Delay and Capacity in Ad Hoc Mobile Networks with f-cast Relay Algorithms

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Abstract—The 2-hop relay algorithm and its variants have been attractive for ad hoc mobile networks, because they are simple yet efficient, and more importantly, they enable the capacity and delay to be studied analytically. This paper considers the 2-hop relay with $f$-cast (2HR-$f$) under the independent and identically distributed (i.i.d.) mobility model, a general 2-hop relay algorithm that allows one packet to be delivered to at most $f$ distinct relay nodes. The 2HR-$f$ algorithm covers the available 2-hop relay algorithms ($f=1$, $\sqrt{n}$) as special cases. Closed-form analytical models rather than order sense ones are developed for the 2HR-$f$ algorithm with a careful consideration of important medium contention and queuing delay issues, which enable an accurate delay and capacity analysis to be performed for ad hoc mobile networks employing 2HR-$f$. Based on our models and some typical settings of $f$ (say, $f = 1$, $\sqrt{n}$), one can easily derive the corresponding order sense results.

I. INTRODUCTION

Since the seminal work of Grossglauser and Tse (2001) [1], the 2-hop relay algorithm and its variants have become a class of attractive routing algorithms for ad hoc mobile networks, because they are simple yet efficient, and more importantly, they enable the capacity and delay to be studied analytically. The 2-hop relay algorithm defines two phases for packet transmission, where in phase 1 a packet is transmitted from its source node to an intermediate node (relay node), and then in phase 2 the packet is transmitted from the relay node to its destination node. Since the source node can directly transmit a packet to its destination node every time such transmission opportunity arises, every packet goes through at most 2 hops to reach its destination in a 2-hop relay network.

By now, extensive order sense results of delay and capacity have been reported to illustrate the scaling laws of 2-hop relay ad hoc mobile networks under various mobility models. Grossglauser and Tse (2001) [1] showed that it is possible to achieve a $\Theta(1)$ throughput per node under i.i.d. mobility model. Later, Gamal et al. [2] showed that the $\Theta(1)$ throughput is also achievable under the random walk model, but which comes at the price of a $\Theta(n \log n)$ delay. Mammen et al. [3] proved that the same throughput and delay scaling are also achievable even with a variant of the Grossglauser-Tse 2-hop relay and a restricted mobility model. The delay and throughput trade-off has been further widely studied under different mobility models, like the i.i.d. mobility model [4], hybrid random walk and discrete random direction models [5], Brownian motion model [6], [7], and correlated mobility model [8]. These order sense results are helpful for us to understand the general scaling laws of delay and capacity in a 2-hop relay ad hoc mobile network, but they tell us a little about the real end-to-end delay and capacity of such networks. In practice, however, the real delay and capacity results are of great interest for network designers.

It is notable that the relay algorithms discussed above can be regarded as the basic 2-hop relay without packet redundancy (i.e., without redundant copies of each packet). As throughput and delay can be traded with each other in a multi-hop way, this trade-off can also be achieved to somewhat extent via delivering redundant copies for each packet. Neely and Modiano [9] considered a modified version of the Grossglauser-Tse 2-hop relay algorithm for ad hoc mobile networks, and proved that under i.i.d. mobility model it achieve $O(1/\sqrt{n})$ throughput and $O(\sqrt{n})$ delay with exact $\sqrt{n}$ redundancy for each packet. Sharma and Mazumdar [10] explored the order sense delay and capacity trade-off in ad hoc mobile networks under random way-point mobility model and with multiple redundancy for each packet. The idea of using packet redundancy has also been adopted to reduce average packet delivery delay in intermittently connected mobile networks (ICMNs) [11]–[13].

In this paper we consider the 2-hop relay with $f$-cast (2HR-$f$) under i.i.d. mobility model, a general 2-hop relay algorithm that allows one packet to be delivered to at most $f$ distinct relay nodes, and develop closed-form analytical models rather than order sense ones for an accurate delay and capacity analysis for 2HR-$f$-based ad hoc mobile networks. With such closed-form models and some typical settings of $f$ (say, $f = 1$, $\sqrt{n}$), one can easily derive the corresponding order sense results for delay and capacity.

