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Solar Radiation–The Estimation of the Optimum Tilt Angles for South-Facing Surfaces in Pristina

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Abstract: Solar energy is derived from photons of light coming from the sun in a form called radiation. Solar energy finds extensive application in air and water heating, solar cooking, as well as electrical power generation, depending on the way of capturing, converting and distribution. To enable such application, it is necessary to analyze the horizontal tilt angle of horizontal surfaces – in order that when the solar energy reaches the earth surface to be completely absorbed. This paper tends to describe the availability of solar radiation for south-facing flat surfaces. The optimal monthly, seasonal, and annual tilt angles have been estimated for Pristina. The solar radiation received by the incident plane is estimated based on isotropic sky analysis models, namely Liu and Jordan model. The annual optimum tilt angle for Pristina was found to be 34.7°. The determination of annual solar energy gains is done by applying the optimal monthly, seasonal and annual tilt angles for an inclined surface compared to a horizontal surface. Monthly, seasonal and annual percentages of solar energy gains have been estimated to be 21.35%, 19.98%, and 14.43%. Losses of solar energy were estimated by 1.13 % when a surface was fixed at a seasonal optimum tilt angle, and when it was fixed at an annual optimum tilt angle, those losses were 5.7%.

Keywords: solar energy; gains; estimation; tilt angle; south-facing; surface; Pristina

1. Introduction

Because of a significant increase in energy demands, conventional energy sources are being violently consumed, leading to an increase of pollutants, which are released from the burning of fossil fuels. Knowing that solar equipment's do not have moving parts, they are considered to have a greater lifetime and do not cause pollution compared to other energy sources.

Therefore, solar energy is considered as one of the best solutions.

In most cities of Kosovo, the maximum global solar radiation is reached during July, whereas the minimum during December.

The performance of solar equipment's (i.e. solar collectors for water and air heating, power generation, photovoltaic systems, etc.), is closely related to the inclined angle from the surface that absorbs the light of the sunshine (i.e. with their placement plane).

Sun tracking systems are used to increase the acceptance of solar radiation. The usage of these systems has a significant cost to the normal operation of an entire system, due to the consumption of a considerable energy generated.

Hence, the estimation of the optimal tilt angle has a crucial effect on the solar technology. Additionally, trackers need periodic maintenance and calibration and require input energy for their operation which is in the range of 5–10% of the energy produced (Eldin et al). (2016) [1]. Further, trackers are made up of sophisticated mechanical parts which add to capital cost and an increase in cost of absolute power produced from solar PV panels (Sinha and Chandel, 2016) [2]. Other method readily suggested by researchers is to optimize the orientation of flat surfaces at optimum tilt

inclination (β_{opt}) (Yakup et al., 2001) [3]. Vieira et al. (2016) [4] performed an experimental study which suggested that the sun tracking panel exhibited a low average gain in power generated, relative to the fixed panel. In another study conducted by Sinha and Chandel [2], it was reported that the horizontal axis weekly adjustment tracking systems and vertical axis continuous adjustment tracking systems, produced less energy annually than the existing PV systems at fixed tilt. Optimum tilt inclination can be adjusted daily, monthly, seasonally, bi-annually or annually for maximizing the performance of the device in use (Ahmad and Tiwari, 2009) [5].

In the city of Shterpca, the annual solar radiation is estimated to be 1333.7 kWh/m²/year, while in the city of Gjakova 1495.1 kWh/m²/year. Furthermore, knowing the geographical position of the aforementioned cities, we can accept an average value of solar radiation of 1400 kWh/m²/year for the climate conditions of Kosovo (Ministry of Energy and Mining of Kosovo, 2010) [6].

This solar radiation potential can be utilized in desalination, solar-thermal collectors, building heating, day-lighting, and Photovoltaic (PV) Cells etc.. Researchers are therefore concerned to maximize the amount of useful energy that can be extracted through the incoming solar radiations. It is believed that proper installation of these devices can make a remarkable change in the observed performance. Hence, climatology, latitude, orientation, tilt angle, azimuth angles and the usage over a period in a specific geographical region affect the performance of the abovementioned devices (Yakup and Malik, 2001) [3].

