A New Perspective of the Proportional Sampling Strategy

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To compare the performance of different testing strategies, P-measure and E-measure are two effectiveness measures used in previous analytical studies. P-measure, which is defined as the probability of detecting at least one failure, is a measure of how likely it is that failure-causing inputs are selected at least once as test cases. E-measure, which is defined as the expected number of failures detected, is a measure of how frequently failure-causing inputs are selected as test cases. However, we have no a priori knowledge of how many failure-causing inputs there are, or where they may lie. In this paper, we study P-measure and E-measure in terms of how much attention an arbitrary input receives. In the context of P-measure, the attention received by an arbitrary input is the probability that the input is selected at least once as a test case, while in the context of E-measure, the attention is the expected number of times that an input is selected as a test case. The attentions received by an input using the proportional sampling strategy and random testing are then compared. The attention received by an input is found to be the same under the two testing strategies for E-measure, whereas for P-measure the attention is always higher for the proportional sampling strategy than for random testing. This new perspective allows us to provide simpler proofs of some known results. Furthermore, we are able to show that the difference in the expected number of distinct test cases considered by the proportional sampling strategy is larger than that of random testing by at most 2k, where k is the number of partitions and is independent of the number of test cases selected.

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1. INTRODUCTION

Exhaustive testing is not usually feasible because input domains are very large. Hence, various test case selection strategies have been developed. Many of these strategies can be classified as subdomain testing, which divides the input domain into subdomains and then selects test cases from these subdomains. The intuition of subdomain testing is to define the subdomains so that each one either gives rise to all correct outputs or all incorrect outputs. Hence, when test cases are selected from every subdomain, program faults should be detected if they exist. However, it is almost impossible to devise subdivision schemes with such an ideal characterisation. Hence, we would like to compare the effectiveness of subdomain testing, and random testing where test cases are simply chosen randomly from the entire input domain.

Most of the investigations on subdomain testing deal with disjoint subdomains only (hereafter referred to as partition testing). The empirical results show that although partition testing usually performs better than random testing, the difference is not very significant [1, 2]. Motivated by these empirical findings, Weyuker and Jeng [3], Chen and Yu [4, 5, 6] and Leung and Chen [7] have conducted analytical investigations of partition testing strategies. Several sufficient conditions for partition testing to perform better than random testing were found.

In this paper, we use the same mathematical model as in previous analytical studies [3, 4, 5, 6]. Test cases are selected independently with replacement based on a uniform distribution over the input domain. Consequently, test cases may be repeated. Sampling with replacement makes the mathematical analysis more tractable and would not produce noticeably different results. This has motivated Leung and Chen [7] to introduce the notion of the expected number of distinct test cases to investigate the properties of random testing and partition testing. They observe that partition testing does not necessarily have more distinct test cases than random testing. However, they do present a sufficient condition for partition testing to ensure that it does.

We now present some definitions and notation to facilitate our discussion. The elements of an input domain are said to be failure-causing inputs if they produce incorrect outputs. For an input domain D, we use d, m and n to denote its size, number of failure-causing inputs and number of test cases respectively. The sampling rate σ and failure rate θ are defined as n/d and m/d respectively.

For any partition scheme P, D is divided into k (k ≥ 2)
disjoint subsets denoted as $D_1, \ldots, D_k$. For any subdomain $D_i$, we use $d_i$, $m_i$ and $n_i$ to denote its size, number of failure-causing inputs and the number of test cases respectively; the corresponding sampling rate $\sigma_i$ and failure rate $\theta_i$ are defined as $n_i/d_i$ and $m_i/d_i$ respectively.

For a fair comparison, we assume that random testing and partition testing select the same number of test cases, that is $n = \sum_{i=1}^{k} n_i$. Furthermore, we assume that $n \leq d$.

