Who Volunteers? A Theory of Firms where Agents are Motivated to Work

Werner Güth* Ernst Maug†

April 8, 2003

Abstract

While effort averse agents provide only minimum effort if not monitored, other agents may be committed to the organizational goal. Our study allows for a continuum of types, representing different degrees of intrinsic work motivation. This results in two different organizational forms, "volunteer organizations" and "companies," which may also attract less committed employees who require output-dependent incentive contracts to elicit effort. Generally, only organizations with significant likelihood to attract motivated volunteers can survive as volunteer organizations whereas "firms" should result when monitoring costs are moderate and effort is critical to the organization's success.

*Max Planck Institute for Research into Economic Systems, Strategic Interaction Group, Kahlaische Strasse 10, 07745 Jena, Germany. email: gueth@mpiew-jena.mpg.de, +49 (3641) 686 620 (phone), +49 (3641) 686 623 (fax).

†Humboldt-Universität zu Berlin, School of Business and Economics, Institute of Konzernmanagement, Spandauer Str. 1, D-10178 Berlin. email: maug@wiwi.hu-berlin.de, +49 (30) 2093-5641 (phone), +49 (30) 2093-5643 (fax).

1 Introduction

Why do organizations exist that rely entirely on volunteers where some members of the organization supply not only (typically unmonitored) effort, but also make other contributions (like membership fees) to the organization? How do organizations like these compare to traditional "firms," where employees have their behavior monitored and often receive wages tied to output for incentive purposes? Examples of such organizations are pervasive and inlcude political parties, churches, environmental pressure groups, some sports clubs, and a wide array of charitable organizations.

We try to address these questions in a uniform framework. We analyze a principal-agent setting where the agent's work motivation (or "type") is his private information. While some agents pursue an agenda different from the organization and provide only minimum effort if not monitored, other agents are committed to the organizational goal and supply the highest level of effort without compensation or monetary incentives. The principal can employ contracts contingent on output as a screening device. To employ such contracts the principal must pay an additional monitoring cost for observing output, otherwise she can only pay a fixed wage. In our model the managerial task rests with the principal. We have in mind an asymetric form of labor division, both in volunteer organizations and in firms. One type of actors (principals) is assumed to take the initiative whereas the other is invited to join.

We find that even though we assume the existence of a continuum of agent-types that differ with respect to their motivation, which we interpret as congruence with the organizational goal, two distinct organizational forms emerge:

- "Volunteer organizations" like parties and charities rely entirely on the commitment of the people who work for them. They do not provide monetary rewards. People have various degrees of commitment to their organization, measured by their work motivation. The principal accepts the fact that only a limited number of volunteers are available. However, the volunteers who get hired provide the maximum level of effort. The principal even expects a certain amount of commitment and may turn down volunteers who are not willing to pay a minimum fee. In this organizational form, wages are low and incentive contracts are never employed. The probability of not producing any output because no volunteers are available may be high.

- "Companies," where people join mainly in order to earn a living, and only if offered a monetary compensation in return. The principal offers a wage and balances wage costs against the probability of hiring more often. Some people who are hired would join for a lower compensation but the wage is set by the marginal agent who is indifferent between accepting and declining the contract offer. Also, some people enjoy their work and provide the maximum amount of effort, whereas others provide only the minimum. Here we see incentive payments and some agents supplying effort only because they are rewarded. The probability of not producing because no agent was hired is small.

As a rule, organizations producing a high level of output prefer to organize themselves as companies, whereas those with less productive technologies prefer to be organized as volunteer organizations. Interestingly, a small change in productivity may lead to a switch from an organization without incentive payments and low wages to a firm with incentive payments and distinctly higher wages, even if monitoring is costless.

While our approach to analyzing volunteer organizations is to the best of our knowledge new, there are a number of related strands of literature.

Volunteer organizations can result from private initiatives by individuals or groups with entrepreneurial skills or by social movements and are mainly studied in sociology (see Opp, 1984, 1986, 1988 and 1989). We cannot discuss in detail sociological aspects although our rational choice-approach is nowadays also popular in sociology like in the other social sciences.

A crucial assumption of our study is the possibility of intrinsic work motivation, here in the specific sense that the principal and agent are driven by the same motives. Intrinsic motivation seems to be initially discussed in (social) psychology, mainly when arguing that extrinsic incentives like the incentive contracts in principal-agent theory may crowd out intrinsic motivation. The debate has been imported into economics (see Frey, 1997) by specifically challenging that incentive contracts should be recommended (Frey and Osterloh, 1997 and Sadowski, Pull and Schneider, 1998). Note, however, that the problem is here more whether and when intrinsic and extrinsic motives can coexist whereas we are interested in the institutional arrangements implied by one type of motivation versus the other.

The debate has inspired a lot of empirical research both, in firm organization theory and in labor economics. It has been established by several experimental studies of principal-agent situations that trust of principals (in the form of decent salaries) and reciprocity (in the sense of full effort in case of no incentives but decent salary) of agents can be as efficient as incentive schemes.[1] A large literature has explored how the pay of CEOs is related to firm performance (see Murphy, 1998, for a survey). This literature has always taken the incongruence between the motives of managers and shareholders (as principals) for granted.

Section 2 introduces the model. The analysis proceeds in two steps: Section 3 first treats the case without monitoring and incentive contracts. Section 4 allows for incentive contracts by paying a monitoring cost for observing output. Finally, in Section 5 we ask under what circumstances the principal would choose a contract with costly monitoring. Section 6 concludes. All proofs are collected in an Appendix.

[1] On the first point see Gächter and Fehr, 2001, and Güth, Willinger and Ziegelmeyer, 2002, for surveys. See also Akerlof, 1982, and Akerlof and Yellen, 1990.

2 The Model

The firm is owned by the principal P who can hire a single agent, denoted by A.[2] Agent A may or may not enjoy his work intrinsically. This is captured formally by his cost of effort $ce^2/2$, which can be both positive $(c > 0)$ or negative $(c < 0)$. The parameter $c \in [-1, 1]$ is agent A's private information. P's expectations concerning c are expressed by the density $\psi(\cdot)$ with

$$\psi(c) = \begin{cases} > 0 \text{ if } c \in [-1, 1] \\ = 0 \text{ otherwise,} \end{cases} \quad , \tag{1}$$

respectively, by its cumulative distribution function

$$\Psi(c) = \int_{-1}^{c} \psi(x)\, dx \quad , \tag{2}$$

with $\Psi'(c) = \psi(c)$ for all $c \in (-1, 1)$. In the tradition of Harsanyi (1967/68) we assume that P's expectations are common knowledge. Thus, by fictitiously assuming that c is randomly chosen according to $\psi(\cdot)$ we have transformed the situation with incomplete information into a stochastic game whith commonly known rules.

Whereas in usual principal-agent models the principal cannot infer effort by observing output (hidden action), we assume output to be a deterministic linear function of effort,

$$\pi(e) = \alpha + \beta e \quad ; \qquad \alpha \geq 0, \beta > 0, \tag{3}$$

where $e \in [\underline{e}, \overline{e}]$ and $\overline{e} > \underline{e} > 0$. Thus, if the principal can observe output levels she can also infer the effort level. However, observing or monitoring output is not free of charge. The costs of monitoring are denoted by $K > 0$. Let

[2] Whether P will actually employ this single agent can be more or less likely. An alternative interpretation would assume that this hiring probability measures the size of the work force which the principal employs (similar to interpreting a demand function as a belief function, compare, for instance Güth and Ritzberger, 1998, with Güth, Kröger and Normann, 2002).