The tilt of a surface (β) is one of the significant factors that considerably affect the availability of solar radiation on a flat surface. Optimization of the performance of solar-based devices requires option-like solar tracking equipment, which follow trajectories of Sun's motion to enhance incident radiation (Ahmad et al., 2016) [7] and (Okoye et al., 2016) [8]. However, these options are not always economical.

Optimization of the tilt angle has been performed for various locations in European countries including Turkey (Bakirci, 2012) [9], (Ertekin, 2008) [10], Romania (Stanciu et al., 2016) [11], Austria and Germany (Hartner et al., 2015) [12], Italy (Calabr, 2013) [13], and Greece (Mehleri et al., 2010) [14]. For Middle Eastern countries such as Oman (Kazem et al., 2013) [15], (Jafari and Javaran, 2012) [16], (Jafarkazemi et al., 2012) [17] (Moghadam et al., 2011) [18], Abu Dhabi (Jafarkazemi and Saadabadi, 2013) [19], Saudi Arabia (Riyadh) (UAE; Tamimi and Sawayan, 2012) [20], Saudi Arabia (Madinah) (Benghanem, 2011) [21], Egypt (Elminir et al., 2006) [22], Joran (Alatarawneh et al., 2006) [23], Jordan (Shariah et al., 2002) [24], Syria (Skeiker, 2009) [25]. For Asian Countries, such as Pakistan (Khahro et al., 2015) [26], Bangkok (Krishna et al., 2015) [27], Indonesia (Handoyo et al., 2013) [28], Hong Kong (Li and Lam, 2007) [29], China (Tang and Wu, 2004) [30], and in Nigeria (Eke, 2011) [31]; finally including Canada, a study that was conducted by Siraki and Pillay, 2012) [32].

1.1. Location under study

Pristina – the capital city – (42.65° N, 21.15° E and 573 m a.s.l.) is situated in the north-east of Kosovo. Pristina has a humid continental climate with maritime influences.

The city features warm summers and relatively cold snowy winters. According to a study conducted by the Ministry of Energy and Mining of Kosovo of Kosovo MEM (2010) [6] for zones with solar radiation potential, Kosovo has been divided into four zones of rough solar radiation.

Population density in Kosovo is greater in the central and western parts compared to the eastern parts. Hence, the division into three zones in an acceptable approximation since the solar radiation in Zone 3, and Zone 4 do not change much, as well as the opportunities to widely use solar energy in Zone 4 (which is smaller regarding surface area) are practically not that greater. The highest value of solar radiation are shown in Zone 1 while the lowest appear in Zone 4. The division of municipalities by area of solar radiation intensity is presented in Table 1. MEM (2010) [6].

Table 1. Division of Kosovo municipalities according to climate sub-zones MEM (2010) [6]

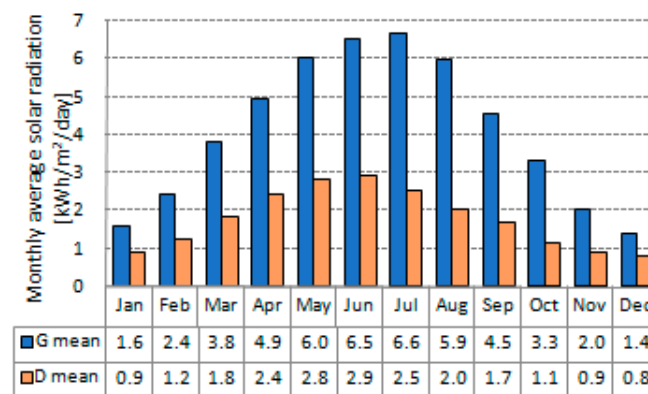
Nr	Zone 1	Zone 2	Zone 3	Zone 4
1	Peje	Prizren	Podujeve	Gjilan
2	Deçan	Dragash	Kamenice	Viti
3	Gjakove	Mitrovice	Istog	Kaçanik
4	Kline	Skenderaj	Zubin Potok	Shterpce
5	Rahovec	Lipjan	Leposaviç	
6	Suhareke	Pristine	Zveçan	
7	Malisheve	Ferizaj	Vushtrri	

2. Methodology of Analysis Concepts

2.2 Solar radiation data

In this paper, we used meteorological data for daily mean global and diffuse radiation on a horizontal plane, which are taken from the Meteorological Institute of Kosovo and are presented in Figure 1. The surface reflection factor is assumed to be 0.20.

