LMS-LIKE TWO DIMENSIONAL ADAPTIVE FILTER
WITH A t-DISTRIBUTION ASSUMPTION AND
NONSYMMETRICAL HALF-PLANE SUPPORT

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ABSTRACT
This paper will focus on proposing a new objective function of the two-dimensional adaptive filters. The objective function is derived by assuming that the obtained error signal is identically and independently distributed with \( t \)-distribution probability density function. Small portions of error signals with large amplitude is given a small weighting factor and large portions of error signals with small amplitude is assigned a large weighting factor. By doing so, the effect of large amplitude error signal is suppressed and the adaptation step is less affected. The simulation results for image enhancement show that when the input is impulsive noise contaminated images, the obtained processed image by using \( t \)-distribution with small \( \alpha \) degrees of freedom is much better than that when large \( \alpha \) is applied.

1. INTRODUCTION
Two dimensional adaptive filter is one of the important parts in many image processing applications [1]. The parameter of the model is solved by using a certain objective function which is a function of the error signal. One of the important factor which determine the performance of the adaptive system is the obtained parameter. It is necessary to develop a method to accurately adapt the parameter of the adaptive system. The nature behavior of the error signal has to be carefully considered in determining the best objective function to precisely adapt the parameter of the adaptive system.

Conventionally, the parameter of the adaptive system is determined to minimize the sum of the square of the error signal. All portions of the error signal is assigned the same weighting factor. By doing so, small portions of the error signal with large amplitude have more effect to the obtained parameter than that of large portions of error signal with small amplitude [2]. Therefore, the obtained parameter of the adaptive system is very much affected by large amplitude error signal portions.

There have been many efforts done in the past to reduce the effect of large amplitude error signal by applying a nonlinear weighting function. One of the most popular objective functions is derived by assuming that the error signal is identically and independently distributed with Huber’s distribution [3]. By doing so, the small portions of large amplitude errors are assigned a small weighting factor. On the other hand, large weighting factor is applied for large portions of small amplitude error signals. Recently, we proposed the usage of \( t \)-distribution assumption to reduce the effect of large amplitude for speech analysis[2]. We have shown and proved that the effect of large amplitude error is less by applying \( t \)-distribution assumption than that by utilizing the Huber’s distribution assumption [2].

Extending the already proposed method in [2], in this paper we propose a two-dimensional adaptive filter based on \( t \)-distribution assumption to reduce the effect of large amplitude error signals.

The proposed adaptive filter has been applied for image enhancement. The input image is additive noise contaminated. The simulation results show that when the input image is impulsive noise contaminated, we can get better improvement by applying the proposed method than that by using the conventional adaptive method.

2. THE PROPOSED METHOD

\[ e(m,n) = d(m) - \sum_{i,j \in S} a(i, j)x(m - i, n - j) \]  \hspace{1cm}(1)

We consider a transversal two-dimensional adaptive filter as depicted in Figure 1. The input signal is \( x(m,n) \) and \( d(m,n) \) is the desired signal. The error signal is denoted as \( e(m,n) \). The error signal can be calculated from the input signal \( x(m,n) \), the desired signal \( d(m,n) \) and the parameter of the adaptive filter by the above equation (1).
The region of support $S$ is selected to be nonsymmetrical half plane (NSHP). The considered region of support is selected to be NSHP to achieved better results, but the output is still possible to be recursively calculated [4]. The desired coefficient is $a(i,j)$, $(i,j) \in S$. The size of the input image is assumed to be $M \times N$. The order of the transversal filter $p$ and $q$ has to be predetermined based on the applications.

In the conventional adaptive system, it is assumed that the error is IID with Gaussian probability density function. In this case, the optimal adaptive parameter is calculated to minimize the sum of the error signal. All portion of the error signal are assigned the same weighting factor, so that large amplitude error signals have more effect on the obtained adaptation step than that of small amplitude error signals.

In this paper, we propose that the usage of the $t$-distribution assumption to derive the objective function. By doing so, we assume that the probability density function of the error signal is

$$f_\alpha(x) = K_\alpha \left\{w_\alpha(x)\right\}^{(\alpha+1)/2}$$  \hspace{1cm} (3)

Where $K_\alpha$ is a constant depend on the degree of freedom $\alpha$ and it is defined as

$$K_\alpha = \frac{\Gamma\left(\frac{\alpha+1}{2}\right)}{\Gamma\left(\frac{\alpha}{2}\right)}$$  \hspace{1cm} (4)

and

$$w_\alpha = \frac{1}{1 + \frac{\gamma^2}{\alpha}}$$  \hspace{1cm} (5)

The optimal adaptive system coefficient is selected to maximize the log likelihood of the error signal in (6)

$$L = \log \left( K_\beta - \tilde{L}(e(m,n)) \right)$$  \hspace{1cm} (6)

where

$$L(e(m,n)) = \frac{\alpha+1}{2} E\left\{\log \left( w_\alpha \left( x(m,n) \right) \right) \right\}$$  \hspace{1cm} (7)

and $E\{.\}$ is the mean operator. The robust scale estimate is

$$\tilde{\gamma} = \text{median} \left\{ |e(m,n)| \right\}$$  \hspace{1cm} (8)

along a certain specified window $0 \leq m, n \leq (scl - 1)$.

