))$ is independent of the path $u_{1,t}$. Thus, we will use

$$\max_{h(0)} \log p(u_{[a(0),b(0)]}, r_{[a(0),b(0)]}, r_p | h(0))$$

(5) to approximate (4) in the metric computation, and refer to it as the segment metric.

Now we compute the segment metric in (5). For the ease of representation, we use $h, v, s$ and $r$ to denote $h(0), u_{[a(0),b(0)]}, x_{[a(0),b(0)]}$ and $r_{[a(0),b(0)]}$, respectively. Using Bayes’ rule, we get

$$p(v, r, r_p | h) = p(r, r_p | v, h)P(v | h),$$

(6) where $v$ is independent of $h$, we have $P(v | h) = P(v)$. Conditioned on the channel gain and the transmitted signal sequence, each of the received signals is Gaussian distributed and statistically independent of one another. Substituting the Gaussian PDF of $p(r, r_p | v, h)$ into (6) yields

$$p(v, r, r_p | h) = \frac{1}{(\pi N_0)^{J}} \exp \left( - \frac{|r - h \sqrt{E_s} s|^2 + |r_p - h \sqrt{E_s} 1|^2}{2 N_0} \right) P(v),$$

(7) where $1$ is a vector of the same length as $r_p$ with each entry equal to 1, and $J$ is the total number of received signals in $r$ and $r_p$. Expressing the probability mass function of independent 0-1 random variables in terms of log-likelihood ratio (LLR) yields

$$P(v) = \prod_{k=0}^{t} C(k) e^{(1-u(k)) L_s(k)},$$

(8) where $L_s(k) = \log \frac{p(u(k)=0)}{p(u(k)=1)}$ is the a priori LLR of the input bit $u(k)$ and $C(k) = (1 + e^{L_s(k)})^{-1}$ is a constant, which is independent of the realization $v$. From [24], for a given hypothesis $v$, the value of $h$ maximizing (7) is given by

$$\hat{h}(s) = \frac{s^H r + 1^H r_p}{\sqrt{E_s (|s|^2 + |1|^2)}}$$

(9) Substituting (8) and (9) into (7), and after simplification, the segment metric can be written as a function of $v$ as

$$M^*(v)$$

$$= \frac{|s^H r + 1^H r_p|^2}{N_0(|s|^2 + |1|^2)} + \sum_{k=a(0)}^{b(0)} \left(1 - u(k)\right) L_s(k)$$

$$+ \left( \log \frac{1}{(\pi N_0)^{J}} - \frac{|r|^2 + |r_p|^2}{2 N_0} + \sum_{k=a(0)}^{b(0)} \log C(k) \right).$$

The last term above is a path-independent constant and thus can be ignored. Finally, we obtain the simplified segment metric as

$$M(v) = \frac{|s^H r + 1^H r_p|^2}{N_0(|s|^2 + |1|^2)} + \sum_{k=a(0)}^{b(0)} \left(1 - u(k)\right) L_s(k).$$

(10)
In the SOVA-NCSI decoder, the reliability of the hypothesized path at time \( t \) is obtained by adding the segment path metric (10) with the previously stored path metric at time \( t - L \). The difference in the path metrics of two contending paths indicates the reliability of the survivor selection, based on which the reliability of each input bit is obtained using the same procedure as in the SOVA [23]. Finally, soft decisions are obtained from the reliability values and the hard-decision decoding outcome.

**IV. COMPLEXITY ANALYSIS**

In this section, we compare the computational complexity of the proposed SOVA-NCSI with the conventional SOVA [23] and the BCJR algorithm [25]. Again, we consider the rate \( 1/n \) convolutional code with constraint length \( \nu \). The complexity measured by the average number of operations per unit time is tabulated in TABLE I.

For the SOVA and the BCJR algorithm (log-domain), we follow the analysis in [26, Sec.12.5] and [26, Sec.12.6], respectively. In addition, we assume that the decision delay in the SOVA is \( 5\nu \), i.e., all survivors will have the same history prior to five constraint lengths in the past [23]. Note that the exponential and logarithmic operations are required in the BCJR algorithm because the reliability information is required to be converted from the log-domain to the probability-domain for scalar addition. Although it is possible to carry out the BCJR algorithm in probability-domain, the complexity is usually higher due to the exponential operations taken to convert each input branch reliability into a vector of the a posteriori probabilities.

The SOVA-NCSI differs from the conventional SOVA only in the computation of the path metric. Using the notations in this paper, the metric in the SOVA can be expressed as

\[
M(\nu) = \frac{2\sqrt{E_s}}{N_0} R[H^*(r \odot \hat{H}^*)] + \sum_{k=1}^{L} (1 - u(k)) L_s(k),
\]

where \( \hat{H}^* \) denotes the conjugate of the estimated channel gain vector, and \( \odot \) denotes the point-wise vector multiplication. By storing the numerical results from the previous time instances with the survivor paths and using them to simplify the current metric computation, it can be deduced that the computation of the path metric in the SOVA-NCSI requires only \( 2 \times 2^\nu \) additional multiplications and \( 6 \times 2^\nu \) additional additions per unit time, compared with that of the SOVA.