$\delta \in \{0,1\}$ be a dummy variable for monitoring ($\delta = 1$) and not monitoring ($\delta = 0$). P can offer only contracts $\mathcal{C} = (W, s, \delta)$ where $W \in \mathbb{R}$ is a fixed wage, $s \in [0,1]$ is $A's$ share of the surplus and δ is the decision to monitor. Whenever P does not monitor ($\delta = 0$) she does not observe output so that $s = 0$. Note that we deliberately refer to the principal's "payoff" and to "surplus" and do not use the term "profits" here, which would be to narrow for our concept of a "firm." Also, our modeling strategy does not depend on the benefits from production to be shared in this game to be pecuniary only. Finally, the participation threshold of agent A is assumed to be type-independent and normalized to 0.[3] The decision process is as follows:

1. Chance selects $c \in [-1, +1]$ randomly according to $\psi(\cdot)$. The result c is revealed to A but not to P.

2. P makes a contract offer $\mathcal{C} = (W, s, \delta)$ to A. Whenever $\delta = 0$ then also $s = 0$.

3. A either accepts the contract offer $\mathcal{C}$ or not. If A does not accept $\mathcal{C}$, then the game is over with payoff level 0 for P and 0 for A. If A accepts $\mathcal{C}$ the game continues.

4. A chooses his effort level $e \in \{\underline{e}, \overline{e}\}$ with $\overline{e} > \underline{e} > 0$ and output $\pi(e) = \alpha + \beta e$ is realized and observed by P (only if $\delta = 1$).

Let u denote agent A's payoff and v the payoff of P. Since u depends on agent A's type $c \in [-1, +1]$, the payoff u_c is the payoff for the type c of agent A. Payoffs are

- $u_c = 0$ for all c and $v = 0$ if $\mathcal{C}$ is rejected,

- $u_c = W + \delta s \pi(e) - \frac{c}{2} e^2$ and $v = (1 - \delta s) \pi(e) - W - \delta K$ otherwise.

This completes the description of the model.

[3] Our interpretation of this participation threshold is that of a benchmark wage.

3 Analysis

We first explore the case where P does not monitor ($\delta = 0$) before analyzing the more complex case with monitoring ($\delta = 1$) where incentive contracts are feasible in the next section.

3.1 The case without monitoring

If the principal does not monitor we have $\delta = s = 0$ and the contract consists just of a fixed wage W. Then agent A chooses his effort level e^* as:

$$e^* = e^*(c) = \begin{cases} \overline{e} \text{ if } c \leq 0 \\ \underline{e} \text{ if } c > 0 \end{cases} \tag{4}$$

if he has accepted the contract. Whether a certain agent type $c \in [\underline{c}, \overline{c}]$ accepts the contract depends, of course, on the wage offered. If $W \leq 0$, then only c types with

$$-\frac{c}{2}\overline{e}^2 + W \geq 0 \text{ or } c \leq \frac{2W}{\overline{e}^2} \tag{5}$$

accept. For $W \geq 0$ the corresponding condition is

$$-\frac{c}{2}\underline{e}^2 + W \geq 0 \text{ or } \frac{2W}{\underline{e}^2} \geq c. \tag{6}$$

Due to $\overline{e} > \underline{e} > 0$ the critical level

$$\begin{aligned} \hat{c}(W) &= \begin{cases} \frac{2W}{\overline{e}^2} \text{ for } W \leq 0 \\ \frac{2W}{\underline{e}^2} \text{ for } W > 0 \end{cases} \\ &= \frac{2W}{(e^*)^2} \end{aligned} \tag{7}$$

is a continuous and increasing function of W.[4] The second line uses (4). When anticipating the consequences of $\mathcal{C} = (W, 0, 0)$ the principal's payoff depends on W as follows:

$$Ev(W) = \begin{cases} \Psi(\hat{c}(W))(\alpha + \beta\overline{e} - W) & \text{for} \quad c\overline{e}^2/2 \leq W \leq 0 \\ \Psi(0)\beta(\overline{e} - \underline{e}) + \Psi(\hat{c}(W))(\alpha + \beta\underline{e} - W) & \text{for} \quad W \geq c\underline{e}^2/2 > 0 \end{cases} . \tag{8}$$

[4] It is also diffentiable except for $W = 0$.

where expectations have been taken with respect to the probability density function $\psi(c)$ and $\hat{c}$ marks the cut-off point so that all agents with types $c \leq \hat{c}$ accept the contract, where $\hat{c}$ is given from (7).

The principal chooses W so as to maximize (8) with respect to W. Solving the first order conditions gives:

$$W^* + \frac{\Psi(\hat{c}(W^*))}{\psi(\hat{c}(W^*))} \frac{(e^*)^2}{2} = \pi(e^*) \quad if \quad W^* > 0 \text{ or } W^* < 0 \quad . \tag{9}$$

$Ev(W)$ is differentiable except at $W = 0$. If $W^* = 0$, then $\frac{\partial Ev(W)}{\partial W} > 0$ for all $W < 0$ and $\frac{\partial Ev(W)}{\partial W} < 0$ for all $W > 0$. In order to characterize the solution further we need to put restrictions on Ψ.

Lemma 1 *$\Psi(c)$ is log-concave in c, if and only if $\Psi(c)/\psi(c)$ is monotonically increasing in c.*

The log-concavity requirement is equivalent to imposing a monotone hazard rate condition found in many applications in mechanism design and information economics. This condition is satisfied by a large number of common distribution functions, including the normal and the uniform distribution.[5] The left hand side of (9) generally does not admit a closed form solution. However, we can use Lemma 1 to characterize the solution as follows:

Proposition 2 ***(Uniqueness):*** *Assume that $\Psi(c)$ is log-concave. There are at most three candidate solutions W^* of the first order necessary conditions: (i) Condition (9) is satisfied for $e = \overline{e}$ and $\hat{c}(W^*) < 0, W^* < 0$; (ii) Condition (9) is satisfied for $e = \underline{e}$ and $\hat{c}(W^*) > 0, W^* > 0$; (iii) $\frac{\partial Ev(W)}{\partial W} > 0$ for all $W < 0$ and $\frac{\partial Ev(W)}{\partial W} < 0$ for all $W > 0$. Then $W^* = \hat{c} = 0$. Generically, the solution is unique.*

[5] See Fudenberg and Tirole (1992), p. 267 for a discussion of log-concavity and monotone hazard rates. They also provide additional references.

Based on Proposition 2 we can legitimately perform comparative static analysis. We can also see that we should generally assume W^* to be piecewise continuous since discontinuities arise only in two cases: Either we have a solution at the point where $\hat{c}$ and $Ev(W)$ are not differentiable ($W = 0$), or, the objective $Ev(W)$ happens to have the same value for positive and negative solutions of (9). Abstracting from these two cases W^* is increasing in both, α and β, if Ψ is log-concave, other things being equal. The reason is that the principal has to be indifferent at the margin between increasing W so that more agent types accept and reducing W to reduce the expected wage bill. The higher the potential output $\pi(e)$ (the right hand side of (9)), the higher are the cost of agents not accepting the contract. Hence, higher potential output levels imply higher opportunity costs from declined offers and therefore higher wage offers to the agent. Note that there is no difference here between higher output levels because of higher fixed output levels α, or because of a higher responsiveness to the agent's effort as measured by β.

