Quantum Neural Network-Based EEG Filtering for a Brain–Computer Interface

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Abstract—A novel neural information processing architecture inspired by quantum mechanics and incorporating the well-known Schrodinger wave equation is proposed in this paper. The proposed architecture referred to as recurrent quantum neural network (RQNN) can characterize a nonstationary stochastic signal as time-varying wave packets. A robust unsupervised learning algorithm enables the RQNN to effectively capture the statistical behavior of the input signal and facilitates the estimation of signal embedded in noise with unknown characteristics. The results from a number of benchmark tests show that simple signals such as dc, staircase dc, and sinusoidal signals embedded within high noise can be accurately filtered and particle swarm optimization can be employed to select model parameters. The RQNN filtering procedure is applied in a two-class motor imagery-based brain–computer interface where the objective was to filter electroencephalogram (EEG) signals before feature extraction and classification to increase signal separability. A two-step inner–outer fivefold cross-validation approach is utilized to select the algorithm parameters subject-specifically for nine subjects. It is shown that the subject-specific RQNN EEG filtering significantly improves brain–computer interface performance compared to using only the raw EEG or Savitzky–Golay filtered EEG across multiple sessions.

Index Terms—Brain–computer interface (BCI), electroencephalogram (EEG), recurrent quantum neural network (RQNN).

I. INTRODUCTION

Brain–Computer Interface (BCI) technology is a means of communication that allows individuals with severe movement disability to communicate with external assistive devices using the electroencephalogram (EEG) or other brain signals. In motor imagery (MI)-based BCIs, the subject performs a mental imagination of specific movements. This MI is translated into a control signal by classifying the specific EEG pattern that is characteristic of the subject’s imagined task, e.g., movement of hands and/or foot. These raw EEG signals have a very low signal-to-noise (SNR) ratio because of the interference from the electrical power line, motion artifacts, electromyogram (EMG)/electrooculogram interference. Preprocessing is carried out to remove such unwanted components embedded within the EEG signal and good preprocessing results in increase in signal quality resulting in better feature separability and classification performance. Very recently, integrated with feature extraction stage, novel spatial filtering algorithms based on Kullback–Leibler [1] common spatial pattern (CSP) [2] and Bayesian learning have been investigated to account for very low SNR EEG [3], [4]. The KLCSP-based approach is investigated on several EEG data sets in [3] and showed significant performance improvement compared to CSP and stationary CSP. Similarly, [4] reports an extensive study of Bayesian learning-based spatial filtering approach and its application using publicly available EEG data. Neural networks and self-organizing fuzzy neural network have also been applied to increase signal separability in motor imagery BCIs [5]–[7]. This paper focuses on EEG signal preprocessing utilizing the concepts of quantum mechanics (QM) and neural network theory in a framework referred to as recurrent quantum neural network (RQNN).

EEG signals can be considered a realization of a random or stochastic process [8]. When an accurate description of the system is unavailable, a stochastic filter can be designed on the basis of probabilistic measures. Bucy in [9] states that every solution to a stochastic filtering problem involves the computation of a time-varying probability density function (pdf) on the state-space of the observed system. The architecture of RQNN model is based on the principles of QM with the Schrodinger wave equation (SWE) [10] playing a major part. This approach enables the online estimation of a time-varying pdf that allows estimating and removing the noise from the raw EEG signal.

In quantum terminology, the state is represented by \( \psi \) (a vector in the Hilbert space \( \mathcal{H} \)) and referred to as a wave function or a probability amplitude function. The time evolution of this state vector \( \psi \) is according to SWE and is represented as

\[
i\hbar \frac{\partial \psi(x,t)}{\partial t} = H \psi(x,t).
\]

(1)

Here \( H \) is the Hamiltonian or the energy operator and is given as \( i\hbar (\partial / \partial t) \) where \( 2\pi \hbar \) (i.e., \( \hbar \)) is the Plank’s constant\(^1\) [11]. Here is the wave function

\( \text{1 The Planck’s constant is an atomic-scale constant that denotes the size of the quanta in quantum mechanics. The atomic units are a scale of measurement in which the units of energy and time are defined so that the value of the reduced Planck constant is exactly one.} \)
The remainder of this paper is organized into nine sections. Section II describes the theoretical concepts of RQNN model. Section III describes the RQNN signal filtering approach. Sections IV and V discuss the datasets and the methodology for EEG filtering with the RQNN model respectively. Section VI details the feature extraction (FE) and classification methodology utilized in this paper. The parameter selection approach for the subject-specific RQNN model is discussed in Section VII. Section VIII discusses the Savitzky–Golay filtering methodology utilized for comparative analysis. The results are presented and discussed in Section IX. Section X concludes this paper.

