Winding Turn-to-Turn Faults Detection of Fault-Tolerant Permanent-Magnet Machines Based on a New Parametric Model

Guohai Liu *, Wei Tang * and Wenxiang Zhao *

Abstract – This paper proposes a parametric model for inter-turn fault detection in a fault-tolerant permanent-magnet (FTPM) machine, which can predict the effect of the short-circuit fault to various physical quantity of the machine. For different faulty operations, a new effective stator inter-turn fault detection method is proposed. Finally, simulations of vector-controlled FTPM machine drives are given to verify the feasibility of the proposed method, showing that even single-coil short-circuit fault could be exactly detected.

Keywords: Fault-tolerant permanent-magnet machine, Fault detection, Short-circuit, Current harmonic

1. Introduction

Due to high efficiency, high power density and high reliability, fault-tolerant permanent-magnet (FTPM) machines have been widely studied for many applications such as aerospace and electric vehicle (EV) [1]-[3]. The major faults of the FTPM machines can be classified as air-gap eccentricity, bearing failures, demagnetized magnet and stator winding faults (open-circuit and short-circuit faults). Inter-turn short-circuit fault is one of the most common fault types which are usually related to insulation degradation and could develop into more serious results [4]. Also, it is one of the most different failures to diagnose, especially when single-turn short circuit occurs.

Many effective techniques have been presented to detect inter-turn short-circuit fault in the past years, and majority of them focused on induction machines [5]. Recently, some effective diagnostic methods for PM machines have been proposed such as electromagnetic torque waveform and the summation of phase voltages [6], reference voltages [7], high-frequency (HF) injection [8]. In [9], the change of the flux was used to identify the short-circuit fault, but an extra search coil should be placed in the stator teeth for this fault detection. The model of self-inductance and mutual-inductance was investigated in [10], however, the resistance of the shorted part was not considered. Very recently, a new five-phase FTPM machines were proposed for four-wheel independently driven EVs as shown in Fig. 1 [11], offering high fault-tolerant capability. Although there is a number of PM machine model with inter-turn fault [12], [13], the model of five-phase fault-tolerant machine with stator inter-turn fault is unavailable.

In this paper, a new dynamic model of the five-phase FTPM machine under short-circuit fault will be developed. The mathematical equations are derived in detail. Finally, simulated results are given for the faulty machine to verify the proposed inter-turn short-circuit fault detection method.

![Cross-section of five-phase FTPM machine.](Fig. 1)

2. Stationary and Rotating Frame Models

In this section, the five-phase FTPM machine models under healthy and inter-turn short-circuit fault conditions will be derived in stationary and rotating frames, respectively. The following assumptions are made.

1. The unbalance of machine itself is neglected.
2. The magnetic saturation is ignored.
3. The permeability of iron is infinite.
4. The symmetry condition is ideal.
2.1 Healthy Machine in Stationary Frame

The stator equation for a healthy five-phase FTPM machine in stationary frames is given as:

\[
[V_{sh}] = [R_{sh}] [i_{sh}] + \frac{d}{dt} [\psi_{sh}] \tag{1}
\]

The stator magnetic flux is:

\[
[\psi_{sh}] = [L_{sh}] [i_{sh}] + [\psi_{psm}] \tag{2}
\]

where \(\psi_{psm}\) is the PM flux linkage.

From (1) and (2), the general equation is calculated as:

\[
[V_{sh}] = [R_{sh}] [i_{sh}] + [L_{sh}] \frac{d}{dt} [i_{sh}] + [e_{sh}] \tag{3}
\]

where \([V_{sh}] = [V_a, V_b, V_c, V_d, V_e]^T\), \([i_{sh}] = [i_a, i_b, i_c, i_d, i_e]^T\),

\[
[R_{sh}] = \begin{bmatrix}
R_o & 0 & 0 & 0 & 0 \\
0 & R_o & 0 & 0 & 0 \\
0 & 0 & R_o & 0 & 0 \\
0 & 0 & 0 & R_o & 0 \\
0 & 0 & 0 & 0 & R_o
\end{bmatrix},
\]

\([L_{sh}] = \begin{bmatrix}
L & M_o & M_i & M_i & M_o \\
M_o & L & M_i & M_i & M_o \\
M_i & M_i & L & M_o & M_i \\
M_i & M_i & M_o & L & M_i \\
M_o & M_i & M_i & M_o & L
\end{bmatrix}\]

and \([e_{sh}] = [E_a, E_b, E_c, E_d, E_e]^T\), in which \(R_o\) is the stator resistance, \(L\), \(M_o\) and \(M_i\) are the phase self-inductance and mutual-inductance, the subscript, ‘0’ and ‘1’, represent the adjacent phase and nonadjacent phase, respectively. However, the machine used for fault-tolerant applications has a minimized mutual-inductance. The structure of the machine has been designed as shown in Fig. 1, and its fault-tolerant teeth could reduce the mutual-inductance. Hence, their relations are given as: \(M_0 = M_1 = M = 0\).