The maximum values of monthly mean solar radiation are reached in July and the minimum values in December.

**Figure 1.** Monthly means daily global radiation G_m and diffuse radiation D_m on horizontal plane of Pristina**Figure 2.** Global horizontal radiation in Kosovo**Figure 3.** Direct normal radiation in Kosovo

Source: <http://globalsolaratlas.info/downloads/kosovo> [33]

In Figure 2 is presented the annual average global radiation on a horizontal plane for Kosovo. As can be seen from Figure 1, the highest value of average global solar radiation from all cities of Kosovo is Gjakova. In Figure 3 is presented the annual average direct normal radiation on a horizontal plane for Kosovo cities.

3. Estimation of Solar Radiation on the Inclined Surface

Data on monthly mean global solar radiation for an inclined surface are necessary for the designing of solar energy systems.

These data are often not available. Therefore, we need tend to estimate the data by considering the monthly average global solar radiation on a horizontal plane, (presented in Figure 3) which is the most important parameter for estimating the optimum tilt angle.

In this study, we present a simple and universal method for determining the mean monthly global radiation based on the methodology released by Surface *Meteorology and Solar Energy* (SSE) (2016) [34]. The total solar energy received on an inclined surface is the sum of the beam, diffuse, and reflected radiation.

Thus, the monthly average total solar radiation (in kWh/m²-day) for an inclined surface is given by this equation:

$$\bar{H}_T = \bar{H}_B + \bar{H}_D + \bar{H}_R \quad (1)$$

Where mathematically, the optimal value of the tilt angle (β_{opt}) is determined by differencing Eq. 1 depending on the angle of inclination (β).

$$\frac{d}{d\beta}(\bar{H}_T) = 0 \quad (2)$$

$$\bar{H}_T = \bar{R} \cdot \bar{H} \quad (3)$$

\bar{R} is the ratio between the mean monthly global radiation on an inclined surface than for the horizontal one. \bar{R} ratio, defined by Bakirci (2012) [9], Liu and Jordan (1962) [35]:

$$\bar{R} = \left(1 - \frac{\bar{H}_D}{\bar{H}}\right) \cdot \bar{R}_b + \frac{\bar{H}_D}{\bar{H}} \cdot \left(\frac{1 + \cos \beta}{2}\right) + \rho \cdot \left(\frac{1 - \cos \beta}{2}\right) \quad (4)$$

\bar{H} and \bar{H}_d are the monthly average daily global and diffuse radiations on a horizontal surface; \bar{R}_b is the ratio of the mean daily direct radiation on an inclined surface to that on a horizontal surface, and the parameters correlated by ground reflection coefficient (albedo $\rho=0.2$); β is the tilt angle. The beam radiation falling on an inclined surface is given by Liu and Jordan (1962) [35]:

$$\bar{H}_B = (\bar{H} - \bar{H}_d) \cdot \bar{R}_b \quad (5)$$

Reflected radiation is the part of total solar radiation that is reflected by the surface of the earth, and by any other surface intercepting object such as trees, terrain or buildings on to a surface exposed to the sky is termed as ground reflected radiation [36]. Reflected radiation on an inclined surface is given by:

$$\bar{H}_R = \bar{H} \cdot \bar{R}_r \quad (6)$$

\bar{R}_r is the reflected conversion factor:

$$\bar{R}_r = \rho \cdot \left(\frac{1 - \cos \beta}{2}\right) \quad (7)$$

where ρ is the constant which depends on the type of ground surrounding tilted surfaces and is called the ground reflectance or albedo. The value of albedo most commonly used is $\rho= 0.2$ for hot

and humid tropical locations, $\rho=0.5$ for dry tropical locations, and $\rho=0.9$ for snow covered ground [37]. The ground reflection coefficient is assumed 0.2 for the climate conditions in Pristina.

