ERROR PERFORMANCE OF BICM-ID IN IMPULSIVE NOISE

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Abstract

The application of bit-interleaved coded modulation with iterative decoding (BICM-ID) was recently considered to improve both the spectral efficiency and error performance in Class-A impulsive noise environment. Such a noise model is widely used in power line communications, where impulsive noise interference is mainly generated by electrical appliances. This paper provides the EXIT chart analysis to illustrate the convergence properties of different mappings of 8PSK in BICM-ID. The superior performance of BICM-ID compared to the OFDM technique is also demonstrated.

Keywords—Impulsive noise, power line communications, bit-interleaved coded modulation, iterative decoding, signal mapping, EXIT chart, OFDM.

1 Introduction

The birth and growth of Internet have accelerated the demand for digital telecommunications services to every site. If the electricity distribution network can provide such services, it will make a tremendous breakthrough in communications. Every premises, factory, office, and organization will be interconnected and form a truly global information superhighway network.

However, electrical distribution grids were not primarily built for communications purposes. Varying levels of impedance and attenuation due to electrical hardware configurations are frequent. Such variations and other interferences from various sources lead to a very poor performance of power line communications (PLC) systems. Those interferences, referred to as man-made noise, have statistical characteristics much different from that of classical Gaussian interference. Man-made noise is typically impulsive. A relatively simple model that incorporates background noise and impulsive noise is suggested in [1] and known as Middleton’s Class-A noise. This noise model corresponds to an independent and identically distributed (i.i.d.) discrete-time random process whose probability density function is an infinite weighted sum of Gaussian densities, with decreasing weights and increasing variances for Gaussian densities [2]. Middleton’s Class-A noise model has been used widely in performance analysis of PLC systems (see [3] and the references therein).

In general, an optimal or suboptimal receiver designed for an additive white Gaussian noise (AWGN) channel does not work well for neither uncoded nor coded systems that are disturbed by impulsive noise [3, 4]. In particular, a recent work in [4] studies bit-interleaved coded modulation with iterative decoding (BICM-ID) for impulsive noise environment. It is shown in [4] that by appropriately designing the soft-output demodulator in the presence of impulsive noise, BICM-ID with 8PSK constellation can provide significant gains in both spectral efficiency and error performance compared to the conventional coded systems that use binary modulation.

This paper further extends the work in [4] by applying the extrinsic information transfer (EXIT) chart [5] to analyze the convergence behaviors of different mappings. Moreover, performance comparison between BICM-ID and orthogonal frequency-division multiplexing (OFDM) technique in impulsive noise is also made.

2 System Model

The block diagram of BICM-ID is shown in Fig. 1. The transmitter is a serial concatenation of an encoder, a random bit interleaver $\pi$ and a modulator. The information bits $\{u_k\}$ are first encoded by a convolutional code to produce a coded sequence $\{c_k\}$. The coded sequence $\{c_k\}$ is then interleaved by a random interleaver. The interleaved sequence $\{v_k\}$ is then mapped by the modulator into the symbol sequence $\{s_k\}$ for transmission. Each symbol $s_k$ is chosen from a two-dimensional $M$-ary constellation $\Psi$ according to some mapping rule $\mu(\cdot)$.

The baseband received signal over the $k$th symbol period can be written as,

$$r_k = s_k + n_k$$  \hfill (1)

where the random variable $n_k$ represents additive white complex Class-A impulsive noise. Its probability density function (pdf) is given by,

$$p_A(n) = \sum_{m=0}^{\infty} e^{-A^{m}/m!} \frac{1}{2\pi \sigma_m^2} \exp \left( -\frac{|n|^2}{2\sigma_m^2} \right)$$  \hfill (2)

where $\sigma_m^2 = \sigma^2 n^2 / |n|^2$ is the $m$th impulsive power, $A$ is known as the impulsive index, $\sigma^2$ is the total noise power (including the powers of impulsive noise and Gaussian noise), and $\Gamma = \sigma_G^2 / \sigma_I^2$ is the Gaussian-to-impulsive noise power ratio (GIR) with $\sigma_G^2$.
and $\sigma^2_I$ are the powers of Gaussian and impulsive noise, respectively. When $A$ is increased, the impulsiveness reduces and the noise comes close to the Gaussian noise.