The electromagnetic torque equation in stationary frame is given as

\[
T_e = \frac{E_a i_a + E_b i_b + E_c i_c + E_d i_d + E_e i_e}{\Omega} \tag{4}
\]

where \(\Omega\) is the mechanical angular speed and \(E_i\) is back-EMF of phase-\(i\).

2.2 Healthy Machine in Rotating Frame

The Concordia/Clarke and Park transforms used to convert magnitudes from stationary frame to rotating frame have been successfully applied to the existing machine control schemes. It could be extended to multi-phase FTPM machines and the reference frame theory from n-phase to two-phase is given in [14].

Fig. 2 shows the coordinate transformation diagram. The transformation equation of convert variables from the stationary reference to the synchronously-rotating frame can be expressed as:

\[
[ X_{sh} ] = T(\theta) [ X_{abcde} ] \tag{5}
\]

where \(T(\theta)\) is the Extended Park transformation matrix, which is given as:

\[
T(\theta) = \begin{bmatrix}
\cos \theta & \cos(\theta - 2\pi/5) & \cos(\theta - 4\pi/5) & \cos(\theta - 6\pi/5) & \cos(\theta - 8\pi/5) \\
-\sin \theta & -\sin(\theta - 2\pi/5) & -\sin(\theta - 4\pi/5) & -\sin(\theta - 6\pi/5) & -\sin(\theta - 8\pi/5) \\
0 & 0 & 1 & 0 & 0 \\
\frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}}
\end{bmatrix}
\]

where \(\theta\) is the rotor electrical position.

By applying the Extended Park transformation matrix to the equations in stationary, the stator equations can be written as:

\[
V_{sh} = R_i i_j - w(L - M) i_j + (L - M) \frac{d i_j}{dt} \tag{7}
\]

\[
V_{sh} = R_i i_j + w(L - M) i_j - (L - M) \frac{d i_j}{dt} + w\psi_{psm} \tag{8}
\]

The electromagnetic torque equation is expressed as:

\[
T_e = \frac{5}{2} p [i_q \psi_{psm} + (L_q - L_y) i_y i_q] \tag{9}
\]

where \(p\) is the number of the pole pairs.
The effect of saliency on the rotor yields could be neglected in the Interior PM machine, the electromagnetic torque equation can now be evaluated as:

$$T_r = \frac{5}{2} P i_2 w_{PM}$$  \hspace{1cm} (10)

The mechanical equation is as follows:

$$T_c = J \frac{d\omega}{dt} + B\omega + T_L$$  \hspace{1cm} (11)

where $B$ is the frictional coefficient and $J$ is the moment of inertia.

### 2.3 Faulty Machine in Stationary Frame

An inter-turn fault is usually related to insulation failures between two adjacent windings. Fig. 3 shows a five-phase FTPM machine with inter-turn fault in phase $a$, in which $a_1$ and $a_2$ represent the healthy turns and the faulted turns, respectively, $r_f$ is used to simulate the fault insulation resistance.

![Diagram of the five-phase fault-tolerant PMSM with stator inter-turn faults in phase $a$.](image)

The relations of the parameters in the fault phase are expressed as:

$$\begin{cases}
R_{a_1} + R_{a_2} = R_a \\
E_{a_1} + E_{a_2} = E_a \\
L_{a_1} + L_{a_2} + 2M_{a_1a_2} = L_a
\end{cases}$$  \hspace{1cm} (12)

The stator equation for five-phase fault-tolerant FTPM machine with inter-turn fault in stationary frame is given as:

$$\begin{bmatrix} V_s' \\ I_s' \end{bmatrix} = \left[ \begin{bmatrix} R_s & 0 & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 & 0 \\ 0 & 0 & R_s & 0 & 0 \\ 0 & 0 & 0 & R_s & 0 \\ -\mu R & 0 & 0 & 0 & \mu R + r_f \end{bmatrix} \right] \begin{bmatrix} I_s' \\ V_s' \end{bmatrix} + \left[ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\mu L \end{bmatrix} \right]$$  \hspace{1cm} (20)

From the short-circuit loop in the Fig. 3, the equations could be obtained as:

$$\begin{cases}
V_{a_1} + V_{a_2} = V_a \\
V_{a_2} = r_f i_f \\
E_{a_2} = E_f
\end{cases}$$  \hspace{1cm} (14)

It is assumed that $\mu$ is the ratio of fault turns $n$ out of the total turns $N$ in phase $a$, resulting in $\mu = n/N$.