Diffused radiation (\bar{H}_D) is that fraction of total solar radiation which is received from the sun when its direction has been changed by atmospheric scattering [38]. The direction of diffused radiation is highly variable and difficult to determine. It is a function of the condition of cloudiness and atmospheric clearness which is extremely unpredictable. The diffused radiation fraction is the sum of three components namely isotropic, circumsolar, and horizon brightening. The isotropic diffuse radiation component is received evenly from the entire sky dome. The circumsolar diffuse part is received from the onward dispersion of solar radiation and concentrated in the section of the sky around the sun [39]. The horizon brightening component is concentrated near the horizon and it is most obvious in the clear skies [40]. In general, the diffuse fraction of radiation on inclined surface is composed of isotropic, circumsolar, and horizon brightening factors. In Liu and Jordan (1960) [41] model, it was assumed that the diffuse radiation is isotropic only; whereas, circumsolar and horizon brightening were taken as zero.

Diffuse radiation falling on an inclined surface is given by:

$$\bar{H}_D = \bar{H}_d \cdot \bar{R}_d \quad (8)$$

where \bar{R}_d is the diffuse conversion factor presenting the ratio of diffuse solar radiation on an inclined surface to diffuse solar radiation on a horizontal surface, given as:

$$\bar{R}_d = \left(\frac{1 + \cos \beta}{2} \right) \quad (9)$$

Data on monthly average daily global radiation which were taken from Figure 3., are used in the following expressions for determination of other parameters. The following expression determines the monthly average daily diffuse radiation [34]:

$$\frac{\bar{H}_d}{\bar{H}} = 0.96268 - (1.45200 \cdot K_T) + (0.27365 \cdot K_T^2) + (0.04279 \cdot K_T^3) + (0.000246 \cdot (SSHA)) + (0.001189 \cdot (NHSA)) \quad (10)$$

where SSHA is the sunset hour angle in degrees on the "monthly average day (n)", which can be calculated from the following equation:

$$SSHA = \omega'_s = \cos^{-1}(-\tan(\phi - \delta) \tan \delta) \quad (11)$$

NHSA is the noon solar angle from the horizon in degrees on the "monthly average day (n)" and can be calculated from the following relation:

$$NHSA = 90^\circ - |\phi - \delta| \quad (12)$$

The monthly average clearness index \bar{K}_T is the ratio of monthly average daily radiation on a horizontal surface to the monthly average daily extraterrestrial radiation and can be obtained from:

$$\bar{K}_T = \frac{\bar{H}}{H_0} \quad (13)$$

where, \bar{H}_0 is the monthly mean daily extraterrestrial radiation on a horizontal surface (Alboteanu et al., 2015) [42], which are calculated using:

$$\bar{H}_0 = \frac{24 \times 3600}{\pi} \cdot I_0 \cdot \left(1 + 0.033 \cos \left(\frac{360n}{365} \right) \right) \cdot \left(\cos \phi \cos \delta \sin \omega_s + \frac{2\pi\omega_s}{360} \sin \phi \sin \delta \right) \quad (14)$$

where I_0 is the solar constant (1367 W/m^2); n is the number of daily readings of the month and is counted from 1 January (1–365); ϕ is the geographic latitude of the location; ω_s is the sunrise (or

sunset) angle on a horizontal surface; δ is the declination of the sun. The “monthly average day” is the day of the month, whose solar declination is closest to the average declination for that month [43]. Declination angle (δ) ranges from a maximum value of $+23.45^\circ$ on June 21st–22nd, to a minimum value -23.45° on December 20th–21st. The declination value is zero on March 22nd and September 22nd of the year, see Table 2.

Table 2. Monthly representative day and its corresponding declination (δ in degrees)

Month	January	February	March	April	May	June
Day (n)	17	47	75	105	135	162
Declination (δ)	-20.92	-12.05	-2.43	9.41	18.79	23.09
Month	July	August	September	October	November	December
Day (n)	198	228	258	288	318	344
Declination (δ)	21.18	13.46	2.22	-9.60	-18.91	-23.05

According to Cooper (1969) [44], declination angle is calculated using the following relation:

$$\delta = 23.45 \cdot \sin \left[360 \frac{(284 + n)}{365} \right] \quad (15)$$

where n is the day of the year.