The rest of the paper is organized as follows. In Section II, we provide the network model, interference model and mobility model considered in our analysis. Section III introduces the the 2-hop relay algorithm $f$-cast (2HR-$f$) and the corresponding scheduling scheme. We develop the closed-form models in Section IV to analyze the expected end-to-end delay and capacity, and finally we conclude the paper in Section V.
II. SYSTEM MODELS

A. Network Model

We assume the network is a square region of unit area, with \( n \) mobile nodes which are initially independently and uniformly distributed inside the network region. The unit square is evenly divided into \( \sqrt{n} \times \sqrt{n} \) cells each of which has an area of \( 1/n \). All the \( n \) mobile nodes are independently roaming from cell to cell, and time is assumed to be slotted so that each node remains in its current cell for one time slot.

We consider a bi-dimensional i.i.d. mobility model, or so-called reshuffling model in this paper. At the beginning of each time slot, every node independently selects a destination cell and stays in it for the whole time slot, thus the position of each node is updated every time slot. The destination cell is chosen uniformly and randomly among all \( n \) cells, so each cell would be chosen with same probability \( 1/n \).

We assume a time-scale of fast mobility [8] in this paper, i.e., the mobility of nodes is at the same time-scale as the data transmission. Thus, only one-hop transmissions are feasible during any single slot, and the total number of bits that can be transmitted in a time slot is a fixed constant independent of \( n \). We normalize this constant to 1 here.

B. Interference Model

Similar to [1], we assume a uniform communication range \( r = \Theta(1/\sqrt{n}) \) for all nodes, and adopt the protocol model introduced in [14] to account for interference among simultaneous transmissions. Suppose node \( i \) is transmitting to node \( j \) at some time slot \( t \), and their Euclidean distance is \( d_{ij}(t) \). According to the protocol model, this transmission can be successful if and only if the following two conditions hold:

1. \( d_{ij}(t) \leq r; \)
2. \( d_{kj}(t) \geq (1+\Delta)r \) for every other node \( k \) which transmits simultaneously, where \( \Delta \) is a protocol specified guard-factor to prevent interference.

C. Traffic Model

Similar to previous works we consider permutation traffic patterns, in which each node is a source and at the same time a destination of some other node. Hence there are \( n \) source-destination pairs in the network. For convenience of expression, we use \( S(K) \) and \( D(K) \) to denote the source node and destination node of node \( K \), respectively. We further assume a homogeneous scenario in which the traffic originating at each node is a Poisson stream with rate \( \lambda \) (packets/slot). We also assume that the packet arrives at the beginning of time slots, and the arrival process at each node is independent of its mobility process.

III. 2HR-\( f \) ALGORITHM AND SCHEDULING SCHEME

A. 2HR-\( f \) Algorithm

We consider a generalization of the 2-hop relay algorithm [9] with \( f \)-cast (2HR-\( f \)), which allows up to \( f \) (\( 1 \leq f \leq \sqrt{n} \)) relay nodes for a single packet. As we keep one copy at the source node, so there are at most \( f + 1 \) copies of a single packet to coexist in the network.

As a node can be a potential relay for any of other \( n-2 \) flows (except the two flows originating at and destined for itself), we assume that every node maintains \( n \) individual queues at its buffer, one local-queue for storing its locally generated packets waiting for copy-distribution, one already-sent-queue for storing packets whose \( f \) replicas have already been distributed but reception status are not confirmed yet (from destination node), and \( n-2 \) parallel relay-queues for storing packets of other flows (one queue per flow). We further assume all packets of one flow are labeled with sequence numbers, so that a packet can be efficiently retrieved from the queue buffers of its source node or relay node(s) according to its sequence number and destination information. During each time slot, a TCP-style handshake is proceeded before packet transmission to indicate which packet the receiver currently needs. Now we are ready to formally define the 2HR-\( f \) algorithm.