A partition testing strategy satisfying $\sigma_1 = \sigma_2 = \ldots = \sigma_k$ is known as the proportional sampling strategy. This strategy has received the most attention because it is the most practically applicable sufficient condition to guarantee partition testing to have at least the same P-measure, E-measure or expected number of distinct test cases as random testing. However, in practice the proportional sampling strategy may not be strictly applied. This has motivated Chan et al. [8] and Chen and Yu [9] to investigate approximation methods for the proportional sampling strategy. For example, assuming that $n$ test cases are to be used, Chen and Yu’s approximation algorithm [9] initially sets $n_1$ equal to $1$ for every $n_1$, and then repeatedly adds $1$ to an $n_j$ that has the smallest $\sigma_j$, until $\sum_{i=1}^{k} n_i = n$. In this paper, following the same assumption as previous analysis, we assume that the proportional sampling strategy can be applied without approximation. That is, for each $1 \leq i \leq k$ we have $n_i/d_i = n/d$, which implies that $n_i = nd_i/d$ is an integer.

All previous analysis uses the probability of detecting at least one failure (hereafter referred to as the P-measure) or the expected number of failures detected (hereafter referred to as the E-measure) as the effectiveness metric. For random testing, the P-measure is $1 - (1 - \theta)^n$, whereas the E-measure is $n\theta$. For partition testing, the P-measure is $1 - \prod_{i=1}^{k} (1 - \theta_i)^{n_i}$, whereas the E-measure is $\sum_{i=1}^{k} n_i\theta_i$.

To compare the performance of different testing strategies, P-measure and E-measure are two effectiveness measures used in previous analytical studies. P-measure, which is defined as the probability of detecting at least one failure, is a measure of how likely it is that failure-causing inputs are selected at least once as test cases. E-measure, which is defined as the expected number of failures detected, is a measure of how frequently failure-causing inputs are selected as test cases. However, we have no a priori knowledge of how many failure-causing inputs there are, or where they may lie. In this paper, we study P-measure and E-measure in terms of how much attention an arbitrary input receives. In the context of P-measure, the attention received by an arbitrary input is the probability that the input is selected at least once as a test case, while in the context of E-measure, the attention is the expected number of times that an input is selected as a test case. Chen and Yu [10] have formulated the E-measure similarly but focusing on the failure-causing inputs only. The attentions received by an input using the proportional sampling strategy and random testing are then compared. The attention received by an input is found to be the same under the two testing strategies for E-measure, whereas for P-measure the attention is always higher for the proportional sampling strategy than for random testing. This new perspective allows us to provide simpler proofs of some known results. Furthermore, it helps obtain new results that are difficult to derive with the previous perspective. For example, when the number of test cases increases, the relative difference in the expected number of distinct test cases between the proportional sampling strategy and random testing decreases.

Section 2 gives the mathematical preliminaries and Section 3 presents the main results of the paper.

2. PRELIMINARIES AND NOTATION

We present some well known results which will be needed in the next section. Let $f(x) = (1 - 1/x)^{a}$ and $g(x) = (1 - 1/x)^{a-1}$. It is well known that $f(x)$ is monotonically increasing for $x \geq 1$ and $\lim_{x \to \infty} f(x) = 1/e$. Therefore, $f(x) < 1/e$ for $x \geq 1$. Following from the fact that the second derivative of $g(x)$ is

\[ \left( \frac{1}{x^2(x-1)} + \left( \frac{1}{x} + \log \left(\frac{1 - 1/x}{x}\right)^2 \right) \right) (1 - 1/x)^{-1} \]

which is positive for $x > 1$, $g(1) = 1$ and $\lim_{x \to \infty} g(x) = \lim_{x \to \infty} f(x)$ $\lim_{x \to \infty} 1/(1 - 1/x) = (1/e)1 = 1/e$, we conclude that $g(x) > 1/e$ for $x \geq 1$.

3. A NEW PERSPECTIVE

In this section, we analyse random testing and the proportional sampling strategy from the perspective of how frequently an arbitrary input is selected as a test case. This perspective provides a better insight into the foundations of random testing and the proportional sampling strategy. In addition to providing some new results, this approach makes the proofs of some known results simpler and more natural. As an example let us provide another proof of a result of Chen and Yu [5] that the proportional sampling strategy and random testing perform equally well with respect to the E-measure. Consider random testing where $n$ test cases are selected from a domain of size $d$. The expected number of times that an arbitrary input is selected, that is the attention that the input receives, is $\sum_{i=1}^{n} (1/d) = n/d$. Next, for the proportional sampling strategy, the expected number of times that an input is selected among the $n_i$ test cases from subdomain $D_i$ of size $d_i$ is $\sum_{i=1}^{n} (1/d_i) = n_i/d_i = \sigma_i = \sigma = n/d$. Therefore, the attentions received by an input under the two testing methods are the same. The E-measure is defined in the same way for both methods as the summation of the attention received by the failure-causing inputs. Therefore we deduce that the two E-measures are the same, no matter how many failure-causing inputs there are or where the failure-causing inputs lie.

