Worst-Case Optimal and Average-Case Efficient Geometric Ad-Hoc Routing
What is Geometric Routing?

- A.k.a. location-based, position-based, geographic, etc.
- Chapter 18 in Handbook of Wireless Networking and Mobile Computing [Stojmenovic 2002]
- Famous representative: GPSR [Karp & Kung, MobiCom 2000]

- Each node knows its own position and position of neighbors
- Source knows the position of the destination
- No routing tables stored in nodes!

- Geometric routing makes sense
  - GPS/Galileo, local positioning algorithm, overlay P2P net, geocasting
  - Most importantly: Learn about general ad-hoc routing
Greedy Routing

- Each node forwards message to “best” neighbor
Greedy Routing

• Each node forwards message to “best” neighbor

• But greedy routing may fail: message may be stuck in a “dead end”
• Needed: Correct geometric routing algorithm
Overview

• Introduction
  – What is Geometric Routing?
  – Greedy Routing

• Correct Geometric Routing: Face Routing

• Efficient Geometric Routing
  – Adaptively Bound Searchable Area
  – Lower Bound
  – Worst-Case Optimality
  – Average-Case Efficiency
  – GOAFR+

• Conclusions
Face Routing

• Based on ideas by [Kranakis, Singh, Urrutia CCCG 1999]
• Here simplified (and actually improved)
Face Routing

- Remark: Planar graph can easily (and locally!) be computed with the Gabriel Graph, for example.
Face Routing
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Face Routing

- Theorem: Face Routing reaches destination in $O(n)$ steps
- But: Can be very bad compared to the optimal route
Bounding Searchable Area
Adaptively Bound Searchable Area

- What is the correct size of the bounding area?
  - Start with a small searchable area
  - Grow area each time you cannot reach the destination
  - In other words, adapt area size whenever it is too small

Theorem: Algorithm finds destination after $O(c^2)$ steps, where $c$ is the cost of the optimal path from source to destination.

- Proof: Not in this presentation.
Algorithm is worst-case optimal

- Can we do any better?

- No, with example
  - Destination is central node
  - Source is any node on ring
  - Any spine can go to middle
  - Geometric routing: no routing tables \(\rightarrow\) test many spines
  - Best path of size \(O(c)+O(c)\)
  - Test \(\Omega(c)\) spines of length \(\Omega(c)\)
  - Cost \(\Omega(c^2)\) instead of \(O(c)\)

Theorem: Algorithm is asymptotically worst-case optimal.
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GOAFR – Greedy Other Adaptive Face Routing

- Algorithm is not very efficient (especially in dense graphs)

- Combine Greedy and (Other Adaptive) Face Routing
  - Route greedily as long as possible
  - Circumvent “dead ends” by use of face routing
  - Then route greedily again

Theorem: GOAFR is still asymptotically worst-case optimal…
…and it is efficient in practice, in the average-case.

- What does “practice” mean?
  - Usually nodes placed uniformly at random
Average Case

- Not interesting when graph not dense enough
- Not interesting when graph is too dense
- **Critical density range** ("percolation" $\rightarrow$ IP 1)
  - Shortest path is significantly longer than Euclidean distance

![Network Density Examples](image)
Simulation on Randomly Generated Graphs

Network Density [nodes per unit disk]

Performance

greedy success

Connectivity

GPSR

GOAFR+

better

worse

Frequency
A Word on Performance

• What does a performance of 3.3 in the critical density range mean?

• If an optimal path (found by Dijkstra) has cost \( c \), then GOAFR+ finds the destination in \( 3.3c \) steps.

• It does not mean that the path found is 3.3 times as long as the optimal path! The path found can be much smaller...

• Remarks about cost metrics
  – In this talk “cost” \( c = c \) hops
  – We have other results, for instance on distance/energy/hybrid metrics
  – In particular: With energy metric there is no competitive geometric routing algorithm
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Conclusions

• The first geometric routing algorithm that is correct and (using a lower bound argument) worst-case optimal [KWZ DialM 2002]

• Improved algorithm (GOAFR+) still worst-case optimal but also average-case efficient [KWZ submitted]

• Using percolation theory there is a new way to evaluate ad-hoc routing algorithms in general (not only geometric algorithms)

• Arguably one of the first tight results in ad-hoc routing
Future Work

• Overlay P2P networks
  – Distributed P2P-style information system for position of destinations

• Stable form of source routing
  – Mobility of intermediate nodes are not a problem anymore

• Non-geometric routing
  – How do we route with sparse routing tables?
  – New source routing algorithms

• Other network layer structures
  – First constant-time local (Connected) Dominating Set approximation
  – See chapter 20 of Stojmenovic’s Handbook…
Questions?
Comments?

Distributed Computing Group