

# Three Mathematical Models for Bucking-to-Order

Hamish D. Marshall, Glen Murphy and Kevin Boston

---

**Marshall, H.D., Murphy, G. & Boston, K.** 2006. Three mathematical models for bucking-to-order. *Silva Fennica* 40(1): 127–142.

The aim of this paper is to investigate different mathematical approaches to buck-to-order log merchandizing. A new bucking-to-order planning model using mixed integer programming was developed to determine the optimal production from a stand given different market constraints and forest inventory data. Three different approaches: market prices, target cutting patterns and adjusted price list were tested for generating cutting instructions to fulfill the plan created by the new planning model. The three approaches were evaluated in four test stands. The market prices approach simply applied the market prices to each stand. The target cutting patterns approach applied the sample cutting patterns generated from the planning model to the stand. The adjusted price list used a dynamic programming algorithm embedded in a search heuristic to adjust both the prices and small end diameters of log products to achieve the production goals of the planning models. The results showed that developing a buck-to-order plan is important in obtaining good order fulfillment. The target cutting patterns and adjusted price list approaches certainly out performed the market prices approach. This paper shows that these two approaches are capable of achieving excellent order fulfillment. Further development and testing is needed to determine which method is the best at generating cutting instructions for buck-to-order merchandizing.

**Keywords** mechanical harvesting/processing, optimal bucking, mixed integer programming, dynamic programming, buck-to-value

**Author' addresses** *Marshall*, Ensis Forests, Private Bag 3020, Rotorua, New Zealand; *Murphy*, Forest Engineering Department, Oregon State University, Corvallis, Oregon 97331, USA; *Boston*, Forest Engineering Department, Oregon State University, Corvallis, Oregon 97331, USA **E-mail** hamish.marshall@ensisjv.com

**Received** 22 December 2004 **Revised** 9 November 2005 **Accepted** 23 November 2005

**Available at** <http://www.metla.fi/silvafennica/full/sf40/sf401127.pdf>

---

# 1 Introduction

The adoption of highly mechanized, timber harvesting systems is increasing worldwide (Raymond 1988, Nordlund 1996, Godin 2000). With these systems, stems are delimbed, bucked and sorted by a single machine. There are a number of factors causing this shift from the traditional motor manual harvesting systems to mechanized harvesting systems. These include economic (the need to continually increase productivity) and social pressures along with the continuing need to improve the safety record of forestry operations. A sometimes overlooked aspect of economic improvement in harvesting is value recovery. Value or revenue can be lost in numerous places along the forest to mill value chain. One process that has been identified as having a large potential for value loss is the process of bucking trees into logs. Recent surveys of value recovery studies have shown that on average, manual log making systems were losing 11% and mechanical log making systems 18% of potential value (Murphy 2003a, Marshall 2005).

These kinds of figures have spurred significant research in the area of optimal log bucking. Optimal bucking is an effective means of making informed decisions before mistakes are made that result in value loss. A number of mathematical formulations and computer models have been developed to optimize the value in each individual stem, this is commonly referred to as buck-to-value (Pnevmaticos and Mann 1972, Briggs 1980, Geerts and Twaddle 1984, Sessions et al. 1989). The objective of buck-to-value optimal bucking is to obtain the maximum monetary value from an individual stem. A stem can be cut up into logs in numerous ways; each set of logs will yield a different financial return. However, there is, in many cases one unique bucking pattern that yields the maximum value. The value and logs cut using the optimal bucking pattern depends on the species, diameter, taper rate and quality of the stem plus the properties and relative market values of log grades being cut.

The problem operationally is what is optimal for individual stems may not meet the market and operational constraints at a harvest unit or forest level. To maximize the value coming from a forest, these in-the-field bucking algorithms

must be given log specifications which take into account market and operational constraints. Constraints can be in the form of the following: target volumes, minimum percentage of volume must be of greater than a certain length, minimum average small end diameter (SED) for a product, and minimum percentage of the volume must be of a certain grade. There may also be constraints on the amount of volume that can be bought from and sold to the open market. Buying volume from, and selling volume to, the open market may incur additional costs. In some cases, however, it may be economically better to produce excess volume of a high value product and sell it to the open market while under-producing a low value product.

To account for these operational and market constraints a number of different buck-to-order procedures have been developed. The objective of buck-to-order optimal bucking is to maximize the monetary value at the harvest unit or forest level while meeting market and operational constraints. Although the majority of buck-to-value models were developed in the eighties, it has only been in recent years that these models have been implemented into large scale commercial harvesting operations (Boston 2001).

In the literature there are generally two approaches to developing the in-the-field cutting instructions for buck-to-order bucking:

*Approach 1:* Selecting cutting instructions either before or during the bucking process for each tree that will produce the required volume for each product.

*Approach 2:* Finding the correct price list (in some cases the correct specifications) that will be applied to the stand of trees to produce the required volume for each product.

The first published optimal bucking formulation utilized Approach 1 (Smith and Harrell 1961). It solved the buck-to-order problem using linear programming (LP). However, as Pnevmticos and Mann (1972) stated, the Smith and Harrell LP formulation was restricted by the requirement that all relationships be linear and by the limited number of cutting patterns available for each diameter class.

The limited number of cutting patterns issue was solved by Näsberg (1985), Mendoza and Bare

(1986), Eng et al. (1986) and Laroze and Greber (1997), by using a two stage iterative formulation of the stand-level buck-to-order problem. Arce et al. (2002) applied a similar approach to solve the forest level bucking problem. The first stage, or master problem, uses LP to solve the constrained market problem and the second stage, or sub-problem, uses dynamic programming (DP) or a network algorithm to solve the individual tree problem. The shadow prices from the master problem are used in the second stage to generate new cutting patterns. These are then used to form new columns in the master problem using column generation techniques. This general approach is theoretically correct and computationally efficient (Laroze 1993), but as many authors (Sessions et al. 1989, Laroze 1993 etc) have pointed out, the solutions produced are not particularly practical as they produce large numbers of cutting instructions. Sessions et al. (1989) also noted that the requirement of these techniques to subdivide the stand into stem classes makes these solutions hard to implement.

Approach 2 does not suffer from the same problems, however, it can not guarantee theoretically that maximum revenue is gained from the bucking of the stand. Duffner (1980) is the first published work on adjusting the price list in a bucking algorithm to meet market demands. There is, however, very little detail in the Duffner (1980) paper on exactly how he adjusted the prices.

Sessions et al. (1989) developed a system to adjust the prices iteratively using a shortest path algorithm to solve the sub-problem and a binary search procedure to find the price multipliers which obtain the correct ratio of long logs to short logs. The formulation was designed to overcome the problem of producing too many short logs that plagued optimal bucking in areas where the Scribner volume scaling rules were used.