At the receiver, due to the presence of bit-based interleaving, the true maximum likelihood decoding of BICM is too complicated to implement. Instead, the receiver in Fig. 1 uses a suboptimal, iterative method with soft-output demodulator and the soft-input soft-output (SISO) decoder. The SISO channel decoder is designed to implement. Instead, the receiver in Fig. 1 uses a suboptimal, iterative method with soft-output demodulator and the soft-input soft-output (SISO) decoder. The SISO channel decoder uses the maximum a posteriori probability (MAP) algorithm. Similar to decoding of Turbo codes, here the demodulator and the channel decoder exchange the extrinsic information of the coded bits $P(v^i_k; O)$ and $P(c^i_k; O)$ through an iterative process. The detailed algorithm of the optimal soft-output demodulator is described in [4] for an additive white Class-A impulsive noise (AWAN) channel.

3 Convergence Analysis with EXIT Chart

Following the same notations as in [5], let $A_1$ and $A_2$ denote the log-likelihood values of the a priori information of the coded bits at the inputs of the demodulator and the SISO decoder, respectively. Similarly, let $E_1$ and $E_2$ denote the log-likelihood values of the extrinsic information at the outputs of the demodulator and the SISO decoder, respectively. Also let $I_{A_1}$, $I_{A_2}$, $I_{E_1}$ and $I_{E_2}$ represent the mutual information of the random variables $A_1$, $A_2$, $E_1$ and $E_2$, respectively. Given the real-valued observation $z$ and a coded bit $X_1$, the $A_1$-value can be computed as follows:

$$A_1 = \ln \left\{ \frac{P_A(z|v_k = 1)}{P_A(z|v_k = 0)} \right\} = \ln \left( \frac{\sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m!} \exp \left( \frac{-(z-1)^2}{2\sigma^2_m} \right)}{\sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m!} \exp \left( \frac{-z^2}{2\sigma^2_m} \right)} \right)$$

(3)

where $z = X_1 + n$. With the $A_1$-value computed from (3), the pdf of the random variable $A_1$ can be conveniently determined by means of Monte Carlo simulation. Using this distribution, the mutual information $I_{A_1} = I(X_1; A_1)$ is computed by:

$$I_{A_1} = \frac{1}{2} \sum_{b=0}^{\infty} \int_{-\infty}^{\infty} p_A(\xi|X_1 = b) \times \log_2 \left[ \frac{2 \times p_{A_1}(\xi|X_1 = b)}{p_{A_1}(\xi|X_1 = 0) + p_{A_1}(\xi|X_1 = 1)} \right] d\xi$$

(4)

Similarly, the mutual information $I_{E_1} = I(X_1; E_1)$ can be calculated by the histogram method. Viewing $I_{E_1}$ as a function of $I_{A_1}$ and $E_b/N_0$, where $N_0 = 2\sigma^2$, the extrinsic transfer characteristic of the demodulator is defined as:

$$I_{E_1} = T_1(I_{A_1}, E_b/N_0)$$

(5)

The transfer characteristic of the SISO decoder is defined as:

$$I_{E_2} = T_2(I_{A_2})$$

(6)

which depends only on the convolutional code used in the system, not on the mapping scheme nor the value of $E_b/N_0$.

To visualize the exchange of extrinsic information, the demodulator and decoder characteristics are depicted in a single diagram, which is commonly referred to as the extrinsic information transfer (EXIT) chart [5]. The demodulation/decoding trajectory is then plotted in the EXIT chart to approximately predict the convergence behavior of the iterative receiver.

Now consider the error performance of a BICM-ID system employing 8PSK modulation and a 4-state, rate-2/3 convolutional code, whose generator polynomials are $G = [1001; 0001; 1100]$ [4]. This combination of channel code and modulation scheme yields a spectral efficiency of 2 bits/sec/Hz. Each information block has a length of 3999 bits. Two different mapping schemes, namely Gray and semi-set partitioning (SSP) mappings, shown in Fig. 2, are considered.

Figure 2: 8PSK with two different mapping schemes.