Various researchers have proposed numerous models which are classified as isotropic including Liu and Jordan (1960) [41], Tian (2001) [45], Koronakis (1986) [46] and Badescu (2002) [47] and anisotropic Hay (1979) [48], Reindl et al. (1990) [49], Klucher (1979) [50] models, Skartveit and Olseth (1986) [51], and Steven and Unsworth (1980) [52] to estimate solar radiation on inclined surfaces. However, according to Tang and Wu (2004) [30], measured diffuse solar radiation data gives better estimates of the optimum tilt angle. In this study, measured data of global and diffuse solar radiation has been utilized to calculate the optimum tilt angles. Liu and Jordan (1960) [41] (isotropic) model has been used to estimate total solar radiation on inclined flat surfaces facing south.

The angle of incidence for a surface oriented in any direction can be mathematically expressed by following relation (Duffie and Beckman, 2006) [53]:

$$\begin{aligned} \cos \theta = & \sin \delta \sin \phi \cos \beta - \sin \delta \cos \phi \sin \beta \cos \gamma + \cos \delta \cos \phi \cos \beta \cos \omega + \\ & + \cos \delta \sin \phi \sin \beta \cos \gamma \cos \omega + \cos \delta \sin \beta \sin \gamma \sin \omega \end{aligned} \quad (16)$$

For a surface in northern hemisphere facing south (i.e. $\gamma = 0^\circ$) Eq. (16) can be simplified as:

$$\cos \theta = \sin(\phi - \beta) \sin \delta + \cos(\phi - \beta) \cos \delta \cos \omega \quad (17)$$

For a horizontal surface ($\beta=0^\circ$), the angle of incidence (θ) becomes equal to zenith angle (θ_z). Substituting this value in Eq. (17), zenith angle can be written as:

$$\cos \theta_z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos \omega \quad (18)$$

The total solar energy received on an inclined surface is the sum of beam and diffuse radiations directly incident on a surface and reflected radiations (reflected by the surroundings).

According to Liu and Jordan (1960) [41], the beam conversion factor \bar{R}_b is given as:

$$\bar{R}_b = \frac{\cos(\phi - \beta) \cos \delta \sin \omega + (\pi / 180) \omega \sin(\phi - \beta) \sin \delta}{\cos \phi \cos \delta \sin \omega_s + (\pi / 180) \omega_s \sin \phi \sin \delta} \quad (19)$$

where ω is the sunrise (or sunset) hour angle for the inclined surface.

$$\omega = \min \left\{ \omega_s, \omega'_s \right\} \quad (20)$$

The hour angles at sunrise and sunset ω_s are very useful parameters. Considering that numerically these two angles have the same value the sunrise angle is negative, and the sunset angle is positive. Both can be calculated from the following expression:

$$\omega_s = \cos^{-1}(-\tan \phi \tan \delta) \quad (21)$$

If a surface is inclined from the horizontal, the Sun may rise over its edge after it has risen over the horizon. Therefore, the surface may shade itself for some days. The sunrise and sunset angles for an inclined surface (ω'_s) facing the equator (i.e. facing south for the northern hemisphere) is given by:

$$\omega'_s = \cos^{-1}(\tan(\phi - \beta) \tan \delta) \quad (22)$$

4. Results and Discussions

In this paper, Equations (1-22) are applied to determine the monthly mean daily global solar radiation on the south-facing inclined surface for the current location. By changing the tilt angle from 0° to 90° in steps of 0.1° , the optimal tilt angle is defined by the corresponding value of maximum solar radiation for a given period.

Using the procedure described in the previous Section 3, and based on the Liu and Jordan (1960) [41] model, estimations have been made to obtain the optimum monthly, seasonal, and annual tilt angles by corresponding to global solar radiation on an inclined surface. Table 3 presents the results for determining the optimum monthly, seasonal and annual tilt angles at certain periods.