A number of other procedures have been tried, such as using an LP solution at the upper level, to adjust the prices in the DP lower level, or using an heuristic to find the correct prices so the demand constraints are met in the master problem (Laroze and Greber 1993, Pickens et al. 1992).

Laroze and Greber (1997) used a rule-based stem bucking algorithm combined with a Tabu Search heuristic to generate easy to implement bucking rules that are applicable to the entire

stand, while providing the best feasible solution given a set of log prices and market constraints. Laroze and Greber (1997) compared the solution from their algorithm with a LP/DP formulation and found that it would lead to profits approximately 2.5% below those of the LP/DP algorithm. Laroze (1999) used the rule-based approach described above, in combination with a LP formulation, to solve the forest level bucking optimization problem. Laroze found that his formulation consistently achieved efficiency levels of approximately 97% compared to the optimal solutions for all of the scenarios analyzed.

Kivinen and Uusitalo (2002) developed a fuzzy logic controller to adjust the prices specifically for harvesters. The fuzzy logic controller is a set of rules which changes the price of a log type based on the disparity between the target proportion and the actual proportion in each log class and the rate of change in this error. Kivinen and Uusitalo found that for the four stands tested, the output log distribution derived by the fuzzy logic controlled production price matrix was within 92% of the log distribution produced by the desired (target) price matrix. Kivinen (2004) published another paper outlining a genetic algorithm that searches for price matrices at the forest level. He found similar results to Kivinen and Uusitalo (2002)

Murphy et al. (2004) developed a two level model where the upper level used a threshold accepting heuristic and the lower level used a DP bucking algorithm. The upper level heuristic was designed to find the product prices and minimum SEDs that minimized the difference between the target and the actual log distributions while meeting length proportion and average SED constraints. This algorithm is discussed in more detail later in the paper.

The buck-to-order process can be split into three stages; buck-to-order planning, cutting instruction development and adaptive control during the harvesting. The aim of the research presented in this paper was to develop new, and test existing, algorithms for solving the first two stages of this process. The paper presents a new methodology for creating a buck-to-order plan and tests the effectiveness of the two approaches discussed earlier for creating buck-to-order cutting instructions for implementation of the buck-to-order plan. Three mathematical models have

been designed to optimise the returns to forest owners, selling into constrained markets.

## 2 Methods

The methods section is divided into three parts. The first part describes a new mixed integer programming (MIP) formulation for developing an optimal buck-to-order plan. The second part describes two buck-to-order approaches, along with a buck-to-value approach for implementing the plan. The third part describes the metrics that were used to assess the effectiveness of the approaches described in the second part.

### 2.1 Developing a Buck-to-Order Planning Model

An MIP model was used to maximize the market value of the stand while meeting the market constraints, the customer order book constraints and the spot market constraints. The volume of log products that should be cut from the stand was determined. The model optimizes the projected stand value, given the different market constraints, by determining the optimal bucking patterns for a sample of trees from the stand. The tree data for these sample trees would normally be collected as part of the pre-harvest inventory. The model satisfies the customer order book constraints either by using the volume produced from the stand or buying volume from other sources at an additional cost. In cases where excess volume is produced from the stand the excess is reallocated to other markets at an additional cost. The mathematical formulation of the model is shown below:

$$\text{Max} \sum_{i=1}^p y_i \cdot c_i + \sum_{i=1}^p w_i \cdot d_i + \sum_{i=1}^p z_i \cdot c_i - \sum_{i=1}^p z_i \cdot e_i \quad (1)$$

subject to:

$$\sum_{j=1}^s x_{ij} = y_i + w_i \quad \forall \quad i = 1, \dots, p \quad (2)$$

$$y_i + z_i = b_i \quad \forall \quad i = 1, \dots, p \quad (3)$$

$$w_i \leq \text{US}_i \quad \forall \quad i = 1, \dots, p \quad (4)$$

$$z_i \leq \text{UB}_i \quad \forall \quad i = 1, \dots, p \quad (5)$$

$$x_{ij} - \text{BigN} \cdot \text{cut}_{ij} \leq 0 \quad (6)$$

$$x_{ij} \geq \text{Min}V_{ij} \cdot \text{cut}_{ij} \quad \forall \quad i = 1, \dots, p \quad j = 1, \dots, s \quad (7)$$

$$x_{ij} \leq pV_{ij} \quad \forall \quad i = 1, \dots, p \quad j = 1, \dots, s \quad (8)$$

$$\sum_i x_{ij} \leq \text{CV}_{ij} \quad \forall \quad i \in \{\text{product group}\} \quad j = 1, \dots, s \quad (9)$$

$$\text{cut}_{ij} \in \{0, 1\}$$

where

$p$  = the number of log products

$s$  = the number of stems

$b_i$  = the volume demanded of each product ( $i$ ) from the markets

$x_{ij}$  = the volume cut of each product ( $i$ ) from each sample stem ( $j$ )

$y_i$  = the volume of each product cut from the stand used to fulfill the demand constraints

$w_i$  = the volume of each product “sold” to other markets.

$z_i$  = the volume of each product “bought in” from other sources

$c_i$  = the market price for log-type  $i$

$d_i$  = the “sell off” price for log-type  $i$

$e_i$  = the “buy in” price for log-type  $i$

$\text{UB}_i$  = upper limit on volume that can be bought from other sources

$\text{US}_i$  = upper limit on volume that can be sold to the markets

$\text{cut}_{ij}$  = a binary trigger variable, which has a value of 0 if no log-type  $i$  logs are cut from stem  $j$ , and 1 otherwise

$\text{BigN}$  = a large number, for example 200

$\text{Min}V_{ij}$  = the minimum possible volume for a single log of that log product in that stem (It is found by optimal bucking the stem using only that product and restricting the length of the logs to the smallest possible length for that log product)

$pV_{ij}$  = this is the maximum potential volume that can be cut from stem ( $j$ ) of that product ( $i$ ) (This value is found by bucking the stem using a dynamic programming bucking algorithm, using only the product specifications for that

**Table 1.** An example of downgrade groups.

Product	Minimum small end diameter SED (mm)	Allowable qualities	Member of product downgrade groups
Pruned	350	A	Pruned
Sawlog 1	200	AB	Pruned, Sawlog 1
Sawlog 2	350	ABC	Pruned, Sawlog 2
Sawlog 3	200	ABC	Pruned, Sawlog 1, Sawlog 2, Sawlog 3
Pulp	150	ABCD	Pruned, Sawlog 1, Sawlog 2, Sawlog 3, Pulp
Waste	10	ABCDE	Pruned, Sawlog 1, Sawlog 2, Sawlog 3, Pulp Waste

product and waste, where waste has a value of zero in this model)

$CV_{ij}$  = the maximum constrained volume; this is the maximum volume from a stem when all the products in a particular product’s “downgrade group” are used in the bucking algorithm. A “downgrade group” is defined as those products that can be downgraded based on quality and small end diameter specification into that product. Examples of product groups and their definitions are given in Table 1.