II. Conceptual RQNN Framework

QM theory is extremely successful in describing the process we see in nature [22]. Dawes in [23] and [24] proposed a novel model—a parametric avalanche stochastic filter using the concept of time-varying pdf proposed by Bucy in [9]. This paper was improved by Behera et al. [13], [14], [25] using maximum likelihood estimation (MLE) instead of inverse filter in the feedback loop. Further, Ivancevic in [18] provided an analytical analysis of nonlinear Schrodinger equation and used the closed-form solution for the concerned application. Because the RQNN approach does not make any assumption about the nature and shape of the noise that is embedded in the signal to be filtered, this approach is most suitable for those signals where the characteristics of the embedded noise is not known. EEG signals are one of these types of signals where the characteristics of the embedded noise is not known and hence this paper presented here on EEG signal filtering is strongly inspired by these works.

A conceptual framework of RQNN model is shown in Fig. 1. It is basically a 1-D array of neurons whose receptive fields are initially excited by the signal input reaching each neuron through the synaptic connections. The neuronal lattice responds to the stimulus by actuating a feedback signal back to the input. The time evolution of this average behavior is described by SWE [10]

\[
i \hbar \frac{\partial \psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(x, t) + V(x, t) \psi(x, t)
\]

where \(\psi(x, t)\) represents the quantum state, \(V\) is the Laplacian operator and \(V(x, t)\) is the potential energy.

The neuronal lattice sets up the spatial potential energy \(V(x)\). A quantum process described by the quantum state \(\psi\) which mediates the collective response of neuronal lattice, evolves in this spatial potential \(V(x)\) according to (2). As \(V(x)\) sets up the evolution path of the wave function, any desired response can be obtained by properly modulating the potential energy.

Such RQNN filter used for stochastic filtering is discussed in [13], [14], and [25]. Although this filter is able to reduce noise, because of its stability being highly sensitive to model parameters, in case of imperfect tuning, the system may fail to track the signal and its output may saturate to absurd values.

In the architecture used in this paper (Fig. 2), the spatial neurons are excited by the input signal \(y(t)\). The difference between the output of spatial neuronal network and the pdf associated with the quantum object at spacetime point \((x, t)\).
feedback $|\psi(x,t)|^2$ is weighted by a weight vector $W(x)$ to get the potential energy $V(x)$. The model can thus be seen as a Gaussian mixture model estimator of potential energy with fixed centers and variances, and only the weights are variable. These weights can be trained using any learning rule.

The parameters of RQNN model have been selected using a two-step inner–outer fivefold cross-validation technique for filtering EEG data sets and using PSO technique for simple signals used to validate the method. There are several filtering EEG data sets and using PSO technique for simulating EEG can be time-consuming. In [19], the parameters were heuristically selected and kept the same for all the subjects. This leads to underfiltering or overfiltering for a few subjects without making the system unstable, but for optimal performance, the EEG signal preprocessing should preferably be carried out with subject-specific choice of parameters.

III. RQNN SIGNAL FILTERING

This section describes the RQNN architecture (see Fig. 2). In RQNN, we make the assumption that the average behavior of neural lattice that estimates the signal is a time-varying pdf which is mediated by a quantum object placed in the potential field $V(x)$ and modulated by the input signal so as to transfer the information about pdf. We use SWE to recurrently track this pdf because it is a well-known fact that the square of the modulus of $\psi$ function, the solution of the wave equation (2), is also a pdf. The potential energy is calculated as

$$V(x) = \zeta W(x, t) \phi(x, t)$$  \hspace{1cm} (3)

where

$$\phi(x, t) = e^{-\frac{(x-x_0)^2}{2\sigma^2}} - |\psi(x, t)|^2$$  \hspace{1cm} (4)

where $y(t)$ is the input signal and the synapses are represented by the time-varying synaptic weights $W(x, t)$. The variable $\zeta$ represents the scaling factor to actuate the spatial potential energy $V(x, t)$, and $\sigma$ is the width of the neurons in the lattice (taken here as unity). This potential energy modulates the nonlinear SWE described by (1). The filtered estimate is calculated using MLE as

$$\hat{y}(t) = \text{E}(|\psi(x, t)|^2) = \int x |\psi(x, t)|^2 dx$$  \hspace{1cm} (5)

where $x$ represents the different possible values which may be taken up by the random process $y$. The variable $x$ can be interpreted as the discrete version of quantum space with the resolution within this discrete space being referred to as $\delta x$ (taken as 0.1 in this paper). Thus all the possible values of $x$ will construct the number of spatial neurons $N$ for RQNN model.