Table 3. Monthly, seasonal, and yearly optimum tilt angles (β_{opt} in degrees) for Pristina city

Station	Period	Month					
		January	February	March	April	May	June
Pristina $\phi=42^\circ 39' N$	Monthly	64.8	56.5	43	25	9.2	0.3
	Seasonal	62.1	62.1	25.7	25.7	25.7	8.9
	Annual	34.7	34.7	34.7	34.7	34.7	34.7
Station	Period	Month					
		July	August	September	October	November	December
Pristina $\phi=42^\circ 39' N$	Monthly	4.8	19.2	37.3	54.3	63.8	66.4
	Seasonal	8.9	8.9	50.9	50.9	50.9	62.1
	Annual	34.7	34.7	34.7	34.7	34.7	34.7

For the location under study, the monthly optimal tilt angle has been estimated with the method described in Section 3. The results are shown in Table 4. The daily extraterrestrial radiation on a horizontal surface, clearness index, diffuse solar radiation on a horizontal plane, optimal tilt angle β_{opt} , monthly average daily global radiation in optimal tilt angle, and the comparison with (in percentage), has also been estimated. Table 4 shows the minimum and maximum value of, which correspond with December and July respectively.

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Table 4. \bar{H}_0 , \bar{K}_T , \bar{H}_d , β_{opt} , \bar{H}_T and comparison with \bar{H} in location Pristina city

Month	\bar{H}_0 (kWh / m ² / day)	\bar{K}_T	\bar{H}_d (kWh / m ² / day)	β_{opt} (°)	\bar{H}_T (kWh / m ² / day)	The comparison \bar{H}_d with \bar{H}_T (%)
January	3.60	0.45	0.69	64.8	3.14	95.03
February	5.04	0.49	0.96	56.5	3.89	58.13
March	7.24	0.52	1.36	43	4.81	26.58
April	9.17	0.54	1.76	25	5.33	7.24
May	11.34	0.53	2.29	9.2	6.07	0.83
June	11.62	0.56	2.30	0.3	6.52	0.00
July	11.98	0.56	2.36	4.8	6.70	0.15
August	10.32	0.58	1.90	19.2	6.23	4.01
September	7.84	0.58	1.36	37.3	5.46	19.21
October	5.86	0.57	1.00	54.3	5.12	53.75
November	4.14	0.50	0.77	63.8	3.97	92.72
December	3.38	0.42	0.65	66.4	2.90	104.23
Average	7.63	0.53	1.45	-	5.01	21.34
% Gain	-	-	-	-	21.35	-

For all the months and different tilt angles, monthly average total solar radiation was calculated using Eq. (1). The results have been plotted and are shown in Figure 4 for Pristina, where the tilt angle has been varied in the range of 0–90° (in steps of 10°). Figure 4 (a) shows the total solar radiation versus tilt angle for the months from January to June while Figure 4 (b) shows the same for the remaining months of July to December. It is apparent from the Figure 4 that the solar radiation is an intensive function of tilt angle. The calculated solar energy radiation incident on the flat surface is increased, with the increasing of horizontal position from 0° to an angle of inclination, but a further increase of the tilt angle of the flat surface will result in the decreasing of solar radiation received.

The result also indicated that the optimum angle varies with the months of the year. The maximum solar radiation is achieved for every month with unique optimum tilt angle. The optimum tilt angle increases in winter months and decreases to the minimum value in summer & autumn months, (see Figure 5). Figure 6 compares the average annual solar radiation on a tilted surface fixed at monthly, seasonal and annual optimum tilt angles for Pristina. As observed from the Figure 6, the annual average total solar radiation estimated at monthly optimum tilt angle is found to be the highest, followed by average annual solar radiation estimated at seasonal and annual optimum tilt angles value.