The constraints shown above ensure that:

- Eq. 2 The sum of the volumes cut from all the stems for each log product, is equal to the volume produced from the stand that is being used towards fulfilling the order, plus the volume being sold onto the open market.
- Eq. 3 The sum of the volume produced from the stand that is being used towards fulfilling the order, plus the “buy in” volume, is equal to the demand requirement for each log product.
- Eq. 4 The amount of volume that can be sold on the open market is limited.
- Eq.5 The amount of volume that can be bought from the open market is limited.
- Eq. 6 A binary trigger is set to either 0 or 1. If  $x_{ij}$  is greater than zero then  $cut_{ij}$  must be 1. Combining this constraint with the constraint in Eq. 7, requires  $x_{ij}$  to be greater than  $MinV_i$ .
- Eq. 7 The volume of log product ( $i$ ) cut from a stem is greater than integer multiples of the minimum log product volume.
- Eq. 8 The total volume of all logs of log product ( $i$ ) cut from stem ( $j$ ) is less than or equal to the maximum potential volume for that log product in that stem.
- Eq. 9 The total volume of the “downgrade group”

is less than or equal to the maximum potential volume for that “downgrade group” in that stem.

The MIP model was formulated in AMPL mathematical programming language and solved using CPLEX 8.0. The default CPLEX optimizing settings were used. The model was solved on a Pentium 4 laptop with 1 GB of memory. Most of the models took less than one minute to solve.

The solution provides projected volumes that should be 1) cut from the stand, 2) be sold on to the open market, and 3) purchased from the open market to satisfy the order book constraints. Solving the MIP formulation also creates cutting patterns for each of the sample trees included in the formulation. These cutting patterns, which are in the form of volume per product ( $x_{ij}$ ) and the maximum potential volumes per product ( $pV_{ij}$ ) for each tree, are used in the Cutting Pattern approach described in the next section.

## 2.2 Methods for Implementing the Buck-to-Order Plan

### 2.2.1 Approach 1: Market Price

In this approach the market prices were applied using an individual stem optimal bucking DP algorithm. The basic formulation has been published by a number of authors in the past (Pnevmaticos and Mann 1972, Briggs 1980, Geerts and Twaddle 1984 and others). This algorithm was developed by the first author of this paper and is similar to that described by Deadman and Goulding (1979). Its aim was to maximize the total value of each stem by determining the optimal allocation of the log

products on a stem. The log products are specified in terms of maximum and minimum SED (small end diameter) and LED (large end diameter), feasible lengths and minimum quality requirements. The quality requirements were specified as a single character code which represents a combination of branch size and sweep characteristics. The algorithm was applied to all the trees in the stands using the product market prices given in Table 3. This is effectively a buck-to-value approach and will optimize the market value for each stem in the forest. The product volume for the stand is determined by adding up the volume for each product for all the trees in the stand.

### 2.2.2 Approach 2: Target Cutting Patterns

This algorithm was formulated to solve the problem of determining which bucking pattern should be applied to each tree, by allocating the target cutting patterns to a tree using the maximum potential volume ( $pV_{ij}$ ) of each log product. The theory is that two trees that are similar in terms of size and quality characteristics will have similar maximum potential volumes and hence should be bucked in a similar manner.

The theory behind this formulation is that each of the target trees in the inventory sample represents the same proportion of trees in the total stand and the target cutting pattern will therefore, be applied to the same proportion of trees in the stand. The target cutting pattern to be used on the current stem is found by determining which of the sample trees is most closely matched in terms of maximum potential volumes. A simple distance function, as shown in Eq. 10, was used to determine the nearest neighbours:

$$d = \sum_{i=1}^n |PTV_i - PAV_i| \tag{10}$$

where

- $d$  = the distance between trees in terms of total potential volume for each product
- $i$  = products (1, ...,  $n$ )
- $PTV_i$  = the maximum potential target volume for product  $i$
- $PAV_i$  = the maximum potential actual volume for product  $i$  for the current stem.

Ponsse harvesters store information on the previous 80 stems harvested and use the “nearest” eight trees as the basis for predicting stem taper rather than the taper from the single closest tree. It is possible to use  $k$ -nearest neighbours to determine the best cutting pattern to apply to each tree in the stand. The same distance function (Eq. 10) was used to determine the  $k$ -nearest neighbours for the current candidate tree. Trials using different numbers of nearest neighbours ( $k$ ) showed that no gains were made by using more than 4 of the closest trees (in terms of  $d$ ) for this application.

The target volumes for each product are then calculated from the  $k$ -nearest neighbours. The target volumes are calculated in proportion to the distance each of the  $k$ -nearest neighbours is from the current stem. The following equation is used to calculate the target volumes for the current stem:

$$TV_i = \sum_{m=1}^k \left( TV_{im} \cdot \left( 1 - d_m / \sum_{m=1}^k d_m \right) / (k-1) \right) \tag{11}$$

where

- $i$  = products (1, ...,  $n$ )
- $m$  = nearest neighbours (1, ...,  $k$ )
- $d$  = the distance between trees in terms of total potential volume (as calculated in Eq. 10)
- $TV_i$  = the target volumes for the current stem
- $TV_{im}$  = the target volumes from the  $k$ -nearest neighbours.

The target volumes for each product are then used in a heuristic allocation model that uses the same structure as a forward recursive DP algorithm to minimize the deviation from these target volumes. The decision whether to cut a product at a particular state in this problem depends on what products have been cut before, hence breaking the principle of optimality. The algorithm attempts, as closely as possible, to cut the same volumes out of the current tree as the sample cutting pattern. This is achieved by replacing the maximize revenue objective function with a minimize the weighted volume deviation from the target volume objective function. The minimize volume deviation requires the addition of  $i$  more state variables; where  $i$  represents the number of products. These new state variables contain the volume of each product that has already been cut at the state.

The volume deviations from the target of each product are weighted to encourage the algorithm to meet the targets for higher value products. This is done using the market prices for the products. If the current cut volume of that product at that stage is less than the target volume for that product, the market price for that product is used as the weight. However, if the current cut volume of that product at that stage exceeds the target volume then the market price list is applied in reverse.