2HR-\( f \) Algorithm: Every time a node (say \( K \)) gets a transmission opportunity, it operates as follows:

**Step 1:** (Source-to-Destination) Check if node \( D(K) \) is among its one-hop neighbors. If so, a handshake goes as follows: \( D(K) \) first sends its current request number \( RN \) to \( K \), then \( K \) compares \( RN \) with the send number \( SN(P_h) \) of the packet \( P_h \) at the head of its local-queue.

- If \( SN(P_h) > RN \), \( K \) retrieves from its already-sent-queue the packet with \( SN = RN \), and deletes all packets with \( SN \leq RN \) inside the already-sent-queue;
- If \( SN(P_h) = RN \), \( K \) sends \( P_h \) directly to \( D(K) \), moves ahead remaining packets waiting at its local-queue and deletes all packets with \( SN < RN \) in its already-sent-queue;
- If \( SN(P_h) < RN \) (then \( RN = SN(P_h) + 1 \)), \( K \) sends the packet behind \( P_h \) with the send number equal to \( RN \) to \( D(K) \), moves ahead remaining packets inside its local-queue (by two packets) and empties its already-sent-queue.

**Step 2:** Otherwise, \( K \) flips a unbiased coin, and choose either one from the following:

- (Source-to-Relay) \( K \) randomly selects one node as relay from its current one-hop neighbors, and a similar handshake between them is proceeded to indicate whether the selected node, say \( R \), has received one copy of \( P_h \), i.e., the packet for which node \( K \) is distributing copies. If so, \( K \) remains idle for this time slot. Otherwise, \( K \) sends a copy of \( P_h \) to \( R \), and checks whether \( f \) copies have already been delivered out for \( P_h \), if yes, \( K \) moves ahead its local-queue and put \( P_h \) to the end of its already-sent-queue. At the relay node, \( R \) adds \( P_h \) to the end of its relay-queue dedicated to node \( D(K) \).
- (Relay-to-Destination) \( K \) acts as a relay and randomly selects one node as receiver from its one-hop neighbors. The selected receiver, say \( V \), sends its request number \( RN(V) \) to \( K \), and \( K \) checks whether a packet with
SN = RN(V) exists inside its relay-queue destined for V. If yes, K sends it directly to V, and deletes all packets with SN ≤ RN(V) from its relay-queue for V. Otherwise, K remains idle for this time slot.

Notice that in the above two steps, every time a node moves ahead its local-queue by one packet (or receives a packet destined for itself), it increases its send number (or request number) by one.

Remark 1: For any packet, each copy travels at most two hops. Actually only the copy (at some relay) that reaches the destination first will travel two hops, the other f copies will be flushed out from their queue buffers after receiving confirmation from the destination node. Thus, the queue size of the already-sent-queue and n−2 relay-queues will not grow indefinitely. According to the request number sequence, all packets are received in order by their destinations.

Remark 2: It takes at most f+1 transmission opportunities for each packet to reach its destination, and nearly every packet received at its destination will consume exact f +1 transmission opportunities. There are only two scenarios in which a packet will take less than f +1 transmissions, (1) node D(K) receives this packet directly from node K, (2) D(K) first receives this packet from one of its relay nodes and then crashes into K (to notify K to stop delivering out remaining copies). Notice that either of these two scenarios should happen before K finishes the distribution of all f copies, which happens with a vanishingly small probability (as we show later). Thus, in the actual case, as n scales up every packet takes f +1 transmissions to reach its destination with high probability.

B. A Scheduling Scheme

As we assume a cell-partitioned network and a time slotted system, we make the following restrictions on the transmissions during each time slot:

- For each time slot, we allow at most one transmitter inside each cell. If there are more than one nodes inside a cell, then a transmitter is chosen randomly.
- If some node, say K, wins the transmission opportunity in its cell for the current time slot, it can only send packets to the nodes in the same cell or its eight adjacent cells. Two cells are said to be adjacent if they share a common points. Thus, the maximum distance between a transmitter and receiver is \( \sqrt{8/n} \), and we set the communication range as \( r = \sqrt{8/n} \).
- Every time a node gets a transmission opportunity, it follows 2HR-f algorithm.