Then the faulty resistance $R_{a_2}$ and inductance which related to the fault winding $r_f$ and inductance which could be expressed as follows:

$$\begin{cases}
R_{a_2} = \mu R_s \\
L_{a_2} = \mu^2 L \\
M_{a_1a_2} = \mu (1 - \mu) L
\end{cases}$$  \hspace{1cm} (15)

$$M_{a_1b} = M_{a_1c} = M_{a_1d} = M_{a_1e} = \mu M$$  \hspace{1cm} (16)

Based on (12)-(16), the parameters of the FTPM machine can be derived as:

$$\begin{bmatrix} V_s' \\ I_s' \end{bmatrix} = \begin{bmatrix} V_a & V_b & V_c & V_d & V_e \end{bmatrix}'$$  \hspace{1cm} (17)

$$\begin{bmatrix} I_s' \\ V_s' \end{bmatrix} = \begin{bmatrix} i_s & i_b & i_c & i_d & i_e \end{bmatrix}'$$  \hspace{1cm} (18)

$$\left[ R_s \right] = \begin{bmatrix} R_s & 0 & 0 & 0 & 0 \\
0 & R_s & 0 & 0 & 0 \\
0 & 0 & R_s & 0 & 0 \\
0 & 0 & 0 & R_s & 0 \\
-\mu R & 0 & 0 & 0 & \mu R + r_f \end{bmatrix}$$  \hspace{1cm} (19)

The electromagnetic torque equation with intern-turn fault in stationary frame can be expressed as:

$$T_{sf} = \frac{E_s i_s + E_b i_b + E_c i_c + E_d i_d + E_e i_e - E_f i_f}{\Omega}$$  \hspace{1cm} (21)
2.4 Faulty Machine in Rotating Frame

When the FTPM machine is with the inter-turn fault, the fault current circulates in the short circuit loop only. Hence, the Extended Park transformation in fault condition will change as:

\[
T(\theta) = \frac{2\pi}{5}
\]

By applying the Extended Park transformation matrix to the stator equation (13), the result will be obtained as:

\[
\begin{bmatrix}
V_{d_{eff}}
\end{bmatrix} = T(\theta) \cdot \left[ R_c \right]^{-1} \cdot T^{-1}(\theta) \cdot \left[ i_{d_{eff}} \right]
\]

\[
+ T(\theta) \cdot \left[ L_c \right] \cdot \frac{d}{dt} \left[ i_{d_{eff}} \right] + T(\theta) \cdot \left[ L_c \right] \cdot T^{-1}(\theta) \cdot \frac{d}{dt} \left[ i_{d_{eff}} \right]
\]

\[
+ T(\theta) \cdot \frac{d}{dt} \left[ \Psi_{pm, \theta} \right] + T(\theta) \cdot T^{-1}(\theta) \cdot \frac{d}{dt} \left[ \Psi_{pm, \theta} \right]
\]

where \[ \begin{bmatrix} V_{d_{eff}} & V_{q_{eff}} & V_{0_{eff}} \end{bmatrix} \] is \[ \begin{bmatrix} i_d & i_q & i_0 \end{bmatrix} \].

Then, the stator equations for the five-phase FTPM machine in rotating frame are expressed as:

\[
V_d = R_i i_d - w(L - M) i_q - \frac{2}{5} \mu R_i \cos \theta
\]

\[
+ (L - M) \frac{di_d}{dt} + \frac{2}{5} \mu (L - M) \cos \theta \frac{di_q}{dt}
\]

\[
V_q = R_i i_q + w(L - M) i_d + \frac{2}{5} \mu R_i \sin \theta
\]

\[
+ (L - M) \frac{di_q}{dt} + \frac{2}{5} \mu (L - M) \cos \theta \frac{di_d}{dt} + w\Psi_{pm}
\]