During the early testing of this algorithm it was found that the pulp and waste products were being over produced. In an attempt to reduce this, the target volumes for all the target cutting patterns for pulp and waste were set to zero. This changed the objective function of the algorithm to minimize the volume deviation from the target volumes (for all products except pulp and waste) while minimizing the production of pulp and waste volumes. This change significantly improved the performance of the algorithm.

### 2.2.3 Approach 3: Adjusted Price List

The adjusted price list algorithm that has been used in this paper is FASTBUCK. This was developed by Murphy et al. (2004).

In this algorithm an individual stem optimal bucking DP procedure is imbedded within a threshold accepting algorithm which adjusts relative prices for log products to meet order book constraints. The threshold accepting algorithm is designed to optimize the order fulfillment, not the market value of the volume produced. The objective function is to maximize the apportionment degree, which is a measure of how well the production meets the orders.

Apportionment degree (AD%) is defined as:

$$AD\% = 100 \cdot \left( 1 - \frac{\sum_{i=1}^m |D_{di} - D_{pi}|}{2} \right) \quad (12)$$

where

AD% = apportionment degree (goodness of fit between the demand and production vector/matrix)

$m$  = number of log grades

$D_{di}$  = target proportion demanded for the log grade

$D_{pi}$  = actual proportion produced for the log grade.

A set of “good” relative prices is found through an iterative process of changing the relative prices. The DP bucking algorithm bucks each of the stems in the sample given a set of relative prices. The AD% is calculated for the resulting volumes generated from bucking the set of sample stems. If the AD% is better than, or within a certain threshold of, the current best AD%, the current set of relative prices is kept as the starting point for the next iteration. If the AD% is outside the threshold, the current set of relative prices is discarded. Only one product’s price is changed at any one time. The product is randomly selected and its price changed by increments of \$1. The process stops when a set number of iterations have been completed (Murphy et al. 2004).

In this paper the target proportion of the total volume for each product was determined using the results from the buck-to-order planning model. The projected volume production for each product was divided by the total projected volume. The FASTBUCK algorithm was first applied to the pre-harvest inventory stem data. The resulting relative prices and minimum SED were then applied to the whole stand to simulate the harvesting process.

## 2.3 Calculating the Effectiveness of the Approaches

The production from the simulated harvest from each approach was then adjusted using the projected “buy in” and “sell off” volume from the Buck-to-Order planning model. The effectiveness of the different approaches were measured using 1) the level to which the orders were fulfilled and 2) the monetary return from harvesting the block. The metrics that were used are described below:

### 2.3.1 Order Fulfillment

To evaluate the goodness of fit between the demand and production vector/matrix the apportionment degree (AD% in Eq. 12) was used. It was originally developed by Bergstrand (1989). This

is a commonly used measure for evaluating the fit between an actual output distribution of logs and the desired log distribution.

### 2.3.2 Monetary Return

The monetary return is calculated by determining the gross value gained by harvesting the unit, given that the volume of the over produced products is sold on to the open market at a discounted price (Table 4), and the orders that are undersupplied have to be fulfilled using volume bought from the open market at an inflated price. The monetary return (MR) is determined using the following equation:

$$MR = \text{Max} \sum_{i=1}^p y_i \cdot c_i + \sum_{i=1}^p w_i \cdot d_i + \sum_{i=1}^p z_i \cdot c_i - \sum_{i=1}^p z_i \cdot e_i \quad (13)$$

where the coefficients are the same as those defined for Eq. 1.

The formula gives the monetary return of the solution, given that the log demand distribution has been completely fulfilled, either from the stand, or from the planned sales and purchases from the open market. However, given that perfect information is not available, it is possible that additional volume will have to be purchased from the open market during or after the harvest. These purchases come at a significant cost to the company. In this paper, it has been assumed this cost will be 125% of the original market prices. Any volume produced in excess of the originally projected production is valued at the price of pulp, regardless of its original value.

## 3 Materials

### 3.1 Test Stands

Four stands were used to test and evaluate the performance of the above buck-to-order planning and implementation approaches. The four stands were the same as those used in Murphy et al. (2004). All stands had been pruned and were of similar mean diameter at breast height (DBH), details of which are provided in Table 2. Only one of the

**Table 2.** Characteristics of test stands.

	EVEN	UNEVEN	FROST	WHAKA
Total area (ha)	10.0	10.0	10.0	1.9
Density (stems per ha)	375	375	375	249
Mean DBH (cm)	45.0	44.8	45.2	48.5
Total volume (m <sup>3</sup> )	5990	6136	6554	1066

four stands was a “real-world” stand; this stand (WHAKA) was a *Pinus radiata* plantation stand in the North Island of New Zealand. Every tree in this irregular shaped stand was located, measured and described using the MARVL inventory system (Deadman and Goulding 1979).

The other three stands were virtual stands and were rectangular in shape (500 m × 200 m). These were based on growth and form characteristics of *Pinus radiata* and were generated to represent a variety of forest conditions. The lower limbs had been removed (pruned) from all trees to a height of approximately 6 m in the EVEN stand. Selection for pruning was uneven in the UNEVEN stand; 100% of trees were pruned in the middle of the stand decreasing to 70% at the edges of the stand. This mimicked situations where the pruning contract supervision or funds were inadequate to ensure all final crop trees in the stand were pruned. The EVEN and UNEVEN stands were generated to have diameter distributions which ranged from 20 to 70 cm with a DBH of approximately 45 cm and standard deviation of 5.9 cm.

The FROST stand mimicked a situation where there was a frost effect in the center of the stand; tree size was small in the center and increased toward the edge of the stand. All trees were pruned in the FROST stand.

Fifteen circular pre-harvest inventory plots were systematically located in each of the EVEN, UNEVEN, and FROST stands and five square plots were located in the WHAKA stand. The inventory plots occupied 3% of total area in each stand.

### 3.2 Product Requirements for Test Stands

The same product requirements were applied to all four test stands (Table 3). There were five log-types (Pruned Domestic Sawlogs, Unpruned

**Table 3.** Market requirements and constraints for the four test stands.

Log-types	Lengths (m)	Minimum SED (mm)	Market prices (\$/m <sup>3</sup> )	Sell off prices (\$/m <sup>3</sup> )	Buy in prices (\$/m <sup>3</sup> )	Target proportions (%)
Pruned Domestic Sawlog	3.7–6.1	350	145	138	160	15
Unpruned Export Sawlog 12.2 m	12.2	260	97	92	107	20
Unpruned Export Sawlog 8.2 m	8.2	260	88	83	97	6
Domestic Sawlog #1	3.7–6.1	200	68	65	75	12
Domestic Sawlog #2 Longs	4.9–6.1	200	48	46	53	8
Domestic Sawlog #2 Shorts	3.7–4.6	350	46	44	51	7
Pulp	3.7–6.1	100	25	24	28	24
Waste	0.1–0.6	0	0	–1	1	8

Export Sawlogs, Domestic Sawlogs #1, Domestic Sawlogs #2, and Pulp) plus waste. Most log types allowed multiple lengths; some in multiples of 0.3 m, others in multiples of 0.1 m. A total of 51 lengths were included in the analyses. Each log-type had target proportions of total volume that were required. For example, the demand target proportion for Pruned Domestic Sawlogs was 15%, of the total volume harvested.