As the wireless transmissions interfere with each other, only cells that are sufficiently far away could simultaneously transmit without interfering each other. In our scheme we allow a receiver to be selected among any of the eight adjacent cells, so the maximum number of cells that support a transmitting node during every time slot is finite. Toward this end, similar to the "equivalence class" in the [15] we define here the "transmission-group" of cells such that one node in each cell of the transmission-group can transmit simultaneously without interfering with one another.

Transmission-group: A transmission-group is defined as a subset of cells, which keeps a vertical and horizontal distance of exactly some multiple of an integer number of cells away. We denote such integer number by \( \alpha \). The shaded cells in Fig. 1 represent one such transmission-group.

Now we are able to determine the value of \( \alpha \) for our scheduling scheme. Suppose during some time slot, node V is scheduled to receive a packet. Then, according to the definition of "transmission-group", the closest another simultaneous transmitting node (other than V’s transmitter), say K, is at a distance of at least \((\alpha - 2)/\sqrt{n}\) away from V. The condition that K will not interfere the reception at V is that,

\[(\alpha - 2)/\sqrt{n} \geq (1 + \Delta) \cdot r\]

substituting \( r = \sqrt{8/n} \), we obtain that

\[\alpha \geq (1 + \Delta)\sqrt{8} + 2\]

As \( \alpha \) is an integer, we take \( \alpha = \lceil(1 + \Delta)\sqrt{8}\rceil + 2 \), where \( \lceil x \rceil \) returns the smallest integer not smaller than \( x \).

Notice that there are only a finite number of transmission-groups, i.e., \( \alpha^2 \), and each cell belongs to an individual transmission-group. If all transmission-groups become active (have transmission opportunity) alternatively, then each transmission-group will be active in every \( \alpha^2 \) time slots. Therefore, each cell is activated in every \( \alpha^2 \) time slots.

C. Probability of Medium Contention

We show here explicitly the probability of contending for a transmitting or receiving opportunity.

Lemma 1: For any time slot and an active cell in the slot, as \( n \) approaches infinity, its contention probability for transmitting opportunity approaches \( 1 - 2e^{-1} \), while its contention probability for receiving opportunity approaches \( 1 - e^{-1} - \frac{10}{2} e^{-9} \).

Proof: The proof is omitted here due to space limit. Please refer to [16] for details.
IV. EXPECTED END-TO-END DELAY AND THROUGHPUT CAPACITY

Before presenting our main results, we first establish the following lemma.

**Lemma 2:** For a time slot and a given node $K$ in network adopting the 2HR-$f$ algorithm and our scheduling scheme introduced in Section III, we denote by $p_1$, $p_2$ and $p_3$ the probability that $K$ conducts a packet transmission, the probability that $K$ conducts a source-to-destination transmission and the probability that $K$ conducts a source-to-relay or relay-to-destination transmission, respectively. If we assume the local-queue of $K$ always has waiting packets, then we have

$$p_1 = \frac{1}{\alpha^2} \left( 1 - \left(1 - \frac{1}{n} \right)^n - \left(1 - \frac{9}{n} \right)^{n-1} \right) \quad (1)$$

$$p_2 = \frac{1}{\alpha^2} \left( \frac{8}{n - 1} - \left(1 - \frac{1}{n} \right)^{n-2} \left( \frac{7}{n} + \frac{1}{n^2} \right) \right) \quad (2)$$

$$p_3 = \frac{1}{\alpha^2} \left( \frac{n - 9}{n - 1} - \left(1 - \frac{1}{n} \right)^{n-2} - \left(1 - \frac{9}{n} \right)^{n-1} \right) \quad (3)$$

**Remark 3:** One can easily prove that $p_1 = p_2 + p_3$, and $p_2$ quickly approaches zero as $n$ scales up.

**Lemma 3:** For a time slot and a node $K$ with head packet $P_h$ in its local-queue. Suppose that there are already $m$ ($m \leq f + 1$) copies of $P_h$ in the network at the current time slot and $SN(P_h) = RN(D(K))$. We use $p_o(m)$ and $p_c(m)$ to denote the probability that the node $D(K)$ will receive $P_h$ and the probability that $K$ will successfully deliver out a new copy of $P_h$ (if $m \leq f$) in the next time slot, respectively. Then we have

$$p_o(m) = p_2 + \frac{m - 1}{2(n - 2)} \cdot p_3 \quad (4)$$

$$p_c(m) = \frac{n - m - 1}{2(n - 2)} \cdot p_3 \quad (5)$$

The proofs of Lemma 2 and 3 are omitted here due to space limit. Please refer to [16] for details.