We will now consider a special case of our model in order to be able to derive stronger results. Assume $\psi(\cdot)$ is the uniform density with support $[-1, +1]$, i.e.

$$\begin{aligned} \psi(c) &= \begin{cases} 1/2 \text{ if } c \in [-1, +1] \\ 0 \text{ otherwise,} \end{cases} , \\ \Psi(c) &= \frac{c+1}{2} \text{ for all } c \in [-1, 1] \quad . \end{aligned} \tag{10}$$

After substituting $\hat{c}$ from (7) we can rewrite (9) for the special case of the uniform distribution as

$$W^* = \frac{\pi(e^*)}{2} - \frac{(e^*)^2}{4} \quad , \tag{11}$$

where e^* is again given from (4). Please note also that (11) applies only where $W^* \leq \frac{1}{2}\underline{e}^2$. At $W^* = \frac{1}{2}\underline{e}^2$ even agents with type $c = 1$ (the upper bound of the support) accept the contract and the principal would not benefit from increasing the wage any further.

Finally, we need to evaluate the total surplus of P at the optimal solution W^*. Hence, substitute W^* from (11) back into (8) to obtain:

$$Ev\left(W^*\right)=\begin{cases}\left(\frac{\pi(\overline{e})+\overline{e}^2/2}{2\overline{e}}\right)^2 & if \quad W^*\leq 0\\ \frac{\beta}{2}\left(\overline{e}-\underline{e}\right)+\left(\frac{\pi(\underline{e})+\underline{e}^2/2}{2\underline{e}}\right)^2 & if \quad W^*>0\end{cases}. \tag{12}$$

For every pair of values $\underline{e}$, $\overline{e}$ the function $Ev\left(W^*\right)$ in (12) defines a pair of hyperplanes in $\alpha, \beta-$space. In order to characterize the solution we need to know whether the principal should choose $W^*>0$ or $W^*\leq 0$ and if the solution is feasible, i.e. if $W^*\leq 0$ corresponds to $c\leq 0$ and vice versa for $W^*>0$. We define

$$\overline{\beta} = \frac{\overline{e}}{2}-\frac{\alpha}{\overline{e}}, \tag{13}$$

$$\underline{\beta} = \frac{\underline{e}}{2}-\frac{\alpha}{\underline{e}}. \tag{14}$$

Note that $\overline{\beta}-\underline{\beta}>0$.

Proposition 3 ***(Solution):*** *For every parameterization of the model with uniform distribution (10) we can separate the parameter space into three regions: (i) if $\beta<\underline{\beta}$, then the only candidate solution has $W^*\leq 0$, only agents without effort aversion ($c\leq 0$) are hired, and all agents that are hired provide maximum effort; (ii) if $\beta>\overline{\beta}$, then the only candidate solution has $W^*>0$ and the principal sometimes hires agents with effort aversion who provide only the minimum level of effort. (iii) If $\underline{\beta}\leq\beta\leq\overline{\beta}$, then both solutions are feasible and the principal chooses the solution with the highest payoff $Ev\left(W^*\right)$.*

Hence, candidate solutions can be characterized as follows. Whenever the agent's effort has very little impact on output (unproductive technology), then the principal is better off to relying on "volunteers"only, i.e. agents who are willing to supply maximum effort without a pecuniary reward. Whenever the technology is highly productive, the principal optimally offers a higher

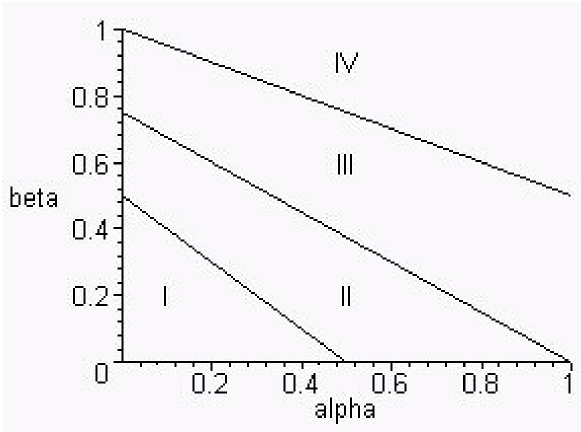

Figure 1: Candidate solutions for $\overline{e} = 2$, $\underline{e} = 1$. Areas I and II are separated by $\underline{\beta}$, Areas II and III by $\hat{\beta}$ and areas III and IV by $\overline{\beta}$.

wage where some agents provide maximum effort and others provide only minimum effort. There is an intermediate region where both are potentially optimal. In this case we have to evaluate the objective $Ev(W^*)$ for both candidate solutions.

Proposition 4 *For every parameterization of the model with uniform distributions where $\overline{e} > \underline{e} > 0$ and $\alpha \geq 0$ there exists a critical cutoff point $\hat{\beta}$ such that the optimal solution has $W^* > 0$ if and only if $\beta \geq \hat{\beta}$.*

In order to obtain explicit algebraic expressions we use $\overline{e} = 2$ and $\underline{e} = 1$. Then we have $\overline{\beta} = 1 - \frac{\alpha}{2}$, $\underline{\beta} = \frac{1}{2} - \alpha$, and $\hat{\beta} = \frac{3}{4} - \frac{3}{4}\alpha$. Figure 1 demonstrates the result graphically. We can distinguish four different regions from Proposition 3. If $\beta < \underline{\beta}$, then only $W \leq 0$ is feasible (case (i), area I). Similarly, if $\beta > \overline{\beta}$, then only $W > 0$ is feasible (case (ii), area IV). In the last case (iii) of Proposition 3, where $\underline{\beta} \leq \beta \leq \bar{\beta}$, we need to distinguish further between $\beta \leq \hat{\beta}$ (area II) and $\beta > \hat{\beta}$ (area III) from Proposition 4. For this example the wage function becomes:

$$W^* = \begin{cases} \frac{\alpha}{2} + \beta - 1 & if \quad \beta < \frac{3}{4} - \frac{3}{4}\alpha \\ Min\left\{\frac{1}{2}; \frac{\alpha+\beta}{2} - \frac{1}{4}\right\} & otherwise \end{cases} . \tag{15}$$

Note that the optimal wage can never exceed $W^* = \frac{1}{2}$. At this point even the type with the highest effort aversion accepts the contract, so the probability of the agent accepting is 1. Also, W^* increases monotonically and continuously except at the point where $\beta = \hat{\beta} = \frac{3}{4}(1-\alpha)$. At this point W^* jumps discontinuously from a strictly negative wage $W^* < -\frac{1}{4}(1+\alpha)$ to a strictly positive wage $W^* > \frac{1}{8}(1+\alpha)$. However, $Ev(W^*)$ does *not* jump discontinuously at $\beta = \hat{\beta}$. Whenever the $\beta = \hat{\beta}-$condition that separates region II (only high-effort agents are hired) and region III (some low-effort agents are hired as well) is satisfied, the principal has two options that generate exactly the same net surplus:

1. Pay a positive wage $W_+^* = \frac{1}{4}(\alpha+1)$ and hire all types with effort aversion $c \leq -\frac{1}{8}(\alpha+1)$; This contract is accepted by the agent with probability $\frac{2\alpha+10}{16}$.