Three different prices were included in the specifications for each log type:

- The market prices, which were the prices for the volume of each log type with confirmed markets.
- The “sell off prices” that can be thought of as, either a transfer cost into log stocks, or the price received for selling excess volume on the open market. These prices were 5% less than the market prices rounded to the nearest dollar.
- The “buy in prices” can also be thought of as a transfer cost out of log stocks, or the price for buying volume from the open market. These prices were 10% greater than the market prices rounded to the nearest dollar.

Waste was given a negative “sell off” price of \$1 and positive “buy in” cost of \$1 to represent the cost of handling under and over production of waste volume.

### 3.3 Market Scenarios

To test the robustness of the different approaches four different test market conditions were used:

- 1) Unconstrained Spot Markets (Unconstrained Spot)  
This is the base scenario; it uses the prices in Table 4. The scenario has no constraints on the volume that can be brought in from, and sold off to, the open (also referred to as “spot”) market.
- 2) High “Buy In” for “Unpruned Export Sawlog 12.2 m” (Hi-Price Exp 12)  
In this scenario the “buy in” price for the “Unpruned Export Sawlog 12.2 m” was increased from \$107 to \$147 which is greater than the “Sell Off” price of “Pruned Domestic Saw”. This is to simulate an increase in price on the spot market due to limitations in the supply of “Unpruned Export Sawlog 12.2 m”.
- 3) Spot Market Availability Constraints (Spot Constraints)  
The volumes for EVEN, UNEVEN and FROST stands of “Unpruned Export Sawlog 12.2 m” and “Domestic Sawlog #2 Shorts” that was available on the spot market is limited to 575 m<sup>3</sup> and 300 m<sup>3</sup> respectively. The available market for the surplus “Pruned Domestic Saw” volume was limited to 175 m<sup>3</sup>. The numbers were reduced to 30, 13 and 70 for the WHAKA stand. These volumes were chosen arbitrarily, solely to constrain the model.
- 4) Minimize “Buy In” and “Selling Off” volume. (Buy/Sell Min)

To test the robustness of the model, the objective function of the buck-to-order planning model was changed from maximizing return to minimizing the amount of the volume that was brought in and sold off. To stop that model just producing lots of “Waste”, the objective function was formulated to

minimize the production of waste as well.

$$\text{Min} \sum_{i=1}^p w_i + \sum_{i=1}^p z_i + y_7 \tag{14}$$

where  $y_7$  = volume of waste.

## 4 Results

### 4.1 Buck-to-Order Planning Model

The buck-to-order MIP planning model was run on all four stands under the four market scenarios. Fig. 1 shows the projected maximum values for each stand and market scenario combination.

The constraint on “buy in” volume of Unpruned Export Sawlog 12.2 m had to be relaxed for the UNEVEN stand under the Spot Constraints scenario, as the model was infeasible with the original constraints. The chance of obtaining an infeasible solution will be significantly increased as the number of hard constraints that are placed on the amount of available “buy in” and “sell off” volume increases.

The market value objective function shows the company’s operational planner and marketers

the trade offs from placing more market constraints on a stand. For example, trying to minimize the “buy in” and “sell off” volumes reduced the market value of the forest for the UNEVEN stand by 5% compared with the unconstrained spot market value.

The effect on the volumes that are cut from the EVEN stand, under the four market scenarios is shown in Fig. 2. The graph shows the change in the proportion of the total stand volume that is projected to be cut for each product from the stand in comparison to the original order book targets given in Table 4. Increasing the “buy in” cost of the Unpruned Export Sawlog 12.2 m caused the model to cut more of that volume from the stand, reducing the volume of Pruned Domestic Saw log, and the overall return from the stand by 7%. Minimizing the amount of “buy in” and “sell off” volume was the most costly scenario of the four market scenarios for all four stands.

### 4.2 Implementing the Buck-to-Order Plan

#### 4.2.1 Order Fulfillment Effectiveness

The AD% from simulating implementation of the different approaches to carrying out the plans for each market scenario is given in Table 4. All three

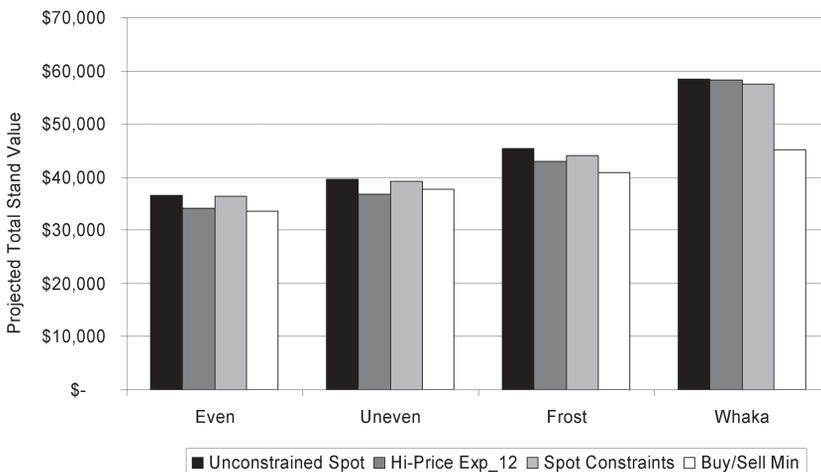


Fig. 1. The projected objective function (\$/ha) from the buck-to-order planning model for each stand under the different market scenarios.