Based on the above basic probabilities $p_o(m)$ and $p_c(m)$, we are ready to derive the overall expected end-to-end delay and achievable throughput per node. Notice that a packet may take at most $f + 1$ transmissions to travel from its source node to the destination, we use $p_d(m)$ to denote the probability that it takes $m$ transmissions to reach its destination, $m = 1, ..., f + 1$. From Remark 2 we can easily see that $\sum_{i=1}^{f+1} p_d(m)$ is much less than $p_d(f + 1)$. Thus, to simplify the analysis, we make the following assumption:

**Assumption 1:** We assume that for each packet waiting in the local-queue, exact $f$ copies of it are delivered to distinct relay nodes before its destination receives the packet.

Obviously, the mean end-to-end delay derived under this assumption serves as an upper bound for the actual expected end-to-end delay of the 2HR-$f$ relay, as shown in the following theorem.

**Theorem 1:** For a network with the 2HR-$f$ relay, let $S_1$ denote the time a node takes to deliver out f copies of the head-of-line packet at its local-queue, $S_2$ denote the time its destination node takes to receive one of the $f + 1$ copies, $E\{T_e\}$ denote the actual expected end-to-end delay per packet, and $\mu$ denote the per-node throughput (i.e., the network can stably support any rate $\lambda < \mu$). Then we have

$$\mu = \min \left\{ \frac{1}{E\{S_1\}}, \frac{1}{E\{S_2\}} \right\} \quad (6)$$

$$E\{T_e\} \leq \frac{E\{S_1\}}{1 - \rho_1} + \frac{E\{S_2\}}{1 - \rho_2} - \frac{\rho_2}{2(1 - \rho_2)} \quad (7)$$

$$E\{S_1\} = \frac{2(n - 2)}{p_3} \sum_{m=1}^{f} \frac{1}{n - m - 1} \quad (8)$$

$$E\{S_2\} = \frac{1}{p_2 + \frac{2}{n(n-2)} \cdot p_3} \quad (9)$$

where $\rho_1 = \lambda E\{S_1\}$ and $\rho_2 = \lambda E\{S_2\}$.

**Proof:** Notice that the expected value $E\{S_1\}$ of $S_1$ is given by

$$E\{S_1\} = \sum_{m=1}^{f} \frac{1}{p_o(m)} \quad (10)$$

After substituting (5) into (10), the (8) follows. Similarly, the expected value $E\{S_2\}$ of $S_2$ is determined as

$$E\{S_2\} = \frac{1}{p_o(f + 1)} \quad (11)$$

After substituting (4) into (11), the (9) follows.

The proof of (7) is similar to the derivation of the standard Pollaczek-Khinchin formula for mean waiting time in an $M/G/1$ queue. We consider some tagged packet arriving to the local-queue of node $K$ at the beginning of a time slot, and use $W_1$ to denote the waiting time of this packet in the local-queue before getting service (i.e., to be replicated and delivered to $f$ distinct relays), hence we have

$$W_1 = \sum_{i=1}^{L_q} X_i + R \quad (12)$$

where variable $R$ is the residual service time, $L_q$ is the number of packets waiting in the queue, and $X_i$ is the service time of the $i$th packet. The service times $\{X_i\}$ are independent, and their expected values are upper bounded by $E\{S_1\}$. Let $\rho$ represent the real probability that the node is busy with delivering copies of some packet, and $E\{X\}$ represent the actual mean time the node takes to serve a generic packet, so we have $R = \rho_t E\{X\}$ and $\rho = \lambda E\{X\}$. Thus, $\rho \leq \rho_1$ and $R \leq \rho E\{S_1\}$. Taking expectations of the both sides of (12) yields