2. Pay a negative wage $W_-^* = -\frac{1}{8}(\alpha+1)$ and hire all types with effort aversion $c \leq \frac{1}{4}(\alpha+1)$; This contract is accepted by the agent with probability $\frac{7-\alpha}{16}$.[6]

While both wages generate the same net surplus, they involve very different strategies, either paying a high wage and generating output with a higher probability, or paying a low (effectively, negative) wage and generating output with a lower probability. At $\beta = \hat{\beta}$, the principal is indifferent between both, hence the net surplus-function is smooth.[7]

The best way to understand these results is to conceive of this model as characterizing two different types of organization. "Volunteer organizations,"

[6] These are not mixed strategies. The probabilities arise because the principal does not know the type of the agent.

[7] This is a "value-matching"-condition. Recall that W^* is piecewise continuous and Ev is continuous in W. Hence, if there were a discontinuity in W, then there would be an infinitesimal change of W^* that could increase the objective, contradicting the result that W^* is optimal.

where the principal relies entirely on self-motivated agents who contribute effort and additional resources to the organization, and "companies" with agents who only work for a wage. Those activities that generate only limited surplus (α, β small) are best performed by volunteer organizations. These organizations rely on highly committed volunteers who provide maximum effort and other inputs that are characterized here by a negative wage. However, these organizations are fragile in that there are only few committed volunteers. On the other hand, if the benefit the organization can create is large, then it is better that the organization stays productive and is not disrupted if it cannot attract volunteers. Then employees who work for a wage are recruited. The organization is more robust, but it does not always perform at its full ($e = \overline{e}$) potential since it has to rely on the involvement of employees whose agenda differs from that of the organization.

4 Monitoring

4.1 Feasible Contracts

In this section we analyze our model for the case where monitoring is feasible ($\delta = 1$), so

$$u_c = W + s(\alpha + \beta e) - \frac{c}{2}e^2 \quad . \tag{16}$$

Define

$$\bar{c} = \frac{2s\beta}{\overline{e} + \underline{e}} \quad . \tag{17}$$

Then the optimal strategy of the agent is:

$$e^* = \begin{cases} \overline{e} & if \quad c \leq \bar{c} \\ \underline{e} & otherwise \end{cases} \quad . \tag{18}$$

We define the $\hat{c}-$function for the monitoring case analogously to (7) as:

$$\hat{c}_M(e^*) = 2\frac{W + s(\alpha + \beta e^*)}{(e^*)^2} \quad , \quad e^* \in \{\underline{e}, \overline{e}\} \quad . \tag{19}$$

The interpretation is the same as before. An agent who would optimally supply effort $e = \overline{e}$ after being hired would accept the contract if and only if $c \leq \hat{c}_M(\overline{e})$, and similarly for $e = \underline{e}$. We define

$$\hat{W} = -s\left(\alpha + \beta \frac{\underline{e}\overline{e}}{\overline{e}+\underline{e}}\right) \quad . \tag{20}$$

We need this threshold level to describe acceptable contracts $\mathcal{C} = (W, s, 1)$:

Proposition 5 ***(Acceptable Contracts):*** *Any contract $C = (W, s, 1)$ will be accepted by the agent if and only if $W \geq \hat{W}$. All agents with $c \leq \bar{c}$ will accept the contract and choose $e^* = \overline{e}$. All agents with $c \in [\bar{c},\ \hat{c}_M(\underline{e})]$ will accept the contract and choose $e^* = \underline{e}$, where $\bar{c} < \hat{c}_M(\underline{e})$ if and only if $W > \hat{W}$. All agents with $c > \hat{c}_M(\underline{e})$ will reject the contract.*

Proposition 5 is intuitive. Low-cost agents accept the contract and supply the high level of effort, high-cost agents find the wage offered unacceptable, and an intermediate range of agents will accept the contract and only supply the minimum level of effort. There exists a lower bound $\hat{W}$ such that all types will reject contracts with a wage lower than this threshold.

4.2 Optimal Contracts

In this subsection we will solve the optimal contracting problem of the principal contingent on monitoring. We will analyze the two regimes of the previous section ($\hat{W} = 0$ and $\hat{W} > 0$) separately. In this section we work only with the uniform distribution (10).

Consider first the case where $W = \hat{W}$ and all agents who are hired supply the maximum level of effort (see Proposition 5). Then the agent with the highest cost of effort who is hired has $c = \bar{c}$ and the wage is equal to $W = \hat{W}$ from (20) by assumption. Hence, the principal's expected payoff can now be written as:

$$Ev_M(C) = \Psi(\hat{c}_M(\overline{e}))\left((\alpha + \beta\overline{e})(1-s) - \hat{W} - K\right) \tag{21}$$

where the subscript M indicates the monitoring case. Note that the objective (21) is independent of W, hence the principal will maximize it only with respect to the share s. The agent's participation constraint ($c \leq \hat{c}_M(\overline{e})$) has already been substituted in (21).

Lemma 6 *If the principal offers a contract where $W = \hat{W}$, then the agent chooses the high effort level $e = \overline{e}$ after accepting the contract. The optimal share s is then:*

$$\frac{\left(\alpha + \beta\overline{e} - K - \frac{\overline{e}^2}{2}\right)(\underline{e} + \overline{e})}{2\beta\,\overline{e}^2} \quad , \tag{22}$$

and the maximized objective equals

$$\frac{1}{4}\left(\frac{\alpha - K}{\overline{e}} + \beta + \frac{\overline{e}}{2}\right)^2 \quad . \tag{23}$$

The situation is a little different if the principal chooses $W > \hat{W}$ so that some agents who are hired supply only the minimum level of effort (see Proposition 5). Then the objective becomes:

$$\begin{aligned} Ev_M(C) \;=\; & \Psi(\overline{c})(\alpha + \beta\overline{e})(1 - s) \\ & + (\Psi(\hat{c}_M(\underline{e})) - \Psi(\overline{c}))(\alpha + \beta\underline{e})(1 - s) \quad . \\ & - \Psi(\hat{c}_M(\underline{e}))(W + K) \end{aligned} \tag{24}$$

The first line in (24) represents the principal's surplus share from agents who supply the high level of effort, where some agents may have $c > 0$ and supply a higher level of effort only because they receive output-dependent payments. The second line represents the likelihood of hiring agents who supply only the minimum level of effort, multiplied by the net surplus obtained from hiring such agents. The last line represents the expected costs from paying a fixed wage and monitoring. We can now characterize the optimal contract of the monitoring firm for the case where the constraint $W \geq \hat{W}$ is not binding:

Lemma 7 *If the principal chooses a contract where $W > \hat{W}$, then*

$$s_+^* = \frac{1}{2} - \frac{\overline{e} + \underline{e}}{4\beta} \quad , \tag{25}$$

$$W_+^* = \frac{1}{4}\left(\frac{\alpha}{\beta}(\overline{e} + \underline{e}) - 2K + \underline{e}\overline{e}\right) \quad . \tag{26}$$

Note how the agent's pay package is broken up in the monitoring firm. Specifically, s_+^* is increasing in β, the responsiveness of output to higher levels of effort, but independent of α: higher levels of output have no impact on performance related pay unless they are connected to the agent's effort choice. This is an intuitive result. Also, the fixed component of pay is increasing in the fixed level of output α, but decreasing in the effort-response β. This is also intuitive: the principal's surplus increases in α, and the principal is more keen to actually hire an agent if production generates a higher surplus. An increase in monitoring costs has the reverse impact: higher monitoring costs K reduce the potential surplus of the principal, who is accordingly more willing to risk losing this surplus altogether by not hiring an agent, so she offers a lower fixed wage. Hence, we have two robust conclusions for the monitoring firm, other things being equal:

- A higher net surplus (higher α, lower K) leads to higher fixed wages as the principal tries to secure the gains from production.