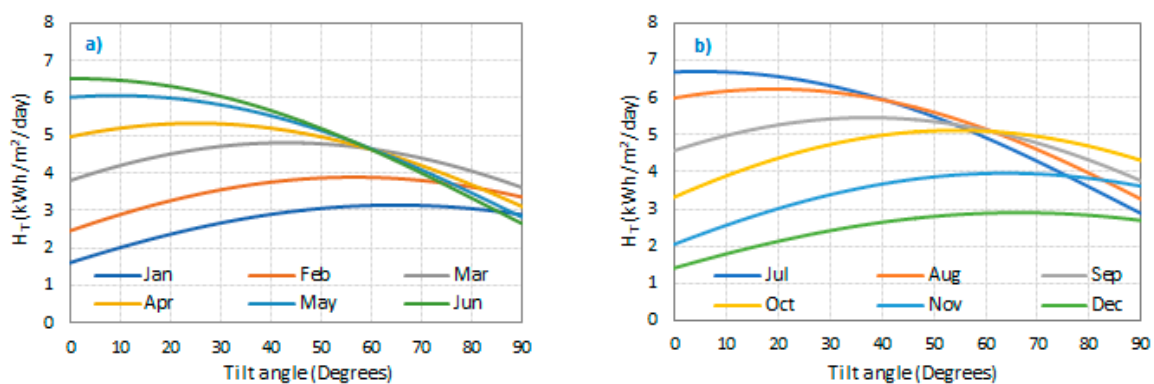


Figure 4. Monthly mean solar radiation data for Pristina when the tilt angle changes from 0° to 90°:
(a) January-June and (b) July-December

The monthly, seasonal and annual optimal tilt angles are also plotted in Figure 5.

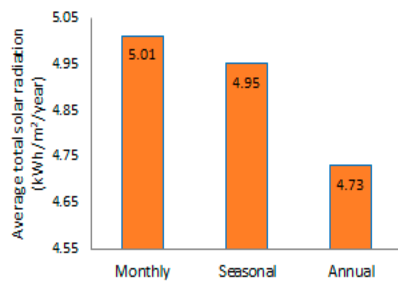


Figure 5. Average maximum total solar annual

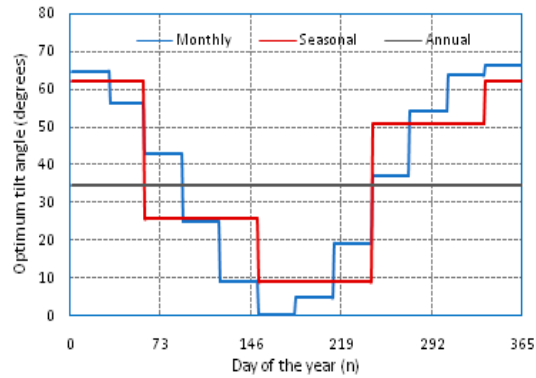


Figure 6. Variations of monthly, seasonal and

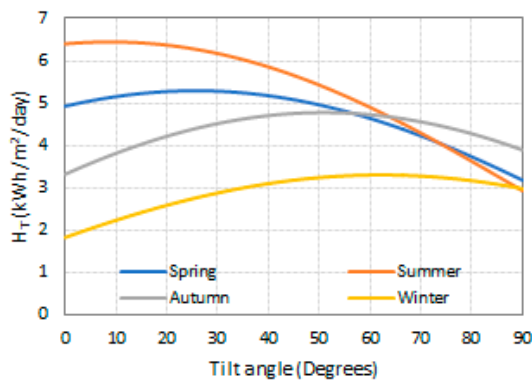


Figure 7. Seasonal mean solar radiation in Pristina for: Season 1: Spring; Season 2: Summer; Season 3: Autumn, Season 4: Winter, when the tilt angle changes from 0° to 90°. (Considering the azimuth angle=0° for maximum solar radiation).

Gains of solar radiation available on an inclined surface are defined by the following equation:

$$Gains (\%) = \left(\frac{\bar{H}_T(\beta = \beta_{opt,i})}{\bar{H}_T(\beta = 0)} - 1 \right) \times 100 \quad (23)$$

where i = monthly, seasonal and annual

Therefore, four seasonal optimum tilt angles were obtained (corresponding to each season) - 62.1° in winter, 25.7° in spring, 8.9° in summer, and 50.9° in autumn. Thus, winters have a higher value of optimum tilt angle, while summers observe a lower tilt angle value. Annual optimum tilt angle was calculated by using expressions from Section 3 and has resulted to be 34.7° for Pristina.

Annual average total gains of solar radiation on an optimally tilted angle of a surface in comparison to a horizontal surface are 21.35% (monthly optimum tilt angle), 19.98% (seasonal optimum tilt angle) and 14.43% (annual optimum tilt angle).

The difference between the total solar radiation at monthly, seasonal, and annual optimum tilt angles is very small. However, there is a loss in solar radiation available, when a surface is fixed on seasonal or annual optimum tilt angle in comparison to a fixed monthly optimum tilt angle.