On the basis of MLE, the weights are updated and a new potential $V(x, t)$ is established for the next time evolution. It is expected that the synaptic weights $W(x, t)$ evolve in such a manner so as to drive the $\psi$ function to carry the exact information of pdf of the filtered signal $\hat{y}(t)$. To achieve this goal, the weights are updated using the following learning rule:

$$\frac{\delta W(x, t)}{\delta t} = -\beta_d W(x, t) + \beta \phi(x, t)(1 + \nu(t)^2)$$  \hspace{1cm} (6)

where $\beta$ is the learning rate, and $\beta_d$ is the delearning rate. Delearning is used to forget the previous information, as the input signal is not stationary, rather quasistationary in nature. The second right-hand side term in the above equation maybe purely positive and so in the absence of delearning term, the value of synaptic weights $W$ may keep growing indefinitely. Delearning thus prevents unbounded increase in the values of the synaptic weights $W$ and does not let the system become unstable. The variable $\nu(t)$ in the second term is the difference between the noisy input signal and the estimated filtered signal, thereby representing the embedded noise as

$$\nu(t) = y(t) - \hat{y}(t).$$  \hspace{1cm} (7)

If the statistical mean of the noise is zero, then this error correcting signal $\nu(t)$ has less impact on weights, and it is the actual signal content in input $y(t)$ that influences the movement of wave packet along the desired direction which results in helping the goal of achieving signal filtering.

A. Numerical Implementation

The space variable $x$ is defined uniformly spaced as $x_n = n\delta x, n = -(N/2), \ldots, + (N/2)$ and the time is spaced as $t_k = k\delta t, k = 1, \ldots, T$. The potential function is approximated as $V(x_n, t_k) = V_n^k$. This potential function excites the nonlinear SWE to obtain the quantum wave function $\psi_n^k$. Various methods, both explicit as well as implicit, have been developed for solving nonlinear SWE numerically, on a finite dimensional subspace [26]. The first approach uses Crank–Nicholson method [27] which is an implicit scheme for solving nonlinear SWE and requires a quasitrdiagonal system of equations to be solved at each step [28]. This scheme, although accurate, requires solving the inverse of a huge $N \times N$ matrix, which is time-consuming. Hence the implementation of the same was carried out using the explicit scheme

$$\frac{\psi_n^{k+1} - \psi_n^k}{\delta t} = -\frac{\psi_n^{k+1} - 2\psi_n^k + \psi_n^{k-1}}{2m\delta x^2} + V_n^k \psi_n^k.$$  \hspace{1cm} (8)

This method is linearly stable for $\delta t/(\delta x)^2 \leq 1/4$, with a truncation error of the order of ($O(\delta t^2) + O(\delta x^2)$). Another point to note is that we need to maintain the normalized character of pdf envelope, $|\psi|^2$, by normalizing at every step, i.e., $\Sigma_{n=1}^N |\psi_n^k|^2 dx$ for all $k$. 

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Fig. 2. Signal estimation using RQNN model.
IV. DATA SETS

The EEG data used in this analysis is data set 2b provided in the BCI competition IV [29] with each subject contributing a single session referred to as *03T for the training phase and two sessions referred to as *04E, *05E for the evaluation phase. The data set is obtained using acue-based paradigm which consists of two classes, namely MI of left hand (class 1) and right hand (class 2). Three EEG channels (C3, Cz, and C4) were recorded in bipolar mode with a sampling frequency of 250 Hz and were bandpass-filtered between 0.5 Hz and 100 Hz, and a notch filter at 50 Hz was enabled. However, for investigation, only two channels C3 and C4 are utilized. As shown in Fig. 3, the trial paradigm started at 0 s with a gray smiley centered on the screen. At 2 s, a short warning beep (1 kHz, 70 ms) was given. The cue was presented from 3 to 7.5 s and the subjects were accordingly required to perform the specific imagination. At 7.5 s, the screen went blank and a random interval between 1.0 and 2.0 s was added to the trial so as to avoid user adaptation. More details of this EEG signal recording methodology are available in [29].