- A higher responsiveness of output to the agent's effort leads to more performance-related pay and a lower fixed wage.

Note also that nothing in Lemma 7 and our assumptions on parameters rules out the possibility of negative incentive payments $s_+^* < 0$ (see equation (25)). The possibility of incentive contracts that are decreasing in output, at least over some ranges, has always puzzled contract theorists. In fact, a number of additional assumptions is necessary in order to obtain monotonically increasing incentive contracts in standard principal agent models.[8] Since our model has two states, contracts are always monotonic, but they can be increasing or decreasing. The principal would use a contract with a negative slope if the response of output to effort, measured by β, is low ($\beta < \frac{\overline{e}+\underline{e}}{2}$). Negative slopes arise since the principal prefers to discourage high effort because it is not sufficiently productive. Rather, she has more agents accepting the contract and supplying little effort: the principal benefits more from having more low-effort agents accepting (high fixed wage) than from having fewer agents exerting maximum effort. The negative incentive payment is therefore a screening device.

We obtain from Lemmas 7 and 6 a description of the contract that depends on the constraint $W \geq \hat{W}$. We therefore need to determine under which conditions this constraint is binding.

Lemma 8 *The constraint $W \geq \hat{W}$ is not binding if $W_+^* > \hat{W}$. This condition is satisfied whenever*

$$K < \alpha + \beta \frac{\underline{e}\overline{e}}{\overline{e} + \underline{e}} \quad . \tag{27}$$

Lemma 8 shows that the constraint $W^* \geq \hat{W}$ is always binding if monitoring costs are high. We can now characterize the solution to the optimal contracting problem, assuming the principal monitors. We do this by looking at the probability of the agent accepting the contract offer $\mathcal{C}$.

Proposition 9 ***(Contracting Equilibrium):*** *For the optimal contract with monitoring we have*

$$\hat{c}_M(e^*) = \frac{\pi(e^*) - K - (e^*)^2/2}{(e^*)^2} \quad . \tag{28}$$

The agent accepts the contract with probability

$$\Psi(\hat{c}_M(e^*)) = \frac{\pi(e^*) - K + (e^*)^2/2}{2(e^*)^2} \quad . \tag{29}$$

[8] See e. g. Grossman/Hart (1983).

The numerator of (28) is the joint surplus produced if the principal hires the agent with the highest costs ($c = +1$), the numerator of (29) is the joint surplus if the principal hires the agent with the lowest costs ($c = -1$). An intuitive approach to understanding these general conditions is to look at the special case where the agent accepts the contract with probability 1. This requires that $\Psi\left(\hat{c}_M\left(e^*\right)\right) = 1$, which is equivalent to:

$$\pi\left(e^*\right) - K - \frac{\left(e^*\right)^2}{2} \geq \left(e^*\right)^2 \quad . \tag{30}$$

The left hand side of condition (30) is the net surplus generated from hiring the worst agent ($c = +1$). Condition (30) indicates a dead weight loss. Even if the surplus is sufficient to cover monitoring costs K plus the costs of effort provision for the worst agent, the contract is rejected with positive probability. To see that this is a general property, denote by T the type of the marginal agent who would be hired with a first-best contract. This contract should ensure that

$$\pi\left(e^*\right) - K = T\frac{\left(e^*\right)^2}{2} \quad , \tag{31}$$

hence, the agent of type T generates sufficient surplus $\pi\left(e^*\right) - K$ to the principal such that it just offsets the costs of her effort.

Proposition 10 ***(Inefficiency):*** *Consider the contracting game where the marginal agent who could supply productive effort is the type with costs $T \in [-1,\ 1]$ characterized by condition (31). The likelihood of hiring this agent in equilibrium is $\frac{T+1}{4}$, whereas the first-best outcome implies hiring this agent with probability $\frac{T+1}{2}$.*

Hence, whenever a productive relationship is possible and $T > -1$, there is a deadweight loss because the principal exercises monopsony power over the agent.

5 The Decision to Monitor

Next,we investigate how the optimal contract depends on the exogenous parameters of the problem. Specifically, when is it better to choose a monitoring contract ($\delta = 1$, $s \neq 0$) instead of a contract without monitoring ($\delta = s = 0$). Since the resulting expressions are not very transparent, we restrict this discussion to the case where $\overline{e} = 2$ and $\underline{e} = 1$.

Lemma 11 ***(Advantage of Monitoring):*** *Assume $\overline{e} = 2$ and $\underline{e} = 1$. Then, whenever (27) holds, the advantage of monitoring over not monitoring is equal to*

$$\frac{1}{12}\left(\beta^2 - 6K\beta + 3K^2 - 3\beta - 6K\alpha - 3K + \frac{9}{8}\right) \quad . \tag{32}$$

If (27) is violated, the advantage of monitoring over not monitoring is equal to

$$-\frac{1}{16}\left(3\alpha^2 - K^2 + 2\alpha K + 4K - 3 + 4\beta\left(K + 1 + \alpha\right)\right) \quad . \tag{33}$$

These expressions are equal for $K = \alpha + \frac{2}{3}\beta$ where (27) holds as an equality.

We use Lemma 11 to characterize the optimal monitoring policy.

Proposition 12 ***(Monitoring Policy):*** *The advantage of monitoring is strictly decreasing in K for all K where the net benefits from contracting are non-negative. There exists a unique $\hat{K}$ such that monitoring is advantageous if and only if $K \leq \hat{K}$.*

Proposition 12 simply states that the advantage of monitoring is directly related to the costs of monitoring. It is more informative to ask what determines the critical cut-off value $\hat{K}$.