A similar treatment can be applied to analyse the two testing strategies under the P-measure. Chen and Yu [4] prove that the proportional sampling strategy always guarantees a higher P-measure than random testing, with the only exception that the two P-measures are the same when all failure rates in the subdomains are equal. With respect to the P-measure, we define the attention that an arbitrary
input receives under a testing method as the probability that the
input is selected at least once among all the random test
cases selected.

Specifically, for random testing the probability that an
arbitrary input \( x \in D \) is not selected among the \( n \) random
choices is \( (1 - 1/d)^n \). Thus, the probability that \( x \) is selected
at least once is \( 1 - (1 - 1/d)^n \). If \( x \) is selected at least
once, then it contributes the value ‘one’ to the expected
number of distinct test cases, otherwise the contribution is
zero. Therefore, the expected number of distinct test cases is
\( \sum_{x \in D} (1 - (1 - 1/d)^n) = d - d(1 - 1/d)^n \). This derivation of
the expected number of distinct test cases for random testing
is much simpler and more natural than that derived by Leung
and Chen [7].

**Lemma 3.1.** Using the proportional sampling strategy,
every input has a higher chance of being selected at least
once as a test case than that for random testing.

**Proof.** The probability that an arbitrary input is selected
at least once as a test case by random testing among
the \( n \) random choices is \( 1 - (1 - 1/d)^n \). On the other
hand, the probability that an arbitrary input in subdomain
\( D_j \) is selected at least once by partition testing using the
proportional sampling strategy is \( 1 - (1 - 1/d_j)^{n_j} = 1 - \left[ (1 - 1/d_j)^{n_j}/d_j \right] \)
\( n_j/d_j > 1 - [(1 - 1/d_j)^{n_j}/d_j] = 1 - (1 - 1/d_j)^n \).

With the above result, since every input under the
proportional sampling strategy has a higher chance of
being selected at least once than in random testing, it
is understandable (though not in the sense of a formal
proof) that the proportional sampling strategy has a higher
probability of detecting at least one failure (P-measure) than
random testing.

**Lemma 3.2.** The proportional sampling strategy selects
the least number of random test cases from each subdomain,
such that the probability that an arbitrary input is
selected as a test case at least once is higher than that for
random testing.

**Proof.** Suppose the proportional sampling strategy is
followed. Now we decrease \( n d_j/d \) (the number of test
cases selected from \( D_j \)) by one. We want to argue that the
probability that an arbitrary input in \( D_j \) is selected at least
once becomes smaller than that of random testing. Since
\( n \leq d \), the probability that an arbitrary input in \( D_j \) is
selected at least once is \( 1 - (1 - 1/d_j)^{n d_j/d - n j/d} \leq
1 - (1 - 1/d_j)^{n d_j/d - n j/d} = 1 - [\left( 1 - 1/d_j \right)^{n d_j/d - n j/d}] <
1 - (1 - 1/d_j)^{n d_j/d} < 1 - (1 - 1/d_j)^{n_j} \). Together with the
result of Lemma 3.1, the proof is completed.

Therefore, using the proportional sampling strategy every
input has a chance of being selected at least once as a test
case close to the chance received by using random testing;
hence, the chances for inputs to be selected as test cases
across the different subdomains are close to each other.

More specifically, the following lemma shows that the
difference in the chances of being selected at least
once between the use of random testing and that of the
proportional sampling strategy in partition testing could be
very small.

**Lemma 3.3.** Let \( d, d_j, n_j \) be integers. Assume that
\( n_j \leq d \) and \( n_j/d_j = n_j/d \). Then \( 1 - (1 - 1/d_j)^{n_j} = 1 -
(1 - 1/d_j)^{n_j} < 2/d_j \).