V. EEG FILTERING WITH RQNN

Fig. 4 shows the position of RQNN model within the BCI system. The raw EEG signal is fed one sample at a time and an enhanced signal is obtained as a result of filtering process. The raw EEG is first scaled in the range 0–2 before it is fed to the RQNN model. During the off-line classifier training process, all the trials from a particular channel of EEG are available. Therefore, the complete EEG is scaled using the maximum of amplitude value from that specific channel. During the online process, the EEG signal is approximately scaled in the range 0–2 using the maximum of amplitude value obtained from the off-line training data of that specific channel. The net effect is that the input signal during the online process is also maintained approximately in the region 0–2, and this enables the tracking of sample using a reduced range of the movement of wave packet. In addition, the number of spatial neurons has also been reduced along the x-axis from an earlier value of 401 to 612 in the present case. The primary assumption in doing this is that the unknown nonstationary and evolving EEG signal during the evaluation stage will stay within the bound of the range of 61 spatial neurons which can cover the input signal range up to three. If the scaling of the input signal is not implemented, then the number of neurons required to cover the input signal range will be larger thereby leading to an increased computational expense. This is an important modification in [19] and the scaling of EEG is now dictated as per the training data set. During the off-line training process, the complete set of scaled EEG signal (here signals from channels C3 and C4 discussed in Section VI) is fed through the two RQNNs, respectively (see Fig. 4), and a filtered estimate of the signal is obtained for the samples from both these channels.

VI. FEATURE EXTRACTION AND CLASSIFICATION

The next task is to obtain the features from this RQNN-enhanced EEG signal which in the present case are the Hjorth [30] and band power features. These combined features are then fed as an input to train the off-line classifier which in this case is the linear discriminant analysis (LDA) classifier. Once the off-line analysis is complete and the classifier is trained, the parameters and weight vector are stored for use with the classifier to identify the unlabeled EEG data during the online analysis. It needs to be clarified here that to capture the dynamic property of the continuous EEG signal, the weight updation process of RQNN filter is continuous (to enhance the EEG signal) during both the off-line and online stages while the classifier parameters are tuned off-line and then kept fixed for the online classification process.

Various FE approaches such as RQNN-generated features, band power, Hjorth, power spectral density (PSD), bispectrum (BSP), time frequency (t−f) features have been utilized by various research groups [15], [16], [31]–[35] to produce a good practical BCI system. Most of the BCI research in signal processing is focused on frequency domain. The band power FE method is based on calculating the squared amplitude of signal over a small window. This approach typically includes two frequency bands. The μ band (8–13 Hz) and the β band (14–24 Hz) for the purpose of FE, although the range of these frequency bands may vary from one subject to the other. The μ and β bands are important as they are more reactive during a cued motor imagery [8], [36]. There is a much larger difference in band power changes [event-related desynchronization (ERD), event-related synchronization (ERS)] within these bands and help differentiate between hand versus foot MI or right versus left hand MI. In addition, it is also possible to convey relevant information about the EEG epochs with the trio of combinations of conventional time-domain-based descriptive statistics Hjorth parameters, namely

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2If the range of the neuronal lattice is $-2$ to $+2$, then with a spacing of 0.1 between each neuron, the total number of neurons covering the range will be $-2, -1.9, -1.8, \ldots, -0.1, 0, +0.1, \ldots, +1.9, 2$ i.e., 41. However, to incorporate the behavior of signal during the unknown evaluation stage, the range has been extended to cover the range up to $+3$ using 61 neurons.
activity, mobility, and complexity [37]. The computational cost in the calculation of Hjorth parameters is considered low as this approach is based on variance [31]. Moreover, Hjorth parameter, especially complexity, is sensitive to noise because their computation is based on numerical differences and their variances [38]. This prompted the authors to evaluate RQNN preprocessing technique by utilizing a combination of Hjorth and band power features.

VII. RQNN PARAMETER SELECTION

This section discusses the possible ways of selecting RQNN parameters to suit an individual subject. Four parameters in the RQNN model have been kept fixed and are explained in Table I. These are obtained heuristically, but after suitable trial and experimentation over a small set of EEG data. The variable parameters are selected from the search space as explained in Table II through the two-step inner–outer fivefold cross-validation method shown in Fig. 5. The first step is to vary the RQNN parameters within the search space shown in Table II and measure the overall performance of the classifier through an inner–outer cross-validation technique with a limited number of trials using the Hjorth and band power features over the standard frequency bands of 8–13 Hz and 14–24 Hz. In this first step, the training data set of EEG is separated into five outer folds. Of these, the raw EEG is filtered using the RQNN on four folds using a specific set of parameters over the event-related MI period 3–7 s. Once the RQNN-enhanced signal is obtained, FE is performed. This feature set is now further divided into five inner folds. A normal fivefold cross-validation (CV) is performed over the complete set of EEG training data. This stage thus gives one best RQNN parameter and frequency band combination and the optimum time-point for performing the classification as per the highest kappa value for each subject. Once these steps are complete, the classifier is chosen at the best time-point so that it can be applied on the unknown evaluation data sets.