Proposition 13 ***(Comparative Statics I):*** *(i) Assume (27) holds. Then the advantage of monitoring is increasing in β if and only if $\beta \geq \frac{3}{2} + 3K$. There exists an interval $[\beta_L, \beta_H]$ such that monitoring is beneficial if and only if β lies outside this interval. (ii) If (27) does not hold, then the advantage of monitoring always increases in β.*

The first part of this result seems somewhat puzzling. We would expect that monitoring becomes more beneficial whenever the potential output from exerting effort is higher. Why should the prospect of a higher potential output make the principal less inclined to monitor the agent, relying on a fixed-wage contract instead? To see this, reconsider our discussion of downward sloping wage contracts. (See the discussion of Lemma 7 above.) We discovered there that contracts may be downward sloping because the principal wishes to discourage agents from exerting costly effort. If the productivity of effort is low, it is better to hire the agent with a higher likelihood, so that the marginal agent who is hired has high costs of exerting effort. Hence, in this case the benefits from monitoring arise from the low productivity of the agent and the wish to discourage agents from exerting effort. This benefit is diminished if productivity increases. At the point where productivity is sufficiently high so that the principal offers a positive performance fee $s > 0$, the benefits from monitoring increase in β, as expected. The costs of monitoring are justified only if the slope of the contract is sufficient. If β is either too high or too low, then s is also close to zero from (22). If the optimal output-contingent contract is low-powered, then the benefits do not justify the costs of monitoring, so it is optimal to pay a fixed wage.

Proposition 14 ***(Comparative Statics II):*** *The benefits from monitoring over not monitoring are strictly decreasing in the fixed component of output α for all $K > 0$ and the benefit of monitoring is reduced more strongly if monitoring costs K are higher.*

Interestingly, whenever the fixed component of output α is higher, monitoring is less beneficial. To see this, consider the basic trade-off of the principal: either pay a higher fixed wage to make employment more attractive and to make acceptance by the agent more likely, or hire an agent who contribute more effort, but with a lower probability. If fixed output α increases, then the principal puts a stronger emphasis on making employment more attractive in order to ensure that the agent is hired and output $\alpha + \beta \underline{e}$ is produced. Then the principal offers a higher fixed wage in order to increase the probability of acceptance. The principal incurs the monitoring costs more frequently, so monitoring output becomes more costly. Holding β constant, this implies that monitoring becomes less attractive. The last observation implies that not monitoring and paying a fixed wage can be attractive, even if monitoring is possible and even if a contract that ensures more productivity through performance-related pay can be written.

For our last result, we define as a charity an organization that is characterized by the following two features. Firstly, there is no monitoring by the principal ($\delta = K = 0$). Secondly, agents do not receive a positive wage ($W \leq 0$). In fact, they may be required to make monetary contributions in addition to the labour they supply to the organization. Recall that $\hat{\beta} = \frac{3}{4}(1 - \alpha)$.

Proposition 15 ***(Charities):*** *For those parameterizations where* $\beta_L \leq \beta \leq \hat{\beta}$*, the optimal organizational form is a charity. Whenever the agent is hired by a charity, she supplies maximum effort* ($e^* = \overline{e}$)*.*

Different allocations are displayed in Figure 2. The downward sloping line β_L separates those parameter combinations where monitoring is optimal ($\beta \leq \beta_L$, area I) from those where monitoring is not optimal. Note that in area I the agent receives a negative performance fee $s < 0$. The horizontal line indicates the value $\hat{\beta} = \frac{3}{4}(1 - \alpha)$ (the figure assumes $\alpha = 0.1$), so all values below $\hat{\beta}$ and above β_L (area II) indicate charities where monitoring is absent and the agent works for a non-positive wage. All parameter combinations above β_L and $\underline{\beta}$ (area III) refer to organizations where the agent works for a positive fixed wage (no monitoring). In area IV ($\beta \geq \beta_H$) all solutions have monitoring and a positive performance fee, so this area represents conventional companies.

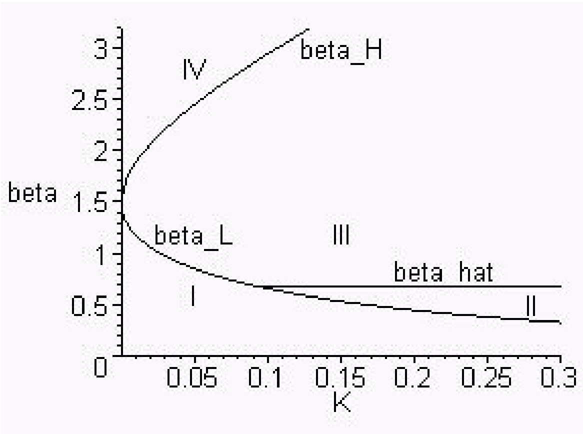

Figure 2: Contracting solutions for $\alpha = 0.1$. The functions β_L and β_H represent the smaller and larger root, respectively, from (54). $\hat{\beta} = \frac{3}{4}(1-\alpha)$.

6 Conclusion

In this paper we have analyzed a simple principal-agent model where the agent's intrinsic work motivation can be positive or negative and is unobserved by the principal. Although parameters vary continuously, two distinct types of organizations emerge:

- "Charities" or volunteer organizations whose members all supply maximum effort, are not monitored, and are required to make a contribution to the organization in addition to their effort.
- Conventional "companies" where all employees are hired for a positive wage, may be monitored, receive incentive payments, and may still not supply the highest level of effort.

Our setup is simple and a number of other issues that are relevant to this subject have been excluded. Clearly, some organizations employ volunteers and paid employees at the same time. In our model, the probability can be interpreted as measuring the size of the labor force which the principal employs. However, we have abstracted from multiple agents. Also, organizations differ in more respects than the classification here that was mainly predicated on productivity and the output-elasticity of effort. For example, organizations may differ with respect to the complementarity between agents. Most likely, organizations where members or employees are close substitutes and total output is less contingent on participation, will find it worthwhile to rely on many employees (who may supply only a small amount of time, e.g. in the form of part-time employment). On the other hand, firms with highly complementary assets need to secure their productivity and may prefer to reward them with output-contingent pay.

Compared to the usual principal-model we have allowed for hidden action and incentive contracts but ruled out stochastic aspects (except for type incertainty capturing incomplete information) and commonly known discrepancies in risk attitudes of principal and agent.[9] Regarding the first aspect one could generalize our model by assuming that high effort increases the probability of higher output levels but that for all effort levels all output levels are possible. If only output can be monitored, this would therefore just allow to update the principal's beliefs concerning the agent's effort rather than revealing it. If effort can be monitored directly, nothing much should change.

[9]Concerning the latter aspect we are sceptical whether discrepancies in risk attitudes will ever be commonly known. Experimentally one would have, for instance, to induce this by employing the binary lottery-technique (see Roth and Malouf, 1979) so that the (by the participants) imported own risk attitudes would be avoided what one may try to make commonly known.

References

[1] Akerlof, G. and J. Yellen (1990): The Fair Wage-Effort Hypothesis and Unemployment, *Quarterly Journal of Economics,* 105, 255-283.

[2] Akerlof, G. (1982): Labor Contracts as Partial Gift Exchange, *Quarterly Journal of Economics,* 97, 543-69.

[3] Frey, B. (1997): *Not Just for the Money. An Economic Theory of Personal Motivation*, Cheltenham: Edward Elgar Publishing.

[4] Frey and Osterloh (1997): Sanktionen oder Seelenmassage? Motivationale Grundlagen der Unternehmensführung, *DBV Deutsche Betriebswirtschaft 3,* 307-321.

[5] Fudenberg and Tirole (1992): *Game Theory.* Cambridge (Mass), MIT Press.

[6] Gächter, S. and E. Fehr (2001): Fairness in the Labour Market - A Survey of Experimental Results, Friedel Bolle and Marco Lehmann-Waffenschmidt (eds.), *Surveys in Experimental Economics. Bargaining, Cooperation and Election Stock Markets.* Physica Verlag.