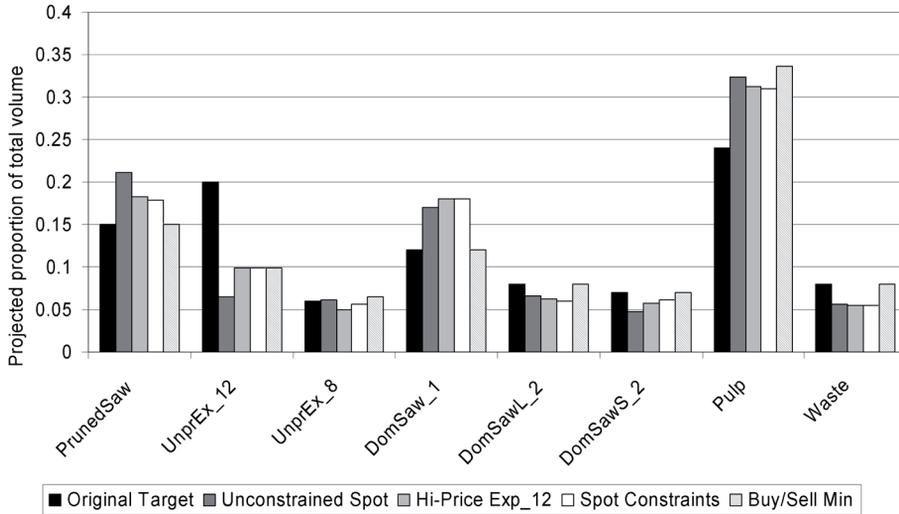


Fig. 2. The projected proportion of the different products being cut from the EVEN stand, under the different market scenarios.

Table 4. Apportionment degree for three approaches for implementing a buck-to-order plan.

Stand	Scenario	Approach		
		Market Price	Cutting Pattern	Adjusted Price List
EVEN	Unconstrained Spot	92.5%	93.6%	95.4%
	Hi-Price Exp_12	88.0%	92.1%	95.1%
	Spot Constraints	87.8%	91.6%	94.0%
	Buy/Sell Min	86.2%	94.1%	96.4%
UNEVEN	Unconstrained Spot	94.6%	92.4%	92.4%
	Hi-Price Exp_12	90.0%	91.3%	95.2%
	Spot Constraints	90.0%	91.3%	96.0%
	Buy/Sell Min	89.5%	94.2%	95.2%
FROST	Unconstrained Spot	91.0%	88.1%	90.5%
	Hi-Price Exp_12	82.4%	86.5%	88.0%
	Spot Constraints	82.4%	84.9%	89.4%
	Buy/Sell Min	84.8%	91.1%	86.6%
WHAKA	Unconstrained Spot	59.0%	74.8%	80.5%
	Hi-Price Exp_12	57.6%	67.6%	75.2%
	Spot Constraints	52.0%	52.3%	77.8%
	Buy/Sell Min	65.3%	73.0%	81.5%

approaches did well in the unconstrained spot market scenario. The order book constraints can simply be fulfilled by buying in needed volume and selling off excess volume to the spot market. As additional market constraints were placed on how the order could be fulfilled, performance of

the Market Price approach decreased while the other two approaches continued to produce high AD%. In the Market Approach there is no way to adapt the cutting instructions to take account of the additional constraints.

Overall the three virtual stands produced sub-

**Table 5.** The best approach in terms of apportionment degree for each market scenario and stand combination.

	Unconstrained Spot	Hi-Price Exp_12	Spot Constraints	Buy/Sell Min
EVEN	Adjusted Price List	Market Price	Adjusted Price List	Adjusted Price List
UNEVEN	Market Price	Adjusted Price List	Adjusted Price List	Adjusted Price List
FROST	Adjusted Price List	Adjusted Price List	Adjusted Price List	Cutting Pattern
WHAKA	Adjusted Price List	Adjusted Price List	Adjusted Price List	Adjusted Price List

**Table 6.** The best approach in terms of monetary return for each market scenario and stand combination.

	Unconstrained Spot	Hi-Price Exp_12	Spot Constraints	Buy/Sell Min
EVEN	Cutting Pattern	Market Price	Cutting Pattern	Cutting Pattern
UNEVEN	Market Price	Adjusted Price List	Adjusted Price List	Adjusted Price List
FROST	Market Price	Cutting Pattern	Cutting Pattern	Cutting Pattern
WHAKA	Adjusted Price List	Cutting Pattern	Adjusted Price List	Cutting Pattern

stantially better AD% than the small “Whaka” stand. This difference in AD% is probably as much a function of the small inventory sample size as a function of the algorithms.

The effect of the increased within-stand variation can be seen in all three approaches. The AD% is much lower in the FROST stand than the EVEN stand. The Adjusted Price List approach generally out performs the other two approaches (Table 5). It produced substantially better results in the WHAKA stand, although this may again be as much a function of the small inventory sample size as a function of the algorithms.

#### 4.2.2 Monetary Return

When monetary return is used as the metric for comparing implementation approaches, the Adjusted Price List approach did not consistently outperform the others. Having the highest AD% does not guarantee the highest monetary return from the stand. This is because the AD% metric treats every log product equally, however, rarely is the importance of fulfilling every order the same.

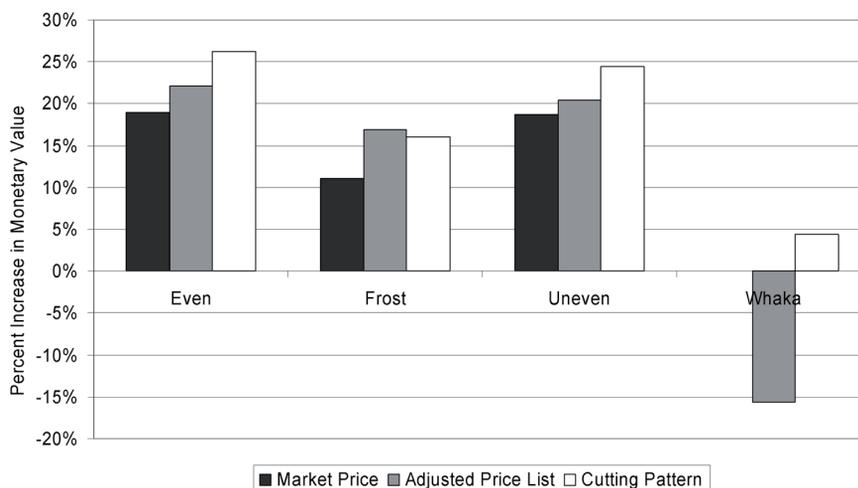
Table 6 summarizes the monetary return results, showing which approach achieved the highest monetary return for each stand under the different scenarios. Using this metric to evaluate the performance of the different approaches, the

Cutting Pattern approach generally outperformed the other approaches but may be stand and market scenario dependent.

The advantage, in terms of monetary return, of developing a buck-to-order plan is shown in Fig. 3. It shows, for the Buy/Sell Min market scenario, the percentage improvement in return of the three different approaches when a buck-to-order plan has been developed compared with when no plan has been developed. Only the Adjusted Price List approach in the WHAKA stand does not produce a positive percentage increase in revenue. This is because, in order to maximize the AD%, the model under-produced the high value products and over-produced the low value products. In this case the two objectives, monetary return and order fulfillment, were in conflict with each other.