Another common approach to handle parameter tuning/selecting issue is to utilize optimization techniques such as PSO or genetic algorithm (GA). However, the RQNN model has several parameters that should be varied in agreement with the frequency bands at the FE stage for EEG classification to suit an individual subject. Applying any optimization technique within a large multidimensional search space would be time-consuming. Therefore, PSO has been applied to select the

3Kappa is a measure of agreement between two estimators and since it considers chance agreement, it is regarded as a more robust measure in comparison to accuracy [58].

3Optimum time-point is an estimate of a point in time within the trial duration of 8 s that produces features with maximum separation that allows for classification with the lowest error.
RQNN parameters for filtering simple example signals while a two-step parameter selection approach has been applied for filtering EEG.

VIII. SAVITZKY–GOLAY FILTER

The performance of RQNN has been compared with the unfiltered EEG as well as with the well-established SG technique [39]. The SG technique has been utilized as a noise removal approach (in a way it is thus similar to the RQNN) in biological signals such as ECG [40] and the EEG [41], [42]. SG filtering can smoothen the signal without destroying the original properties of signal. Hence, the SG approach has been utilized here to compare it with the RQNN model. The RQNN block shown in the EEG framework of Fig. 4 is simply replaced with the SG block.

IX. RESULTS AND DISCUSSION

A. Simple Example Signals

To validate the RQNN technique for filtering the complex EEG signals, we apply it to filter simple example signals in the form of dc, staircase dc, and sinusoidal signals that have been embedded with a known amount of noise. The dc signal of amplitude 2 is embedded with 0 dB noise (i.e., SNR is 1), the staircase dc with amplitude varying from 0 to 2 is embedded with 20 dB noise and the sinusoidal signal of amplitude 3 is embedded with 6 dB noise. The parameters of RQNN model to filter the input dc signal are $\beta = 0.002$, $m = 0.5$, $\zeta = 775.05$, and $N = 400$ while each sample is iterated once, so as to stabilize SWE (Table I). The parameters $\beta$ and $\zeta$ were obtained using the PSO technique [20], [21] by fixing the parameter $m$ at 0.5. The parameters to filter the sinusoidal signal were obtained as $\beta = 5.25$, $m = 0.25$, and $\zeta = 1.75$ and $N = 140$ and each sample was iterated for 60 times before the next sample was fed. The delearning parameter $\beta_d$ has been kept at all places as one. Fig. 6 shows the filtering of these signals using the RQNN approach. A video showing the movement of the wave packet for dc filtering is available at [43]. The root-mean-square error in filtering the dc signal with the proposed RQNN as well as with the Kalman filter [44] is shown in Table III (partially reproduced from [14]) and demonstrates that the RQNN performs better. It can thus be firmly stated from the plots and the figures that the RQNN is able to effectively capture the statistical behavior of the input signal and appropriately track the true signal even when fed with a highly noisy input signal.

It is worth highlighting here that the statistical behavior of noise and signal in terms of pdf is a priori assumed in case of Kalman filter and its variants. However, the proposed RQNN
TABLE III
PERFORMANCE COMPARISON FOR dc SIGNAL OF AMPLITUDE 2

<table>
<thead>
<tr>
<th>SNR (signal-to-noise ratio)</th>
<th>Kalman filter RMSE</th>
<th>Proposed RQNN RMSE</th>
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<td>20 dB</td>
<td>0.015</td>
<td>0.0004228</td>
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<tr>
<td>6 dB</td>
<td>0.037</td>
<td>0.0012071</td>
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<tr>
<td>0 dB</td>
<td>0.090</td>
<td>0.0017837</td>
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</table>

Fig. 7. Representative plot of RQNN filtered and raw EEG.

Fig. 8. Snapshots of the wave packets and MLE that generate the representative plot of the RQNN filtered EEG as shown in Fig. 7.

directly estimates this probability density function without making any such assumption. Thus the proposed model can enhance the EEG signal much better as the noise pdf is naturally non-Gaussian.