[7] Grossman, Sanford J. and Hart, Oliver D.,1983, An Analysis of the Principal-Agent Problem, *Econometrica* 51, 7-45

[8] Güth, W., W. Klose, M. Königstein, and J. Schwalbach (1998): An experimental study of a dynamic principal-agent relationship, *Managerial and Decision Economics* 19, 327-341.

[9] Güth, W., S. Kröger and H. Normann (2002): Durable-Goods Monopoly with Privately Known Impatience, *Max Planck Institute for Research into Economic Systems, Papers on Strategic Interaction # 30-2002*

[10] Güth, W. and K. Ritzberger (1998): On durable goods monopolies and the coase-conjecture, *Review of Economic Design,* 3, 215-236.

[11] Güth, W, M. Willinger and A. Ziegelmeyer (2002): Agency Problems and Market Competition - A Survey of Theoretical and Experimental Results, *Max Planck Institute for Research into Economic Systems, Papers on Strategic Interaction # 31-2002*

[12] Harsanyi, J. C. (1967/68): Games with incomplete information played by 'Bayesian' players, part I: The basic model, part II: Bayesian equilibrium points, part III: The basic probability distribution of the game, *Management Science* 14, 159-182, 320-334, 486-502

[13] Murphy, Kevin J. Executive Compensation, 1998, mimeo, University of Southern California

[14] Opp, K.-D. (1984): Rational Choice and Sociological Man, *Jahrbuch für Neue Politische Ökonomie,* (eds.) E. Boettcher, Ph. Herder-Dornreich, K.-E. Schenk, Tübingen (Mohr Siebeck), 1-16.

[15] Opp, K.-D. (1986): Soft Incentives and Collective Action: Participation in the Anti-Nuclear Movement, *British Journal of Political Science,* vol. 16, 87-112.

[16] Opp, K.-D. (1988): Grievances and Participation in Social Movements, *American Sociological Review,* vol. 53, 853-864.

[17] Opp, K.-D., P. and P. Hartmann (1989): The Rationality of Political Protest. A Comparative Analysis of Rational Choice Theory, *Boulder, Colorado,* Westview Press.

[18] Roth, A. E. and E. F. Malouf (1979): Game-Theoretic Models and the Role of Information in Bargaining, *Psychological Review,* 86:6, 574-94.

[19] Sadowski, D., K. Pull and M. Schneider (1998): Vertrauen: Voraussetzung oder Ergebnis effizienter Arbeitsbeziehungen? - Gutenbergs Solidaritätsaxiom und die institutionenökonomische Unternehmenstheorie, *Band zur Tagung der Erich Gutenberg-Arbeitsgemeinschaft Köln e.V., anläßlich des 100. Geburtstages Erich Gutenbergs.*

7 Appendix: Proofs

Proof of Lemma 1:

Log-concavitiy means that $\ln \Psi(c)$ is concave. Then

$$\frac{\partial^2 \ln \Psi(c)}{\partial c^2} = \frac{\psi'(c)\Psi(c) - \psi(c)^2}{\Psi(c)^2} < 0 \tag{34}$$

However, we also have that $\Psi(c)/\psi(c)$ is monotonically increasing if

$$\frac{\partial}{\partial c}\left(\frac{\Psi(c)}{\psi(c)}\right) = \frac{\psi(c)^2 - \psi'(c)\Psi(c)}{\psi(c)^2} > 0 \tag{35}$$

Clearly, the conditions for (34) and for (35) are both that $\psi'/\psi < \psi/\Psi$. ■

Proof of Proposition 2:

Optimality at any point other than the discontinuity at $W = 0$ requires that the first order condition (9) holds as an equality. Optimality at $W = 0$ requires that the objective is increasing to the left of W and decreasing to the right of W. Generically, only one of the candidate solutions maximizes $Ev(W)$. ■

Proof of Proposition 3:

First, we need to separate the candidate solutions according to the sign of W^*. We obtain:

$$W^* \leq 0 \Longleftrightarrow \beta \leq \frac{\overline{e}}{2} - \frac{\alpha}{\overline{e}} = \overline{\beta} \ , \tag{36}$$

$$W^* > 0 \Longleftrightarrow \beta > \frac{\underline{e}}{2} - \frac{\alpha}{\underline{e}} = \underline{\beta} \ . \tag{37}$$

Then $\underline{\beta} < \overline{\beta}$ whenever $\overline{e} > \underline{e} > 0$ and $\alpha \geq 0$. The remaining statements follow immediately. ■

Proof of Proposition 4:

We need to show that the difference between $Ev(W^*)|_{W^*\leq 0}$ and $Ev(W^*)|_{W^*>0}$ as given in (12) are monotonic in β. Hence, we obtain:

$$\frac{\partial}{\partial \beta}\left(Ev(W^*)|_{W^*>0} - Ev(W^*)|_{W^*\leq 0}\right)$$
$$= \frac{\overline{e}}{4} - \frac{\alpha}{2\overline{e}} + \left(\frac{2\alpha - \underline{e}^2}{4\underline{e}}\right) \ . \tag{38}$$

Evaluate this expression at $\alpha = 0$, then we obtain

$$\frac{\partial}{\partial \beta}\left(Ev\left(W^*\right)|_{W^*>0} - Ev\left(W^*\right)|_{W^*\leq 0}\right)\Big|_{\alpha=0} = \frac{\overline{e}-\underline{e}}{4} > 0 \ .$$

We also obtain:

$$\frac{\partial^2}{\partial \alpha \partial \beta}\left(Ev\left(W^*\right)|_{W^*>0} - Ev\left(W^*\right)|_{W^*\leq 0}\right)\Big| = \frac{\overline{e}-\underline{e}}{2\overline{e}\underline{e}} > 0 \ .$$

Hence, $\frac{\partial}{\partial \beta}\left(Ev\left(W^*\right)|_{W^*>0} - Ev\left(W^*\right)|_{W^*\leq 0}\right)$ is positive for all $\alpha \geq 0$. Therefore, $Ev\left(W^*\right)|_{W^*>0} - Ev\left(W^*\right)|_{W^*\leq 0}$ can change signs only once. Clearly, $Ev\left(W^*\right)|_{W^*>0} > Ev\left(W^*\right)|_{W^*\leq 0}$ for β sufficiently large. If $Ev\left(W^*\right)|_{W^*>0} > Ev\left(W^*\right)|_{W^*\leq 0}$ for $\beta = 0$, then $\hat{\beta} = 0$, otherwise $\hat{\beta} > 0$. ■

Proof of Proposition 5:

We use the definition of $\hat{W}$ from (20) to verify that:

$$\bar{c} \ < \ \hat{c}_M\left(\underline{e}\right) \Longleftrightarrow W > \hat{W} \ , \qquad (39)$$

$$\bar{c} \ < \ \hat{c}_M\left(\overline{e}\right) \Longleftrightarrow W > \hat{W} \ . \qquad (40)$$

All agents with $c \leq \hat{c}_M\left(e^*\right)$ accept the contract and choose $e = e^*$. Then $W \geq \hat{W}$ is a necessary and sufficient condition to accept the contract. We can easily verify that:

$$\hat{c}_M\left(\underline{e}\right) > \hat{c}_M\left(\overline{e}\right) \Longleftrightarrow W > \hat{W} \ . \qquad (41)$$

Hence, for all acceptable contracts we have $\hat{c}_M\left(\underline{e}\right) \geq \hat{c}_M\left(\overline{e}\right)$, where the equality holds only if the constraint $W \geq \hat{W}$ is binding. Hence, we have that

$$\bar{c} < \hat{c}_M\left(\overline{e}\right) < \hat{c}_M\left(\underline{e}\right) \qquad (42)$$

if $W > \hat{W}$, and

$$\bar{c} = \hat{c}_M\left(\overline{e}\right) = \hat{c}_M\left(\underline{e}\right) \qquad (43)$$

if $W = \hat{W}$.