**Proof.** First consider the special case when \( n_j = d_j \). Then
\( n_j = 1 \). Since \( 1 - 1/d_j = 1/e \) for all positive \( d \), we need to show that \( 1/e - (1 - 1/d_j)^{n_j} < 2/d_j \). Let
\( f(d_j) = 2/d_j + (1 - 1/d_j)^{n_j} \). Note that \( f(1) = 2 + 0 - 0.367897 > 0 \), \( f(2) = 1 + (1/2)^2 - 0.367897 > 0 \) and \( \lim_{d_j \to \infty} f(d_j) = 0 + 1/e - 1/e = 0 \). It suffices to show
\( f'(d_j) < 0 \) for all real numbers \( d_j \geq 2 \). Observe that

\[
f'(d_j) = -2/d_j^2 + (1/(d_j - 1) + \log_e(1 - 1/d_j)) \times
(1 - 1/d_j)^{d_j} \leq -2/d_j^2 + (1/e)(1/(d_j - 1) + \log_e(1 - 1/d_j)).
\]

Substituting

\[1/(d_j - 1) = 1/d_j + 1/d_j^2 + 1/d_j^3 + \ldots\]

and

\[
\log_e(1 - 1/d_j) = -1/d_j - (1/2)(1/d_j^2) - (1/3)(1/d_j^3) - (1/4)(1/d_j^4) - \ldots
\]

into the previous expression, we have

\[
f'(d_j) < -2/d_j^2 + (1/e)(1/(d_j - 1)) + (3/4)(1/d_j^3) + (1/e)(1/(d_j - 1)) \times
(1/d_j^4 + 1/d_j^5 + \ldots) \]

\[
< -2/d_j^2 + (1/e)(1/(d_j - 1)) \times
(1/d_j^2 + 1/d_j^3 + \ldots) \]

\[
< -2/d_j^2 + (1/e)(1/(d_j - 1)) \times
(1/d_j^2 + 1/d_j^3 + \ldots) \]

\[
< -2/d_j^2 + (1/e)(1/(d_j - 1)) \times
(1/d_j^2 + 1/d_j^3 + \ldots) \]

\[
= -2/d_j^2 + (1/e)(1/(d_j - 1)) \times
(1/d_j^2 + 1/d_j^3 + \ldots) \]

\[
< 0
\]

since \( d_j \geq 2 \).

Next we consider the case when \( n_j < d_j \). Thus, \( n_j < d_j \),
which is the same as \( n_j \leq d_j - 1 \).

Since \( (1/e)^{n_j/(d_j - 1)} < (1 - 1/d_j)^{d_j - n_j}/d_j - 1 \) and
\( 1 - 1/d_j)^{n_j} = (1 - 1/d_j)^{n_j}/d_j - 1 \) and \( (1 - 1/d_j)^{n_j} < (1/e)^{n_j/d_j} \), it suffices to show that \( (1/e)^{n_j/(d_j - 1)} < 2/d_j \).

Recall that \( (1/e)^x = e^{-x} = \sum_{j=0}^{\infty}(-1/j)x^j \), where
\( t(x, j) = (1/j)x^j \). Since \( t(n_j/d_j, j) < t(n_j/(d_j - 1), j) \),
we have
\[
(1/e)^{n/d_i} - (1/e)^{n/(d_i-1)}
\]
\[
= \sum_{j=0}^{\infty} (-1)^j (t(n_i/d_i, j) - t(n_i/(d_i - 1), j))
\]
\[
< \sum_{j=0}^{\infty} (-1)^{j+1} (t(n_i/d_i, 2j + 1) - t(n_i/(d_i - 1), 2j + 1))
\]
\[
= \sum_{j=0}^{\infty} (t(n_i/(d_i - 1), 2j + 1) - t(n_i/d_i, 2j + 1)).
\]
Thus, it again suffices to show that \(\sum_{j=0}^{\infty} (t(n_i/(d_i - 1), 2j + 1) - t(n_i/d_i, 2j + 1)) < 2/d_i\).