B. EEG-Based BCI

1) Signal Wave Packets: Fig. 8 displays the tracking of EEG signal in the form of snapshots of wave packets. The movement of the wave packet along the x-axis is shown at time instants $t = 5.0$ s, $t = 5.2$ s, $t = 5.6$ s, and $t = 6.0$ s. MLE from the wave packet gives the filtered EEG as shown in Fig. 7. This figure displays the representative plot of the raw EEG and the RQNN-enhanced EEG for a time interval between 5 and 6 s. The effect of filtering can be ascertained through ERD/ERS in the frequency domain as well as through an overall performance enhancement of the classifier outcome.

2) ERD/ERS: Fig. 9 shows a representative ERS obtained with the RQNN-filtered EEG signal and the raw EEG signal for subject four (evaluation set 5 E). The ERD/ERS were obtained for all channels by averaging band power change at each time-point across the time interval 4000–6000 ms (standard activity period) with respect to the reference period from 500 to 1500 ms for all the subjects. The improvements in ERD/ERS with the RQNN-filtered signals for both the evaluation data sets is statistically significant ($p < 0.04$) and enhances the overall BCI performance.

3) Performance Enhancement (CA/Kappa): The list of subject-specific parameters for the RQNN model obtained using the inner–outer fivefold CV (Section VII) is shown in Table IV. Fig. 10 displays the CA plot using the LDA
at the evaluation stage to compare the performance using different methods. From the results displayed in Table VII, specifically observing the performance of subject B03, there seems to be a huge difference in the maximum of kappa values obtained with BSP (0.29)/PSD (0.27) compared to that with the raw (0.84) and the RQNN (0.89) approaches. This may be because, the BSP and PSD techniques are frequency-based, while the raw and the RQNN techniques in this paper have used a combination of frequency (band power) and temporal-based (Hjorth) features. To substantiate this, we implemented the inner–outer fivefold cross-validation using only the band power features for both the raw and the RQNN. The resulting average performance for evaluation stages in terms of CA (and maximum of kappa values) for subject B03 was 61.9 (0.25) and 58.12 (0.16), respectively, with the RQNN and the raw approaches. Thus it may be stated here that the RQNN filtering enhances the performance of BCI when compared to the raw EEG, but the increase in performance when compared to BSP and PSD may also be attributed to the use of a combination of frequency and temporal features. It can therefore be concluded from these results that the RQNN improves the average performance of BCI system for almost all the subjects during both the training and the evaluation stages when compared to the unfiltered EEG, SG-filtered EEG, and even PSD and BSP features-based approaches. The same data sets were also processed and classified by several renowned researchers as competitors of BCI Competition IV 2b-data set [45] which is also discussed in [35]. The performance of RQNN (Table VII) is also significantly better than the ones obtained by the winners of BCI competition in [45]. The competition winner used the filter bank CSP technique for FE along with the Naive Bayes Parzen window classifier. The runner-up group used common spatial subspace decomposition technique for FE followed by LDA classifier. The third group used a CSP followed by log-variance techniques for FE and the best (at training stages) of LDA and SVM classifier. The fourth group used wavelet technique followed by an LDA classifier and it used spectral features before a neural network classifier. The sixth group estimates 75 band power features with their cursive feature elimination technique with a neural network classifier. The second group used the filter bank CSP technique for FE along with a Bayesian LDA classifier. Some of the competitors of Competition IV used only session 3 for training, while some used combined sessions from the three training sessions (combining 1, 3, or 1, 2, or 1, 2, 3) differently for different subjects and evaluated on session 4 and 5 [46]–[51]. In this paper, only session 3 is used for training, while the sessions 4 and 5 are used for evaluation. The results thus show that without prior knowledge of the type of noise characteristics present in EEG, the RQNN can be utilized to enhance EEG signal separability and that the quantum approach-based filtering method can be used as a signal preprocessing method for BCI.

4) Online Real-Time Implementation: The proposed RQNN methodology has also been utilized in online EEG filtering for real-time MI-based robot control task using an intelligent adaptive user interface as shown in the videos at. [52]. A very

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**TABLE V**

<table>
<thead>
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<th>Subject</th>
<th>Training (03T)</th>
<th>Evaluation (04E)</th>
<th>Evaluation (05E)</th>
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<td>RQNN (SG)</td>
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**TABLE VI**