Also, the equation in the shorted turns is written:

\[
0 = \mu \left[ w(L - M) \sin \theta - R_c \cos \theta \right] i_d + (\mu R_c + r_0) i_f
\]

\[
- \mu (L - M) \cos \theta \frac{di_d}{dt} + \mu \left[ w(L - M) \cos \theta + R_c \sin \theta \right] i_q
\]

\[
+ \mu (L - M) \sin \theta \frac{di_q}{dt} + \mu^2 \frac{L d_i}{dt} + \mu w\Psi_{pm} \sin \theta
\]

The electromagnetic torque is given as:

\[
T_{ef} = \frac{\partial W_{co}}{\partial \theta}
\]

where \( W_{co} \) is the co-energy and is defined as:

\[
W_{co} = \frac{1}{2} \begin{bmatrix} i_d & i_q \end{bmatrix} \begin{bmatrix} [L_d] & [i_d] \end{bmatrix} + \begin{bmatrix} i_d & i_q \end{bmatrix} \begin{bmatrix} [\Psi_{pm}] \end{bmatrix}
\]

Considering that there is no saliency on the rotor yields, the torque equation can be derived as:

\[
T_{ef} = P \begin{bmatrix} i_d \end{bmatrix} \begin{bmatrix} \frac{\partial [\Psi_{pm}]}{\partial \theta} \end{bmatrix}
\]

Additionally, \( i_d \) can be expressed as:

\[
i_d = T(\theta)^{-1} \left[ i_{d_{eff}} \right]
\]

Substituting (30) into (29)

\[
T_{ef} = \frac{5}{2} P \begin{bmatrix} \Psi_{pm} \end{bmatrix} i_q - \mu P \begin{bmatrix} \Psi_{pm} \end{bmatrix} i_f \sin \theta
\]

The mechanical equation is as follows:

\[
T_{ef} = J \frac{d\omega}{dt} + B\omega + T_L
\]

3. Verification

The machine is upon the operation of vector-controlled PMSM drives as shown in Fig. 4. The FTPM machine is fed by a five-leg voltage source inverter (VSI), the system contains an outer speed-regulating loop and an inner current-regulating loop.

![Fig. 4. Block diagram of the vector control PMSM drives system.](image)

Table 1 shows the machine specifications. The stator
winding consists of 27 turns of each phase and the machine operates at 1500 r/min, a load torque of 6 Nm was applied to the motor at 0.02s under the shorted turn of 1/27 (3.7%), 2/27 (7.4%) and 3/27 (11.1%), respectively.

Table 1. Specification of the Five-Phase Fault-Tolerant Permanent-Magnet Machine.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed</td>
<td>1500 r/min</td>
</tr>
<tr>
<td>Phase resistance</td>
<td>0.048 Ω</td>
</tr>
<tr>
<td>Torque</td>
<td>6 Nm</td>
</tr>
<tr>
<td>Back EMF (peak)</td>
<td>60 V</td>
</tr>
<tr>
<td>DC supply voltage</td>
<td>200 V</td>
</tr>
<tr>
<td>PM flux linkage</td>
<td>0.034 Wb</td>
</tr>
<tr>
<td>Number of phase turns per phase winding</td>
<td>27</td>
</tr>
</tbody>
</table>

Fig. 5 shows the simulated results of the fault current $i_f$. As the number of shorted turns increase, it can be seen that the magnitude of $i_f$ becomes smaller, since the short fault will change the parameters of the machine and form a short circuit loop. In fact, due to the effect of the short fault, the induced voltage and impedance are related to the number of turns. Moreover, the induced voltage and the resistance are in proportion to the number of turns. However, the reactance is proportional to the square of the number of the turns. As the fault extends, the change of impedance is faster than that of the induce voltage. Therefore, the amplitude of the fault current becomes smaller.

![Simulation result of fault current](image)

Fig. 5. Shorted circuit current $i_f$ for different fault levels.

Fig. 6 shows the simulated results of $q$-axis current obtained from the proposed model. It can be seen from Fig. 6(a) that the inter-turn fault generates ripple in $i_q$ current. Also, the ripple becomes severe as the failure extends. This is because when the fault occurred, the machine would operate in an unbalance condition. As a result, the unbalance effect generates second harmonic components in $q$-axis current, resulting in the oscillation. However, the root-mean-square (RMS) of the $q$-axis current becomes smaller as shown in Fig. 6(b). The actual speed of the machine is regulated to set value by the speed-regulating loop, although there is a little ripple (see Fig. 7). When the fault occurs, the magnitude is small enough to be negligible. Then, it can be deduced that $\frac{d\theta}{dt} \approx 0$. Also, by considering the Equ. (31), the Equ. (32) can be rewritten as:

$$\frac{5}{2} P \left[ \Psi_{\text{PM}} \right] i_q - \mu P \left[ \Psi_{\text{PM}} \right] i_f \sin \theta - B \omega - T_L = 0 \quad (33)$$

When $T_L$ is given, the current $i_q$ is mainly determined by the fault current $i_f$. As stated previously, fault current $i_f$ decreases with the fault expanding, resulting in the reduction of the RMS of $i_q$.