We want to show that \(t(n_i/(d_i - 1), 2j + 1) - t(n_i/d_i, 2j + 1) < 1/(2d_i)\) for \(j \geq 1\). There are two cases. Case 1 is when \(d_i < 2j + 1\). Then
\[
t(n_i/(d_i - 1), 2j + 1) - t(n_i/d_i, 2j + 1)
\]
\[
< t(n_i/(d_i - 1), 2j + 1)
\]
\[
\leq t(1, 2j + 1)
\]
\[
= 1/(2j + 1)!
\]
\[
= (1/(2j + 1))(1/(2j)!)\]
\[
< 1/(d_i)(1/(2j)!)\]
\[
< 1/(2d_i)
\]
for \(j \geq 1\). Case 2 is when \(d_i \geq 2j + 1\). We can rewrite \(t(n_i/(d_i - 1), 2j + 1) - t(n_i/d_i, 2j + 1)\) as
\[
\frac{1}{(2j + 1)!} \left[ \left( \frac{n_i}{d_i - 1} \right)^{2j + 1} - \left( \frac{n_i}{d_i} \right)^{2j + 1} \right]
\]
which is the same as
\[
\frac{n_i^{2j+1} d_i^{2j+1} - (d_i - 1)^{2j+1}}{(2j + 1)! d_i^{2j+1} (d_i - 1)^{2j+1}}.
\]
Next, if we can show that \(d_i^{2j+1} - (d_i - 1)^{2j+1} < (2j + 1)d_i^{2j}\), then the consequence is
\[
t(n_i/(d_i - 1), 2j + 1) - t(n_i/d_i, 2j + 1)
\]
\[
< \frac{n_i^{2j+1}(2j + 1)d_i^{2j}}{(2j + 1)! d_i^{2j+1} (d_i - 1)^{2j+1}}
\]
\[
\leq 1/(2j!d_i)
\]
\[
< 1/(2d_i).
\]
Note that
\[
d_i^{2j+1} - (d_i - 1)^{2j+1}
\]
\[
= d_i^{2j+1} - \sum_{h=0}^{j} (C(2j + 1, 2h)d_i^{2j+1-2h} - C(2j + 1, 2h + 1)d_i^{2j-2h})
\]
\[
= (2j + 1)d_i^{2j} - \sum_{h=1}^{j} (C(2j + 1, 2h)d_i^{2j+1-2h} - C(2j + 1, 2h + 1)d_i^{2j-2h})
\]
where \(C(\alpha, \beta) = \alpha!/(\beta!(\alpha - \beta)!)\) denotes the number of combinations of choosing \(\beta\) items from a collection of \(\alpha\) distinct objects. Since
\[
C(2j + 1, 2h)d_i^{2j+1-2h} - C(2j + 1, 2h + 1)d_i^{2j-2h}
\]
\[
= C(2j + 1, 2h)d_i^{2j-2h} - (d_i - (2j - 2h + 1)/(2h + 1))
\]
\[
\geq C(2j + 1, 2h)d_i^{2j-2h} - ((2j + 1) - (2j - 2h + 1)/(2h + 1))
\]
\[
= C(2j + 1, 2h)d_i^{2j-2h} - ((2j + 2)(2h)/(2h + 1))
\]
\[
> 0,
\]
thus \(d_i^{2j+1} - (d_i - 1)^{2j+1} < (2j + 1)d_i^{2j}\). Therefore,
\[
\sum_{j=0}^{\infty} (t(n_i/(d_i - 1), 2j + 1) - t(n_i/d_i, 2j + 1))
\]
\[
= t(n_i/(d_i - 1), 1) - t(n_i/d_i, 1)
\]
\[
+ \sum_{j=1}^{\infty} (t(n_i/(d_i - 1), 2j + 1) - t(n_i/d_i, 2j + 1))
\]
\[
< t(n_i/(d_i - 1), 1) - t(n_i/d_i, 1) + \sum_{j=1}^{\infty} 1/(2d_i)
\]
\[
= t(n_i/(d_i - 1), 1) - t(n_i/d_i, 1) + 1/d_i
\]
\[
= n_i/(d_i - 1) - n_i/d_i + 1/d_i
\]
\[
= n_i/(d_i(d_i - 1)) + 1/d_i
\]
\[
\leq 1/d_i + 1/d_i
\]
\[
= 2/d_i.
\]
As an immediate consequence, we have the following theorem which extends Leung and Chen’s result that the expected number of distinct test cases considered by a proportional sampling strategy is larger than that of random testing [7].

**Theorem 3.4.** Let \(t_{\text{pss}}\) denote the expected number of distinct test cases considered by partition testing using the proportional sampling strategy. Let \(t_r\) denote the expected number of distinct test cases considered by random testing. Assume that \(n \leq d\). Then \(0 < t_{\text{pss}} - t_r < 2k\), where \(k\) is the number of subdomains.