<table>
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<th>Subject</th>
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<tr>
<td>B03</td>
<td>1.00 0.94 0.99</td>
<td>0.99 0.94 0.98</td>
<td>0.93 0.81 0.73</td>
</tr>
<tr>
<td>B04</td>
<td>0.85 0.61 0.77</td>
<td>0.78 0.53 0.60</td>
<td>0.88 0.53 0.60</td>
</tr>
<tr>
<td>B05</td>
<td>0.71 0.56 0.55</td>
<td>0.63 0.51 0.54</td>
<td>0.78 0.51 0.60</td>
</tr>
<tr>
<td>B06</td>
<td>0.85 0.81 0.78</td>
<td>0.48 0.51 0.41</td>
<td>0.43 0.38 0.33</td>
</tr>
<tr>
<td>B07</td>
<td>0.95 0.75 0.80</td>
<td>0.73 0.71 0.73</td>
<td>0.94 0.83 0.94</td>
</tr>
<tr>
<td>B08</td>
<td>0.75 0.65 0.74</td>
<td>0.74 0.49 0.59</td>
<td>0.76 0.50 0.58</td>
</tr>
<tr>
<td>Avg.</td>
<td>0.82 0.69 0.73</td>
<td>0.63 0.52 0.58</td>
<td>0.71 0.53 0.59</td>
</tr>
</tbody>
</table>

---

5 Two-way analysis of variance (ANOVA) test is performed with the results of the training and the evaluation stages for the RQNN filtered and the raw EEG approach.

6 The average maximum of kappa (across nine subjects) obtained by the first six competitors is 0.6, 0.58, 0.46, 0.43, 0.37, and 0.25 respectively.
TABLE VII
EVALUATION STAGE (*4E, *5E) PERFORMANCE COMPARISON

<table>
<thead>
<tr>
<th>Subj</th>
<th>Average Max.</th>
<th>Kappa</th>
<th>Avg. Max.</th>
<th>Kappa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BSP PSD RQNN</td>
<td>SG RAW</td>
<td>BSP PSD RQNN</td>
<td>SG RAW</td>
</tr>
<tr>
<td>B01</td>
<td>0.54 0.41</td>
<td>0.55 0.30</td>
<td>0.51 0.64</td>
<td>0.59 0.31</td>
</tr>
<tr>
<td>B02</td>
<td>0.29 0.17</td>
<td>0.27 0.28</td>
<td>0.23 0.33</td>
<td>0.21 0.28</td>
</tr>
<tr>
<td>B03</td>
<td>0.22 0.18</td>
<td>0.72 0.53</td>
<td>0.27 0.29</td>
<td>0.89 0.61</td>
</tr>
<tr>
<td>B04</td>
<td>0.93 0.67</td>
<td>0.96 0.88</td>
<td>0.96 0.90</td>
<td>0.99 0.94</td>
</tr>
<tr>
<td>B05</td>
<td>0.64 0.57</td>
<td>0.83 0.53</td>
<td>0.60 0.68</td>
<td>0.73 0.88</td>
</tr>
<tr>
<td>B06</td>
<td>0.69 0.57</td>
<td>0.71 0.51</td>
<td>0.73 0.59</td>
<td>0.78 0.61</td>
</tr>
<tr>
<td>B07</td>
<td>0.50 0.37</td>
<td>0.46 0.45</td>
<td>0.37 0.57</td>
<td>0.39 0.48</td>
</tr>
<tr>
<td>B08</td>
<td>0.82 0.82</td>
<td>0.84 0.77</td>
<td>0.84 0.91</td>
<td>0.94 0.83</td>
</tr>
<tr>
<td>B09</td>
<td>0.74 0.61</td>
<td>0.75 0.50</td>
<td>0.59 0.81</td>
<td>0.71 0.76</td>
</tr>
<tr>
<td>Avg.</td>
<td>0.60 0.49</td>
<td>0.67 0.53</td>
<td>0.59 0.66</td>
<td>0.57 0.73</td>
</tr>
</tbody>
</table>

*Partially reproduced from [28]