Losses of solar radiation available on an inclined surface are defined by:

$$Losses (\%) = \left(1 - \frac{\bar{H}_T(\beta = \beta_{opt,j})}{\bar{H}_T(\beta_{opt}(monthly))} \right) \times 100 \quad (24)$$

where j = seasonal, annual.

Table 5. \bar{H}_d , β_{opt} , \bar{H}_T and the comparison with \bar{H} of Pristina for seasonal and annual tilt angles

Month	Seasons	\bar{H}_d (kWh/m ² /day)	Seasonal	Annual	Seasonal	Annual	The comparison	The comparison
			β_{opt} (°)	β_{opt} (°)	\bar{H}_T (kWh/m ² /day)	\bar{H}_T (kWh/m ² /day)	\bar{H}_T / \bar{H}_o (%) Seasonal	\bar{H}_T / \bar{H}_o (%) Annual
December	Winter	0.65	62.1	34.7	2.90	2.54	104.23	78.87
January		0.69	62.1	34.7	3.14	2.78	95.03	72.67
February		0.96	62.1	34.7	3.88	3.66	57.72	48.78
March	Spring	1.36	25.7	34.7	4.64	4.77	22.11	25.53
April		1.76	25.7	34.7	5.33	5.27	7.24	6.04
May		2.26	25.7	34.7	5.91	5.70	-1.83	-5.32
June	Summer	2.29	8.9	34.7	6.48	5.88	-0.61	-9.82
July		2.35	8.9	34.7	6.69	6.17	0.00	-7.77
August		1.90	8.9	34.7	6.16	6.07	2.84	1.34
September	Autumn	1.36	50.9	34.7	5.34	5.46	16.59	19.21
October		1.01	50.9	34.7	5.11	4.87	53.45	46.25
November		0.76	50.9	34.7	3.88	3.54	88.35	71.84
Average		1.45	-	-	4.95	4.73	19.98	14.43
% Gain		-	-	-	19.98	14.43	-	-

Losses of solar energy were estimated by 1.13 %, when a surface was fixed at the seasonal optimum tilt angle, and when it was fixed at the annual optimum tilt angle, those losses were 5.7 %.

For the location under study, the data have been estimated for the seasonal and annual optimal tilt angles with the method described in Section 3. Table 5 shows the results for the diffuse radiation on a horizontal plane; the annual and seasonal optimal tilt angles for each season; the monthly average daily global radiation for optimal tilt angle; and the comparison of H with HT in percentage.

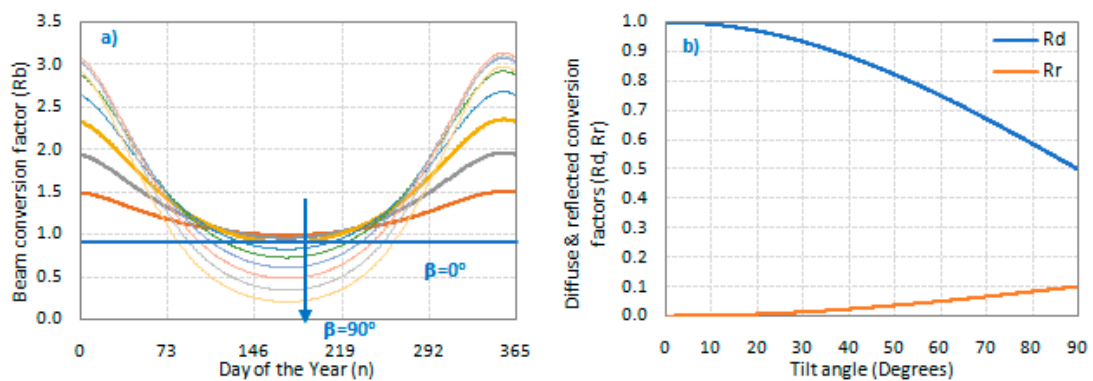


Figure 8. (a) Beam conversion factor at different tilt angles for Pristina,
(b) Diffuse and reflected conversion factors.

Figure 8.a describes the variation of beam conversion factor with days of the year at various tilt angles for the city of Pristina. Also, Figure 8.b shows the variation of diffuse and reflected conversion factors at different tilt angles.

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