Fig. 3 shows the BER performance of 8PSK/SSP mapping when the parameters of impulsive noise are $\{A = 10^{-3}, \Gamma = 10^{-3}\}$, which means that the channel noise is highly impulsive. Also shown in the figure is the asymptotic performance bound, calculated as described in [4]. As can be seen from the figure, performance improvement due to iterations is very significant with SSP mapping. In particular, Fig. 3 shows that with 9 iterations the BER performance of 8PSK/SSP mapping can reach the error bound at SNR of $-24.5$dB, and starting at SNR of $-23.5$dB the error performance of 7, 8 and 9 iterations are practically the same and also reach the asymptotic performance bound. Also shown in this figure are the BER performance curves of 8PSK/Gray mapping and coded BPSK with the same convolutional code. It was observed that iterations do not improve BER performance for Gray mapping of BICM-ID in impulsive noise. On the other hand, it is impressive to see that 8PSK/SSP mapping not only can provide three times higher spectral efficiency but can also provide a coding gain of $1.6$dB at BER level of $10^{-5}$ compared to the coded BPSK system.

Now, Figs. 4 and 5 present the EXIT charts of the iterative decoding for $E_b/N_0 = -26$dB and $E_b/N_0 = -23$dB, respectively. It can be seen that the demodulator characteristics with Gray mapping appear to be constant regardless of the values of $I_{A_1}$. This implies that iterative decoding is useless with Gray mapping, a fact that is also confirmed by the BER performance results mentioned earlier. In contrast, the demodulator characteristic for SSP mappings increases as $I_{A_1}$ increases, thus facilitating the effective operation of the iterative decoding.

Comparing Figs. 4 and 5 shows that for the two mappings of 8PSK, increasing $E_b/N_0$ raises the demodulator characteristics, i.e., opening the tunnel between the demodulator and the SISO characteristics. The wider opening of the tunnel is a crucial factor to provide a faster convergence of the iterative de-
coding. For example, following the decoding trajectories of the SSP mapping shown in Figs. 4 and 5, it can be predicted that about 13 and 7 iterations are required for BER convergence at \( E_b/N_0 = -26 \text{dB} \) and \( E_b/N_0 = -23 \text{dB} \), respectively. The fact that BER convergence happens after about 7 decoding iterations at \( E_b/N_0 = -23 \text{dB} \) can be verified from Fig. 3.

4 Comparison with OFDM

Orthogonal frequency-division multiplexing (OFDM) is also a commonly used technique to combat impulsive noise [6]. It is therefore of interest to compare the performance of BICM-ID and OFDM in the impulsive noise environment.

Let \( N \) be the number of subcarriers. In OFDM, the symbol stream after the \( M \)-ary modulator is passed through a serial-to-parallel converter, whose output is a set of \( N \) \( M \)-ary symbols \( \{s_0, s_1, \ldots, s_{N-1}\} \) corresponding to the symbols transmitted over each of the subcarriers. In order to generate the transmitted signal, the DFT is performed on these \( N \) symbols. Typically, \( N \) is chosen to be a power of 2 and the DFT can be efficiently implemented using the IFFT algorithm. The IFFT yields the OFDM symbol consisting of the sequence \( \{S_0, S_1, \ldots, S_{N-1}\} \) of length \( N \), where

\[
S_k = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} s_i e^{j2\pi ki/N}, \quad 0 \leq k \leq N - 1. \tag{7}
\]

Assuming perfect synchronization and timing, the received symbols after filtering and sampling can be expressed as,

\[
R_k = S_k + n_k, \quad 0 \leq k \leq N - 1, \tag{8}
\]

where, as before, \( n_k \) is additive white complex Class-A impulsive noise, whose pdf is given as in (2).

Now at the receiver, the \( N \)-point FFT is first performed on the the sequence \( \{R_0, R_1, \ldots, R_{N-1}\} \) of \( N \) received symbols to yield:

\[
r_k = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} R_i e^{-j2\pi ki/N} = s_k + \hat{n}_k, \quad 0 \leq k \leq N - 1, \tag{9}
\]

where the noise samples \( \{n_0, \hat{n}_1, \ldots, \hat{n}_{N-1}\} \) are simply the \( N \)-point FFT of the original impulsive noise samples \( \{n_0, n_1, \ldots, n_{N-1}\} \). They are given by,

\[
\hat{n}_k = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} n_i e^{-j2\pi ki/N}, \quad 0 \leq k \leq N - 1. \tag{10}
\]

The outputs of FFT are then passed to the parallel-to-serial converter. Finally, the classical \( M \)-ary demodulator is applied to demodulate each of the sample \( r_k, 0 \leq k \leq N - 1 \). Here the term “classical \( M \)-ary demodulator” refers to the demodulator optimally designed for AWGN, i.e., by assuming that the noise \( \hat{n}_k \) in (9) and (10) is Gaussian.