By assuming that the computational costs of one comparison, one multiplication/division and one exponentiation/logarithm are equivalent to the costs of one addition, two additions and forty additions, respectively, the complexity of the decoders can also be measured in terms of average number of additions (ANA) per unit time. It is observed from TABLE I that the BCJR algorithm requires much higher computational power than the other algorithms. In particular, for the \( 1/3 \) convolutional code, the ANA per unit time for the BCJR algorithm is about 50 times as large as that of the SOVA, and about 30 times as large as that of the SOVA-NCSI.

**V. SIMULATION RESULTS AND DISCUSSIONS**

In our simulation studies, the RA code of message length of 1024 bits is chosen, which constitutes of the 4-regular repetition code, the \( 1/3 \) convolutional code and a pseudo-random interleaver. Iterative decoding is performed with a maximum of 20 iterations. Jakes’s isotropic scattering channel model [27] is chosen to model the quasi-static fading channel.
In particular, the real and imaginary parts of the channel gains are assumed to be independent, each with autocorrelation

$$R_c(k) = \sigma^2 J_0(2\pi f_d T_s k)$$

where $f_d$ is the relative Doppler shift between the transmitter and the receiver, $T_s$ is the symbol period and $J_0(\cdot)$ is the Bessel function of the first kind of order zero. Two normalized fade rates are considered, namely $f_d T_s = 0.001$ and $f_d T_s = 0.005$. The SNR in each graph refers to the normalized effective energy per message bit over the noise spectrum density, i.e.,

$$E_b/N_0 = \frac{B}{R(B-1)} 2\pi^2 E_s/N_0,$$

which accounts for the energy consumed by the pilot signals, where $R$ is the rate of the RA code.

### A. BER comparison

The bit-error rate (BER) performance of the various decoding schemes is compared at different fade rates and pilot symbol spacings in Figs. 4–6. In these simulations, the segmentation length $L$ for the SOVA-NCSI decoder is fixed at 20. It can be observed that the RA-SOVA-NCSI outperforms both the standard decoding with the BCJR and that with the SOVA. The performance gap increases with an increase of the pilot symbol spacing. In particular, at $f_d T_s = 0.005$ and $B = 20$, the RA-SOVA-NCSI achieves a performance gain of 1.6 dB and 2.3 dB over the standard decoding with the SOVA and that with the BCJR, respectively, at the BER of $10^{-5}$.

Convolutional codes tend to produce burst errors, which may deteriorate the performance of the decision-aided estimators. This explains why the performance of the soft-decision-aided (SDA) ICED and the hard-decision-aided (HDA) ICED is worse than that of the standard decoding. This problem can be solved by using a channel interleaver to shuffle the data signal sequence before pilot insertion. It can be observed that the error performance of the ICED schemes is significantly improved when a channel interleaver is used. However, this will further increase the complexity of the ICED receiver.

It is also worthwhile to note that when perfect CSI is available at the receiver, standard decoding with the BCJR outperforms that with the SOVA. This is expected because the SOVA is merely an approximate to the BCJR in the sense of maximum a posteriori probability decoding. Surprisingly, the trend is reversed when the PSAM channel estimator is used, i.e., standard decoding with the SOVA outperforms that with the BCJR under PSAM channel estimation. This observation shows that channel estimation errors have larger impact on the BCJR decoder than on the SOVA decoder. In other words, the SOVA is more robust against channel estimation errors.

### B. Effect of Segmentation Length in SOVA-NCSI

The effect of the segmentation length $L$ of the SOVA-NCSI decoder is investigated in Figs. 7 and 8. It can be observed that at $f_d T_s = 0.001$, the performance improves as $L$ increases. This is attributed to the fact that the implicit channel estimates (9) become more accurate when more data signals are utilized for estimation. However, at the higher fade rate of $f_d T_s = 0.005$, the performance deteriorates after hitting the optimum value at around $L = 20$. This is because the assumption of block fading in the derivation of the SOVA-NCSI becomes inappropriate when the segmentation length $L$ is too large.
Fig. 7: Effect of segmentation length $L$ on the error performance of the RA-SOVA-NCSI at $f_d T_s = 0.001$, $B = 40$

Fig. 8: Effect of segmentation length $L$ on the error performance of the RA-SOVA-NCSI at $f_d T_s = 0.005$, $B = 20$

VI. Conclusion

The proposed RA-SOVA-NCSI scheme for RA decoding offers a good trade-off between error performance and receiver complexity. Although only the regular RA code is demonstrated in the paper, the proposed decoder is generally applicable to other types of RA codes, such as the irregular RA codes considered in [4]–[9], [20]–[22]. Moreover, if the quasi-static channel is frequency-selective, the proposed decoder can still be utilized under the orthogonal frequency-division multiplexing (OFDM).

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