![Simulation result of q-axis current](image)

Fig. 6. Simulated result of $q$-axis current for different shorted circuit number. (a) Current waveforms of $i_q$. (b) RMS of $q$-axis current.
Fig. 8 shows the simulated electromagnetic torque waveforms under single-turn shorted condition. It can be seen that the electromagnetic torque is oscillatory in faulty condition. By comparing the electromagnetic torque Equ. (10) and (31), it can be seen that there is a braking torque component developed by the fault current when shorted fault occurs. Obviously, the direction of the fault current circulated around the shorted loop is opposite to that of the phase current flowed in healthy windings, as shown in Fig. 3. Then the interaction between the EMF and short current emerged in the shorted loop generates a breaking torque component, which causes pulse in the electromagnetic torque.

![Image](image1.png)

**Fig. 7.** Simulated result of speed with intern-turn short fault.

![Image](image2.png)

**Fig. 8.** Simulated result of torque for single-coil short-circuit fault.

As previously discussed, although the amplitude of the different signals depends on the inter-turn fault severity, these signals are usually affected by the given angular speed such as load torque. Therefore, they cannot be used to indicate the fault severity. However, the second-order harmonic components in the q-axis current mainly affected by the fault severity, in other words, it could neglect the effect created by the variation of the given. So, the second-order harmonic components of the q-axis current can be chosen as the fault indicator used to reveal fault and its severity. Further observation from Fig. 9 reveals that the second harmonic components content in q-axis current with different number of turns shorted under a certain given 1500 r/min and 6 Nm. It can be known that as the number of turns in short circuit increase, the content of second harmonic in q-axis current also augments.

Table 2 shows the variation of the second-order harmonic content of q-axis current when the machine operates under fault conditions with different given. It could be seen that, although the second harmonic content will change with the variation of the given, the magnitude of the variation is in a smaller range under the certain fault level. However, the second harmonic changes most significantly with the fault extending, and the range of the change has little relation to the variation of the given.

From the above analysis, it can be concluded that the second harmonic content used to indicate the fault could reveal the short-turn fault severity exactly and almost not affected by the variation of the given.

![Image](image3.png)

**Fig. 9.** Simulated result of the harmonic analysis for q-axis current.

**Table 2.** Second harmonic content of Q-axis current with different given of speed and torque

<table>
<thead>
<tr>
<th>Torque (NM)</th>
<th>Fault Severity</th>
<th>500 (R/min.)</th>
<th>1000 (R/min.)</th>
<th>1500 (R/min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1 Short turns</td>
<td>5.982</td>
<td>6.032</td>
<td>5.974</td>
</tr>
<tr>
<td></td>
<td>2 Short turns</td>
<td>7.297</td>
<td>7.325</td>
<td>7.311</td>
</tr>
<tr>
<td></td>
<td>3 Short turns</td>
<td>7.952</td>
<td>8.032</td>
<td>7.960</td>
</tr>
<tr>
<td>6</td>
<td>1 Short turns</td>
<td>6.013</td>
<td>5.915</td>
<td>6.109</td>
</tr>
<tr>
<td></td>
<td>2 Short turns</td>
<td>7.132</td>
<td>7.342</td>
<td>7.213</td>
</tr>
<tr>
<td></td>
<td>3 Short turns</td>
<td>7.888</td>
<td>7.914</td>
<td>7.885</td>
</tr>
</tbody>
</table>
4. Conclusion

A new parametric model of five-phase FTPM machine have been developed, and the model could reflect the mechanism of the effect caused by inter-turn fault to the performance characteristics of the machine, which can be used as a basis for fault diagnosis and condition monitoring.

Simulated results have verified that the model is effective and detailed analysis has been given to illustrate the effect of inter-turn fault to machines operating performance. The second-order harmonic of $q$-axis has been used to indicate the fault, the proposed fault diagnosis approach is simple and no extra sensor is required, which also could realize on-line detection.

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