Case 1: $W > \hat{W}$. Then all types $c \leq \bar{c}$ have $u_c(e^*) \geq 0$. They will accept the contract and choose $e = \overline{e}$. All types $\bar{c} < c \leq \hat{c}_M(\underline{e})$ will accept the contract and choose $e = \underline{e}$. All types $c > \hat{c}_M(\underline{e})$ will reject the contract.

Case 2: $W = \hat{W}$. Then all types $c \leq \bar{c} = \hat{c}_M(\overline{e})$ will accept the contract and choose $e = \overline{e}$. All types $c > \bar{c} = \hat{c}_M(\underline{e})$ will reject the contract.

Cases 1 and 2 together demonstrate the result. ■

Proof of Lemma 6:

We simply substitute for $\hat{c}_M(\overline{e})$ from (19) and (10) and for $\bar{c}$ from (17) to obtain, after simplification:

$$Ev_M(C) = \left(\frac{s\beta}{\overline{e}+\underline{e}} + \frac{1}{2}\right)\left(\alpha + \beta\overline{e}(1-s) + \beta s\frac{\overline{e}\underline{e}}{\overline{e}+\underline{e}} - K\right) \tag{44}$$

and solve for the first order conditions for s to obtain (22). Substituting the solution for s back into the objective gives (23). ■

Proof of Lemma 7:

We rewrite the objective with $\Psi(\bar{c}) = \frac{s\beta}{\overline{e}+\underline{e}} + \frac{1}{2}$ and $\Psi(\hat{c}_M(\underline{e})) = \frac{W+s\pi(\underline{e})}{\underline{e}^2} + \frac{1}{2}$ and evaluate the first order conditions to obtain (25) and (26) upon solving. ■

Proof of Lemma 8:

The constraint $W_+^* > \overline{W}$ is satisfied (from (20) and (26)) whenever (27) is satisfied from direct computation. ■

Proof of Proposition 9:

Equation (28) follows from (19) by direct substitution for s and W, using the results of Lemmas 7 and 6. Substituting (28) into (10) gives (29). ■

Proof of Proposition 10:

The first best prescribes that all agents $c \leq T$ are hired in equilibrium. With distribution (10), the probability of acceptance of the first-best optimal contract is therefore $\frac{T+1}{2}$. However, from (31) and (29) we obtain

$$\Psi(\hat{c}_M(e^*)) = \frac{\pi(e^*) - K + (e^*)^2/2}{2(e^*)^2} = \frac{T+1}{4} \tag{45}$$

by direct computation. ■

Proof of Lemma 11:

The proof proceeds in three steps.

Step 1: If the principal monitors and $\overline{e} = 2$ and $\underline{e} = 1$, then the maximized objective is:

$$\frac{\beta^2}{3} + \beta\left(\frac{\alpha - K + 1}{2}\right) + \frac{K^2}{4} - K\left(\frac{\alpha}{2} + \frac{1}{4}\right) + \frac{\alpha^2 + \alpha + 1}{4} \quad . \tag{46}$$

Similarly, without monitoring, we obtain:

$$\left(\frac{\alpha + \beta}{2} + \frac{1}{4}\right)^2 + \frac{\beta}{2} \tag{47}$$

from (12).[10] Then (32) is the difference between (46) and (47) by direct computation.

Step 2: From (23), the maximized objective for $K > \alpha + \frac{2}{3}\beta$ is

$$\frac{1}{4}\left(\frac{\alpha - K}{2} + \beta + 1\right)^2 \quad , \tag{48}$$

which gives (33) as the difference between (48) and (47) by direct computation.

Step 3: Finally, for $K = \alpha + \frac{2}{3}\beta$, both expressions, (32) and (33) are equal to

$$-\frac{1}{12}\left(\frac{5}{3}\beta^2 + 5\beta + 6\alpha\beta + 3\alpha^2 + 3\alpha - \frac{9}{8}\right) \quad . \tag{49}$$

This completes the proof. ■

Proof of Proposition 12:

Differentiating (32) with respect to K gives:

$$-\frac{\alpha + \beta - K}{2} - \frac{1}{4} \quad , \tag{50}$$

which is negative for K sufficiently small. Specifically, for $K = \alpha + \frac{2}{3}\beta$ we obtain

$$-\frac{\beta}{6} - \frac{1}{4} \quad , \tag{51}$$

[10] We neglect the case where $W^* < 0$ would be optimal in (12).

which demonstrates the result for those values of K where (27) holds. Also, differentiating (33) with respect to K gives

$$-\frac{1}{8}(\alpha + 2\beta + 2 - K) \quad , \tag{52}$$

which is negative for all $K < \alpha + 2\beta + 2$. For all $K \geq \alpha + 2\beta + 2$ the surplus from production is strictly negative. This shows that the claim is true if (27) does not hold. ■

Proof of Proposition 13:

(i) Assume (27) holds. Differentiating (32) with respect to β gives

$$\frac{1}{6}\beta - \frac{1}{4} - \frac{1}{2} \quad , \tag{53}$$

which is positive if and only if $\beta \geq \frac{3}{2} + 3K$. Also, (32) equals zero for

$$\beta = \frac{3}{2} + 3K \pm \sqrt{12K + 6K^2 + 6K\alpha} \quad . \tag{54}$$

These roots define the interval $[\beta_L, \beta_H]$. Clearly, (32) is negative for all and only those values of β within this interval.

(ii) If (27) is violated, we need to differentiate (33) with respect to β to obtain

$$-\frac{\alpha + 1 + K}{4} \quad , \tag{55}$$

which demonstrates the result. ■

Proof of Proposition 14:

We need to distinguish the cases where (27) holds and where it does not hold. In the former case, differentiating (32) with respect to α, we obtain

$$-\frac{1}{2}K \quad . \tag{56}$$

Otherwise, if (27) does not hold, we differentiate (33) with respect to α to obtain

$$-\frac{3}{8}\alpha - \frac{1}{8}\beta - \frac{1}{8}K \quad , \tag{57}$$

which demonstrates the result. ■

Proof of Proposition 15:

>From Proposition 13 and the definition of $\underline{\beta}$ in (54), any $\beta \in [\beta_L, \beta_H]$ implies an optimal contract without monitoring. Moreover, from Proposition (3), for any $\beta \leq \frac{3}{4}(1-\alpha)$ such a contract has a non-positive wage. From (54), the interval $\beta_L \leq \beta \leq \hat{\beta}$ is non-empty for K sufficiently large and α sufficiently small. ■