**Proof:** Note that
\[
t_{\text{pss}} = \sum_{1 \leq i \leq k } \sum_{x \in D_i} (1 - (1 - 1/d_i)^n_i)
\]
and
\[
t_r = \sum_{1 \leq i \leq k } \sum_{x \in D_i} (1 - (1 - 1/d)^n).
\]
Since \((1 - (1 - 1/d_i)^n_i) > (1 - (1 - 1/d)^n)\), we have
0 < \text{t}_{\text{pss}} - t_r. Next, by Lemma 3.3 we have
\[
\text{t}_{\text{pss}} - t_r = \sum_{1 \leq i \leq k} \sum_{x \in D_i} (1 - (1 - 1/d_i)^n) - (1 - (1 - 1/d)^n) < \sum_{1 \leq i \leq k} \sum_{x \in D_i} 2/d_i = \sum_{1 \leq i \leq k} d_i (2/d_i) = \sum_{1 \leq i \leq k} 2 = 2k.
\]

When used by practitioners as a rough guideline, Theorem 3.4 implies that an additional partition will introduce a difference of less than two distinct test cases.

Since the analysis given in Lemma 3.3 is not very tight, we expect the actual difference in the expected number of distinct test cases between random testing and the proportional sampling strategy to be much smaller than 2k, the upper bound provided in Theorem 3.4. The worst scenario that we can devise is the following. Let the size of each subdomain be one, i.e. \( k = d \). Suppose \( n = d \).

Then the proportional sampling strategy would select one test case from each subdomain. Thus \( \text{t}_{\text{pss}} = k \) and \( t_r = d - d(1 - 1/d)^d \), which approaches \( d(1 - 1/e) = k(1 - 1/e) \) as \( d \) tends to infinity. Therefore, the difference in the two expected numbers is approximately \((1/e)k = 0.367879k\).

Although the upper bound may not be a tight one, our result is still a strong one because the bound is independent of \( n \), the number of test cases selected, despite the fact that \( \text{t}_{\text{pss}} - t_r \) is a function of \( n \). As \( n \) grows, \( \text{t}_{\text{pss}} - t_r \) may be expected to grow as it is a function of \( n \). However, the possibility that \( \text{t}_{\text{pss}} - t_r \) is positively proportional to \( n \) is eliminated by Theorem 3.4 that \( \text{t}_{\text{pss}} - t_r \) is bounded by \( 2k \).

Since there is no evidence or intuition to support that \( \text{t}_{\text{pss}} - t_r \) is inversely proportional to \( n \), Theorem 3.4 provides a good upper bound.

If the subdomains are of varying sizes, in order to achieve proportional sampling where \( \sigma_1 = \sigma_2 = \ldots = \sigma_k \), we normally would have \( n \gg k \). Theorem 3.4 tells us that when \( n \) increases, the relative difference in the expected number of distinct test cases between the proportional sampling strategy and random testing decreases. As a reminder, subdomain testing outperforms random testing significantly when its subdomains are homogeneous (that is, either containing failure-causing inputs or containing none) or nearly homogeneous. However, no effective partitioning scheme of this nature has been devised so far. Also, there is no knowledge of the location and size of failure-causing inputs prior to testing, otherwise more effective test case allocation schemes could then be developed. The proportional sampling strategy is a conservative approach to guarantee that it outperforms random testing irrespective of failure distributions. Obviously, we could not expect the proportional sampling strategy to outperform random testing significantly for all failure distributions. Hence, the proportional sampling strategy should be used along with other fault-based partitioning schemes.

In summary, this paper provides a new perspective for random testing and subdomain testing. This perspective makes the proofs of some known results simpler and more natural, such as the relationship between E-measures for random testing and the proportional sampling strategy, the derivation of the formula for the expected number of distinct test cases for random testing, and the fact that the proportional sampling strategy considers a greater expected number of distinct test cases than random testing for the same number of test cases generated. We believe that in general the new perspective also helps to explain some results that involve failure-causing inputs in a more intuitively appealing way. An example is the simple explanation of why the proportional sampling strategy has a higher P-measure than random testing.

More importantly, it enables us to derive an interesting new result that the difference in the expected number of distinct test cases between the proportional sampling strategy and random testing is smaller than twice the number of partitions.

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