## 5 Discussion and Conclusions

An optimal bucking policy can be produced for a single stem, a single stand, or a set of stands to be successively or concurrently harvested (a forest) (Kivinen and Uusitalo 2002). We have introduced three single-stand models that could be used in the planning for, and implementation of, bucking-to-order procedures on mechanised harvesters.



**Fig. 3.** The percentage increase in return using the different approaches under the “Buy/Sell Min” marketing scenario.

The models were only applied to a single species, *Pinus radiata* which, unlike Norway spruce (*Picea abies*), often exhibits considerable variation in quality along each stem. The models were tested in four stands of similar average size but differing levels of complexity in terms of stand treatment (thinning and pruning) and tree size distribution. Only one of these stands was a real-world stand and it was rather small, less than 2 ha.

A single set of seven log-types and a single target distribution were applied to all four stands. In comparison with Scandinavian markets, where there are many log classes based on combinations of length and diameter, demand constraints were relatively few. Performance of the models was tested under four levels of market restriction, however.

In practice, modern harvesters are able to not only measure length and diameter but also predict the profile of the unknown part of the stem. In a normal operation, the harvester head measures and delimits a portion of the stem length, then the harvester’s computer predicts the profile for the rest of the stem. An optimal solution is then calculated and a log is cut. The process is then repeated until the whole stem has been completely cut into logs. In our models we assume that 1)

all stem measurements are error-free and 2) the entire stem, not just a portion of it, is measured prior to calculating an optimal bucking solution. Marshall (2005) and Murphy (2003b) have shown that both these assumptions are likely to result in log distributions that will differ from those found in practice. Rather than have to decide on what would be appropriate error distributions and scanning lengths to use we decided to control these source of variability for this study.

Given the above limitations of this study we have been able to show that 1) significant gains can be made by first determining the optimal volume that should be cut from the stand; that is, buck-to-order planning, and 2) the Target Cutting Pattern and Adjusted Price List approaches will generally outperform the simple Market Prices approach for implementing the buck-to-order plan.

The buck-to-order planning model gives harvest planners the ability to analyze different market and operational conditions before harvesting the stand. It not only maximizes the value of the stand, given that market constraints exist, but also provides predictions of the surplus volume and the required extra volume before starting to harvest the stand. Having this information means that good markets can be found for the surplus volume

as well as potential sources for the volume that is going to be in short supply. Having to buy volume off the spot market can sometimes be extremely costly. Equally as costly is having surplus volume that can not be sold. Often, unsold volume has to be downgraded to a lower value product that can be sold. The buck-to-order planning model also enables the planner to determine the costs of forcing the stand to produce sub-optimal target distributions.

The buck-to-order plan formulation presented in this paper is for a single harvesting/processing machine operating in a single stand. There are a number of possible extensions to the formulation that would better model the opportunities that may be available to real-world planners. Kivinen (2004) reports that simulations carried out by Imponen (1999) have shown that harvesters linked through mobile communications and sharing data can more rapidly achieve a high AD% than the same harvesters without this real-time connection. Kivinen (2004) also indicated the outputs from multiple stands may complement each other and facilitate meeting market constraints. Since not all logging crews have the same production capabilities, there is the potential to also integrate the crew scheduling concepts presented in Murphy (1998) and Mitchell (2004). These extensions to the buck-to-order problem, that is taking into consideration multiple machines, multiple stands and crew scheduling, could be added to our formulation. They would, however, increase the problem size dramatically, and may require the use of column generation techniques to solve the planning problem.

Implementing the buck-to-order plan is not easy. We have shown that using market prices will generally not result in the best implementation of the plan. In some companies, harvest schedulers adjust the market prices without the use of computer aids to take into account market constraints. Although the basic idea behind the adjustment process is quite obvious, Kivinen and Uusitalo (2002) report that “there have been marked differences in bucking results between regions, contractors, and harvester types” in Finland from using such an approach.

We found that the Adjusted Price List model seems to out perform the Cutting Pattern model when AD% is used as a metric. Although not

entirely unambiguous, it seems that this is reversed when the monetary return is used as the metric of performance. This is largely due to the objective functions in the two algorithms. The Cutting Pattern model weights the volume deviation to place a high importance on cutting the higher value products, whereas the Adjusted Price List model simply tries to maximize the AD%.

AD%’s for the three virtual *Pinus radiata* stands ranged between 84.9 and 96.4% and depended on both market scenarios and stand conditions. These are similar to those reported by Malinen and Palander (2004) for seven virtual, spruce-dominated stands and by Kivinen and Uusitalo (2002) for four real-world, mature Norway spruce stands. Malinen and Palander used DP and a “near-optimum” approach for selecting cutting patterns for each tree. Kivinen and Uusitalo (2002) used DP and fuzzy logic to adapt the price list used for cutting trees within each stand.

The AD%’s for the real-world WHAKA stand ranged between 52.3 and 81.5%. We believe that the low AD%’s for this stand are probably, more a function of the small sample size of the inventory (14 trees in total), than the cutting instructions and targets generated by the buck-to-order plan. Further work is required to determine the optimal sample size for generating the buck-to-order plan and the cutting instructions for fulfilling the plan. The buck-to-order problem is relatively easy to solve with perfect information on all the trees in the stand, however obtaining this information can be extremely costly.

Further improvements to both the Adaptive Price List and Cutting Pattern models are undoubtedly possible. Malinen and Palander (2004) have demonstrated, for example, that using a flexible, penalty-segmented AD% metric can lead to overall improvements in the AD%. It would also allow weighting of higher value products and may lead to higher monetary return values – although we have not tested this. We were able to show, however, that significantly better order fulfillment was achieved for all four market scenarios by the Cutting Pattern model when pulp and waste volume targets were set very close to zero. It is difficult to know, without further testing, whether this is a universal rule or just applies to the stands/market scenarios used in this paper.

Many harvesters on the market have adaptive

buck-to-order systems installed. These systems adjust the cutting instruction while working through the stand. For example, the Ponsse's computer uses an adaptive-price-list where the value of each log grade is changed, as the harvesting progresses through the stand, in accordance with how well the demand for each product is being met (Sondell et al. 2002). Other harvester computer manufacturers, such as Dasa4, Timbermatic 300, Valmet and Motomit, have implemented an approach called "close-to-optimal", where a cutting solution is selected from the top 5 % of the buck-to-value solutions that best fulfills the demand requirements (Sondell et al. 2002, Kivinen and Uusitalo 2002). Sondell et al. (2002) report AD%s of slightly over 80% for these systems.