Larger $scl$ will produce more accurate result. On the other hand, larger $scl$ needs more calculations to solve the median. Compromising both factors, in this paper we select $scl = 64$.

Since the degree of freedom $\alpha$ is a pre-selected value, the first term in (6) is a constant. Thus, maximizing the log like-hood function in (6) is solved by minimizing (7)
using the steepest descent method [6]. It is done by updating the coefficient of the adaptive filter by

\[ a^q(m,n) = a^{q-1}(m,n) - \mu \nabla(m,n) \]  

where the desired coefficient \( a^q(m,n) \) is defined as

\[ a^q(m,n) = \text{col}[a(i,j)] \]  

and \((i,j) \in S\) is the desired optimal system parameter at the \( q \)-th iteration. The step size parameter \( \mu \) has to be experimentally determined based on the nature of the application. The gradient vector is defined as

\[ \nabla(m,n) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \beta \left( \frac{e(m,n)}{\hat{s}} \right) x(m,n) \]  

The iteration number is \( q = (m,M + n). \) The input vector of the input image is defined as

\[ x = \text{col}\left[x(m+i,n+j)\right] \]  

where \((i,j) \in S.\)

The function \( \beta(x) \) is the influence function which determines the suppression ability of the proposed algorithm due to the large amplitude error signal. Practically, the input signal is subsequently available. On the other hand, it is desirable that the adaptive filter reacts instantaneously if there is a change in the input signal. Therefore, it is just impossible to wait all sample of the input signal to be available to evaluate the exact gradient value. Thus, in this paper we propose the usage of the short-term gradient value as an approximation of \( \nabla^q(m,n) \) which is calculated each time the number of the input samples grows [5]. The short gradient is given in (13).

\[ \nabla^q(m,n) = x(m,n)x(m,n) \]  

Therefore we will obtain (14) which is the desired adaptation formula in this paper.

When \( \alpha = \infty \) is used, we get the Gaussian probability density function. In this case, we get exactly the same formula as in the conventional LMS [7]. Thus, the proposed method can be regarded as a generalization of the conventional LMS method.

3. THE SIMULATION RESULTS

The proposed adaptive algorithm has been realized in the form of the two-dimensional adaptive enhancer [7] to process a contaminated image as depicted in Fig. 2. The contaminated image is artificially generated by adding a

![Figure 3 The image enhancement system scheme.](image-url)
random binary impulsive noise [2] into a clean 256 by 256 Lenna image as given in Fig. 3. We adjusted the signal to noise ration (SNR) of the contaminated image to be –3.98 dB. The contaminated image is shown in Fig. 4.

Figure 4. The original Lenna image used in the simulation.

Figure 5. The impulsive noise contaminated image.

Figure 6. The obtained Lenna image after processing using $\alpha = 3$.

The resulted signal to noise ratio (SNR) of the resulted image which is obtained using various convergence factor $\mu$ is depicted in Fig. 8. Those plots show that the obtained image by applying small degree of freedom $\alpha$ is much more cleaner than that by using large $\alpha$. In real applications, it is impossible to know the nature of the noise beforehand and it might be impulsive. Therefore, we recommend the usage of small $\alpha$ degrees of freedom from the fact that the obtained SNR is better than that by using large $\alpha$ assumption.

Figure 7. The resulted Lenna image after processing using $\alpha = \infty$.

We can see from Figs. 5 and 6 that the resulted image by applying $\alpha = 3$ is much more clearer than that by using $\alpha = \infty$. When $\alpha = \infty$ is applied, the resulted image is overflow in some parts which are appeared as black dots in Fig. 6.
4. CONCLUSIONS

We have presented a novel two dimensional adaptive system based on \( t \)-distribution with \( \alpha \) degrees of freedom assumption. The optimal coefficient is solved by the LMS algorithm. In case when the input of the adaptive system is an impulsive contaminated image, the obtained output image has higher SNR than that by using the conventional LMS approach. Further applications of the proposed algorithm is still studied and will be reported somewhere else.

5. REFERENCES


Figure 8. The resulted SNR using various \( \alpha \) when the input is impulsive noise contaminated image.