Observe that the expression of \( r_k \) in (9) is similar to that in (1). The major difference, however, is the characteristic of the additive noise. With FFT operation, the original impulsive noise is
spread over $N$ data symbols in (9). As will be seen shortly, this is the main reason that OFDM can improve the error performance over the uncoded single-carrier system in impulsive noise.

Consider an AWAN channel with $\{A = 10^{-1}, \Gamma = 10^{-3}\}$. Assume that BPSK modulation is used for each subcarrier of OFDM. This means that the spectral efficiency is 1 bit/sec/Hz. Fig. 6 shows the BER performance of OFDM systems with various number of subcarriers. It can be clearly seen that, at high SNR values, OFDM significantly outperforms the uncoded BPSK system that uses a single carrier. In general, a higher coding gain is achieved by using a larger number of subcarriers. Performance improvement achieved with OFDM at high SNR values, however, comes at the expense of degraded performance at the low SNR region. This phenomenon is also observed in [6] and it is merely due to the suboptimality of the classical OFDM demodulator when applied to an impulsive noise environment.

Also plotted in Fig. 6 is the theoretical BER performance of an ideal receiver that can completely remove the impulsive noise, leaving only the Gaussian noise as the additive noise. This BER performance is thus the performance of uncoded BPSK modulation over a classical AWGN channel with noise variance $\sigma^2_G = \frac{\sigma^2}{1+1/\Gamma} = \frac{N_0}{2(1+1/\Gamma)}$. This BER performance is referred to as the uncoded Gaussian bound in Fig. 6 since it serves as the lower performance bound for any practical uncoded systems under the impulsive noise.

Figure 6: Performance Comparison of BICM-ID and OFDM.

The Gaussian bound in Fig. 6 clearly shows that there are still huge performance gaps between the performance of OFDM systems and that of the ideal receiver. This implies that using OFDM with the classical $M$-ary demodulator still does not fully exploit the rich structure of the impulsive noise. Recently, a more advanced and complicated receiver was developed in [6] that can be applied to OFDM. In essence, the work in [6] views OFDM as a “code over complex numbers”. The iterative receiver proposed in [6] consists of two information-exchanging estimators, one in the “codeword” and one in the “information” domain. Such an iterative decoding of OFDM is shown to perform very close to the uncoded Gaussian bound [6].

It is relevant to point out, however, that the iterative decoding of OFDM proposed in [6] is very different from the iterative receiver of BICM considered in this paper. In particular, the iterations in the receiver of [6] are performed between the two estimators of the impulsive noise. In contrast, the iterative processing at the receiver of BICM in this paper is performed between the SISO channel decoder and the soft-output demodulator.

Finally, the BER performance of BICM-ID systems is also provided in Fig. 6 for comparison. Here, 8PSK constellation with Gray and SSP mappings are considered. To have the same spectral efficiency of 1 bit/sec/Hz, a simple 4-state, rate-1/3 convolutional code with generator polynomials $G = [01; 11; 11]$ is used. As in Section 3, each information block contains 3999 bits. It is clear from Fig. 6 that BICM-ID systems perform much better than OFDM systems in impulsive noise. In particular, the BER performance of BICM-ID with 8PSK/Gray mapping (after just one iteration) closely approaches the uncoded Gaussian bound down to the BER level of $10^{-4}$. More interestingly and significantly, the BER performance of BICM-ID with 8PSK/SSP mapping after 9 iterations can even outperform the Gaussian bound. Compared to the Gaussian bound, an SNR gain of about 4dB is achieved at the BER level of $10^{-4}$.

5 Conclusion

EXIT chart analysis was described to predict the convergence behavior of the iterative decoding in BICM-ID under Class-A impulsive noise and was shown to match the BER simulations very well. It was also demonstrated that BICM-ID is much more effective than OFDM technique in combating the impulsive noise.

References