TABLE VIII
RQNN PERFORMANCE ON BCI COMPETITION IV 2A DATA SET

<table>
<thead>
<tr>
<th>Subj</th>
<th>Accuracy</th>
<th>Max. of Kappa</th>
<th>Raw</th>
<th>RQNN</th>
<th>Raw</th>
<th>RQNN</th>
<th>Raw</th>
<th>RQNN</th>
<th>Raw</th>
<th>RQNN</th>
<th>Raw</th>
<th>RQNN</th>
<th>Raw</th>
<th>RQNN</th>
<th>Raw</th>
<th>RQNN</th>
<th>Raw</th>
<th>RQNN</th>
</tr>
</thead>
<tbody>
<tr>
<td>B01</td>
<td>66.60</td>
<td>66.06</td>
<td>0.33</td>
<td>0.32</td>
<td>61.11</td>
<td>61.11</td>
<td>0.22</td>
<td>0.22</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>B02</td>
<td>67.24</td>
<td>69.48</td>
<td>0.36</td>
<td>0.40</td>
<td>61.11</td>
<td>57.64</td>
<td>0.22</td>
<td>0.15</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>B03</td>
<td>77.78</td>
<td>75.02</td>
<td>0.38</td>
<td>0.49</td>
<td>79.17</td>
<td>77.78</td>
<td>0.58</td>
<td>0.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>B04</td>
<td>65.32</td>
<td>63.89</td>
<td>0.30</td>
<td>0.28</td>
<td>60.42</td>
<td>56.25</td>
<td>0.21</td>
<td>0.13</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>B05</td>
<td>70.79</td>
<td>68.03</td>
<td>0.41</td>
<td>0.37</td>
<td>71.51</td>
<td>66.67</td>
<td>0.43</td>
<td>0.35</td>
<td></td>
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<tr>
<td>B06</td>
<td>66.01</td>
<td>62.54</td>
<td>0.32</td>
<td>0.25</td>
<td>61.11</td>
<td>61.11</td>
<td>0.22</td>
<td>0.22</td>
<td></td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>B07</td>
<td>68.67</td>
<td>68.13</td>
<td>0.37</td>
<td>0.37</td>
<td>58.33</td>
<td>56.25</td>
<td>0.17</td>
<td>0.13</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>B08</td>
<td>67.41</td>
<td>65.86</td>
<td>0.35</td>
<td>0.31</td>
<td>67.36</td>
<td>66.67</td>
<td>0.35</td>
<td>0.33</td>
<td></td>
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<td></td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>B09</td>
<td>78.40</td>
<td>73.55</td>
<td>0.36</td>
<td>0.46</td>
<td>79.17</td>
<td>75.00</td>
<td>0.58</td>
<td>0.50</td>
<td></td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Avg.</td>
<td>69.80</td>
<td>68.06</td>
<td>0.40</td>
<td>0.36</td>
<td>66.59</td>
<td>64.27</td>
<td>0.33</td>
<td>0.29</td>
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</tr>
</tbody>
</table>

5) Investigation on BCI Competition Data Set IV Data Set: The RQNN methodology has also been investigated on the BCI competition IV 2a data set [53] as displayed in Table VIII. This data set consists of one training set and one evaluation set for nine subjects with 22 channels and four different MI tasks, namely the imagination of movement of left hand (class 1), right hand (class 2), both feet (class 3), and tongue (class 4). However, RQNN approach has been carried out, as before, using only the two channels, namely C3 and C4 and only for a two-class classification (left hand versus right hand). Therefore, the data was separated into two classes, EEG with left hand and right hand mental imagination task. The same two-step procedure has been applied (Fig. 5) for the parameter selection. The average performance enhancement obtained is > 2% in CA (p < 0.0027) and 0.04 in maximum of kappa (p < 0.0031) when compared with the raw EEG. More details about the subject-specific parameters for this data set can be availed from [54].

X. Conclusion

The RQNN was evaluated with case studies of simple signals and the results show that the RQNN is significantly better than the Kalman filter while filtering the dc signal added with three different noise levels. The learning architecture and the associated unsupervised learning algorithm of RQNN have been modified to take into account the complex nature of EEG signal. The basic approach is to ensure that the statistical behavior of input signal is properly transferred to the wave packet associated with the response of quantum dynamics of the network. At every computational sampling instant, the EEG signal is encoded as a wave packet which can be interpreted as pdf of the signal at that instant. The subject-specific RQNN parameters have been obtained using a two-step inner–outer fivefold cross-validation which results in an enhanced EEG signal that is used further for FE and classification processes. The CA and kappa values obtained from RQNN-enhanced EEG signal show a significant improvement during both the training and the evaluation stages across multiple sessions. This performance enhancement through the RQNN model is superior when compared to that using the raw EEG, Savitzky– Golay filtered EEG or even raw EEG with the PSD or the BSP-based features. Future work will involve developing automated computational techniques such as GA or PSO for selecting subject-specific RQNN model parameters. Improving other stages of signal processing framework as highlighted in [55] will also increase the online performance of BCI for applications in stroke rehabilitation [56] and games [57] among others.

The noteworthy feature of the proposed scheme is that without introducing any complexity at the FE or the classification stages, the performance of BCI can be significantly improved simply by enhancing the EEG signal at the preprocessing stage.

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REFERENCES


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