Our three models used pre-harvest inventory data for each stand to develop the buck-to-order plan, targets, adaptive price, and target cutting patterns. Other stem data sources could also be used. Kivinen and Uusitalo (2002) found that using inventory data in their fuzzy logic model to find the adaptive price list gave the highest AD% for only one of the four spruce stands tested. In the other three stands, using only data from previous stems harvested to adjust the prices produced the best AD%. Similarly, Murphy et al. (2004) found that using pre-harvest inventory data from four *Pinus radiata* stands resulted in AD%s that were 0.7 to 7.6 points lower than when using recently harvested stem data.

Order book constraints that constrain the total volume of the different log products are not the only type of market constraints. Other log mix constraints, such as minimum average SED and percentage long logs within a product group are required by some customers. These types of constraints were not included in the analysis carried out in this paper. However, the FASTBUCK algorithm (Murphy et al., 2004), was developed so that these types of constraints could be included. It is feasible, yet not tested, that these constraints could be included into the formulation of Cutting Pattern model. The minimize volume deviation objective function in the dynamic programming algorithm could be penalized if a particular log caused the constraints to be violated.

Three models for bucking-to-order were described in this paper; one for buck-to-order

planning and two for buck-to-order implementation. The buck-to-order planning and implementation models were shown, in four test stands and a range of market scenarios, to yield log distributions that more closely met target distributions than would be found when using market prices alone. Further testing of these models in a wider range of stands, species and market conditions is required. Recent work in Scandinavia, USA and New Zealand indicates that further development of models such as these is both possible and likely to further improve their utility.

## References

- Arce, J.E., Carnieri, C., Sanquetta, C.R. & Filho, A.F. 2002. A forest level bucking optimization system that considers customer's demand and transportation costs. *Forest Science* 48(3): 493–503.
- Bergstrand, K.G. 1989. Fördelningsaptering med näroptimal. Skogsarbeten. Unpublished internal report. 11 p.
- Boston, K. 2001. Precision log making for plantation operations. Proceedings of the First International Precision Forestry Cooperative Symposium, June 2001 Seattle, Washington, USA. p. 165–170.
- Briggs, D.G. 1980. A dynamic programming approach to optimizing stem conversion. Ph.D. thesis. University of Washington. 409 p.
- Deadman, M.W. & Goulding, C.J. 1979. A method for assessment of recoverable volume by log-type. *New Zealand Journal of Forestry Science* 9: 161–175
- Duffner, W.W. 1980. Decision making from market to stump. Proceedings of Weyerhaeuser Science Symposium, Tacoma, Washington, USA. p. 81–95.
- Eng, G., Daellenbach, H. & Whyte, A.G.D. 1986. Bucking tree-length stems optimally. *Canadian Journal of Forest Research* 16: 1030–1035.
- Geerts, J.M.P. & Twaddle, A.A. 1984. A method to assess log value loss caused by cross-cutting practice on the skidsite. *New Zealand Journal of Forestry* 29(2): 173–184.
- Godin, A.E. 2000. Logging equipment database: 1999 update. Forest Engineering Research Institute of Canada, *Advantage* 1(20). 2 p.
- Imponen, V. 1999. Puutavaralogistiikka pelkistää hankinnan toimintamallit. *Metsätieteen aikakauskirja* 4: 722–726. (In Finnish).

- Kivinen, V-P. 2004. A genetic algorithm approach to tree bucking optimization. *Forest Science* 50(5): 696–710.
- & Uusitalo, J. 2002. Applying fuzzy logic to tree bucking control. *Forest Science* 48(4): 673–684.
- Laroze, A.J. 1993. Development and comparison of stand-level bucking optimization methods. PhD thesis. Oregon State University. 95 p.
- 1999. A linear programming, tabu search method for solving forest-level bucking optimization problems. *Forest Science* 45(1): 108–116.
- & Greber, B. 1997. Using tabu search to generate stand-level, rule-based bucking patterns. *Forest Science* 43(2): 157–169.
- Malinen, J. & Palander, T. 2004. Metrics for distribution similarity applied to the bucking to demand procedure. *Journal of Forest Engineering* 15(1): 33–40.
- Marshall, H.D. 2005. An investigation of factors affecting the optimal output log distribution from mechanical harvesting and processing systems. Ph.D. thesis. Oregon State University. 211 p.
- Mendoza, G.A. & Bare, B.B. 1986. A two-stage decision model for log-bucking and allocation. *Forest Products Journal* 36(10): 70–74
- Mitchell, S.A. 2004. Operational forest harvest scheduling optimisation: A mathematical model and solution strategy. PhD thesis. Auckland University. New Zealand. 272 p.
- Murphy, G.E. 1998. Allocation of stands and cutting patterns to logging crews using tabu search heuristics. *Journal of Forest Engineering* 9: 31–37.
- 2003a. Mechanization and value recovery: worldwide experiences. Proceedings of the Woodfor Africa Forest Engineering Conference, July 2002, Pietermaritzburg, South Africa. Forest Engineering Department, Corvallis, Oregon. p. 23–32.
- 2003b. Procedures for scanning radiata pine stems dimensions and quality on mechanised processors. *Journal of Forest Engineering* 14(2): 11–21
- , Marshall, H.D. & Bolding, M.C. 2004. Adaptive control of bucking on harvesters to meet order book constraints. *Forest Products Journal* 54(12): 114–121.
- Näsberg, M. 1985. Mathematical programming model for optimal log bucking. Linköping Studies in Science and Technology, Dissertation 132. Linköping. 200 p.
- Nordlund, S. 1996. Drivningsteknik och metodutveckling i storskogsbruket. Skogforsk, Resultat 4. 3 p. (In Swedish with English summary).
- Pickens, J.B., Lee, A. & Lyon, G.W. 1992. Optimal bucking of hardwoods. *Northern Journal of Applied Forestry* 9(4): 149–152.
- Pnevmticos, S.M. & Mann, S.H. 1972. Dynamic programming in tree bucking. *Forest Products Journal* 22(2): 26–30.
- Raymond, K. 1988. Mechanised harvesting developments in Australia. Logging Industry Research Association, Rotorua, New Zealand, Report P.R. 37. 47 p.
- Sessions, J. 1988. Making better tree-bucking decisions in the woods. *Journal of Forestry* 86(10): 43–45.
- Sessions, J., Olsen, E.D. & Garland, J.J. 1989. Tree bucking for optimal stand value with log allocation constraints. *Forest Science* 35(1): 271–276.
- Smith, G.W. & Harrell, C. 1961. Linear programming in log production. *Forest Products Journal* 11(1): 8–11.
- Sondell, J., Möller, J.J. & Arlinder, J. 2002. Third-generation merchandising computers. Skogforsk, Results 2. 6 p.

*Total of 32 references*