$$E\{W_1\} \leq E\{L_q\} E\{S_1\} + \rho_t E\{S_1\} \quad (13)$$

We then have

$$E\{W_1\} \leq \frac{\rho E\{S_1\}}{1 - \rho_1} \quad (14)$$

where $\lambda < 1/E\{S_1\}$ and $\rho_1 = \lambda E\{S_1\}$.
Notice that based on the Assumption 1, the whole network can be regarded as two single-server queues in tandem, where a packet enters the second queue as soon as its \( f \) copies have been delivered out (i.e., as soon as this packet finishes its service in the first queue). As the time required to deliver out the \((m + 1)_{th}\) copy of this packet follows a geometric distribution with mean \( 1/p_s\), the service time \( S_1 \) of the first queue can be interpreted as the sum of \( f \) mutually independent and identically distributed geometric random variables. According to the theory of departure processes in [17], the input to the second queuing process can be approximated by a Poisson stream with arrival rate \( \lambda \).

Recall that in the 2HR-\( f \) relay algorithm, the destination node receives packets in order according to the \textit{request number}. Thus, after a packet enters the second queuing process, it has to wait until its \textit{send number} equals the \textit{request number} of its destination node (i.e., the destination node has received all the preceding packets). Let \( W_2 \) denote such a waiting time, then from Pollaczek-Khinchin formula we have

\[
E\{W_2\} = \frac{\rho_2 E\{S_2\}}{1 - \rho_2} - \frac{\rho_2}{2(1 - \rho_2)} \tag{15}
\]

where \( \lambda < 1/E\{S_2\} \) and \( \rho_2 = \lambda E\{S_2\} \).

When a new packet reaches the head-of-line at its local-queue, if its \( SN \) equals the \( RN \) of its destination node, then the time required for the packet to reach its destination is at most \( S_1 + S_2 \). Therefore, we have

\[
E\{T_c\} \leq E\{T_1\} + E\{S_1\} + E\{W_2\} + E\{S_2\} \tag{16}
\]

where \( E\{T_c\} \) is the actual expected end-to-end delay per packet under the 2HR-\( f \) relay. Substituting (14) and (15) into (16), the (7) follows.

Remark 4: Theorem 1 provides a closed-form (rather than order sense) result for the achievable throughput per node and the expected end-to-end delay per packet in 2HR-\( f \)-based ad hoc mobile networks. Based on Theorem 1 and some typical settings of \( f \), one can easily derive the corresponding order sense results. For example, by setting \( f = 1 \) one can obtain a \( \Theta(1/n) \) throughput and \( O(n) \) delay; by setting \( f = \sqrt{n} \), one can easily recover the order sense results (\( O(1/\sqrt{n}) \) throughput and \( O(\sqrt{n}) \) delay) reported in Theorem 6 of [9].

Remark 5: One may also notice that when setting \( f = 1 \), Theorem 1 results in a \( O(n) \) delay and a \( \Theta(1/n) \) throughput, which is lower than the throughput result \( \Theta(1) \) reported in [1]. This is due to the rule of “reception in order” employed in 2HR-\( f \). The restriction of receiving packets according to \textit{request number} ensures that all packets arrive at the destination in order, but it wastes the opportunities of receiving “out of order but fresh” packets (i.e., packets with \textit{send number} larger than the current \textit{request number} of destination node). Thus, the benefits of receiving all packets in order come at the price of a reduced throughput per node.

V. CONCLUSION

We considered a 2HR-\( f \) relay algorithm for ad hoc mobile networks under i.i.d. node mobility, which extends the classic 2-hop relay to a general setting, i.e. \( f \)-cast. Closed-form results were developed to characterize the achievable throughput and the expected end-to-end delay per packet in such a 2HR-\( f \) network. With our closed-form results and some typical settings of \( f \), one can easily derive the corresponding order sense results, like the \( \Theta(1/n) \) throughput and \( O(n) \) delay with \( f = 1 \), and the \( O(1/\sqrt{n}) \) throughput and \( O(\sqrt{n}) \) delay with \( f = \sqrt{n} \).

REFERENCES