Performance of multiple-input and multiple-output orthogonal frequency and code division multiplexing systems in fading channels

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Abstract: In broadband downlink transmission, orthogonal frequency-division multiplexing (OFDM) combined with code-division multiple access (CDMA) is a prospective technique for high-data rate transmission in future wireless communication systems. By adding spatial diversity, multiple-input and multiple-output orthogonal frequency and code division multiplexing (MIMO-OFCDM) offers superior performance relative to both traditional OFDM systems and single-input and single-output OFCDM (SISO-OFCDM) systems. In this study, the authors present an analytical study and investigation of a MIMO-OFCDM downlink system that hires orthogonal variable spreading factor codes to spread each transmitted symbol in both time and frequency domains. Different gain combining schemes are employed in the frequency domain to recover the data symbols of the desired code channels, and space–time block coding is used to achieve spatial diversity. The more general Ricean fading channel is used to model the MIMO channel. The OFCDM system employs Alamouti transmit diversity scheme with multiple receive antennas. For systems without multi-code interference (MCI), analytical bit-error rate results are obtained and compared with simulation results. The authors also investigate the effect of correlation in frequency domain, where we verify that minimum mean-square error frequency combining is more robust to MCI than equal-gain combining.

1 Introduction

Along with the fast growing demand of information exchange, telecommunication systems are required to provide fast and reliable services to high-data rate applications such as video conferences, real-time broadcast and on-line games. Optimising the use of available bandwidth is one of the major challenges in wireless communications. Multiple-access systems provide a solution for optimising frequency use and maximising flexibility. For instance, frequency-division multiple access (FDMA), time-division multiple access (TDMA), spread spectrum multiple access (SSMA) and space-division multiple access (SDMA) are well investigated and implemented in different communication systems [1]. Also, orthogonal frequency-division multiplexing (OFDM) is one useful multiplexing scheme that has attracted many researchers [2–4]. It divides data into a set of parallel streams to be transmitted using mutually orthogonal sub-carriers. Hence the total channel bandwidth is converted into multiple slow rate channels to avoid the effects of intersymbol interference at the receiver side. On the other hand, code-division multiple access (CDMA) is one form of ‘spread-spectrum’ signalling that is broadly used in telecommunication systems [5, 6]. In a CDMA system, users are assigned orthogonal codes so that different users can share the same frequency at the same time with little or no interference.

The integration of OFDM and CDMA takes advantage of both systems. Typical implementations of CDMA into OFDM-based systems include multi-carrier direct-sequence CDMA (MC-DS-CDMA), multi-carrier CDMA (MC-CDMA). MC-DS-CDMA systems spread symbols in time domain only [7, 8], so they cannot achieve frequency domain diversity. On the other hand, MC-CDMA systems spread symbols in frequency domain...
only [8, 9], and therefore cannot adapt to variable transmission rates.

In a different approach for high-data rate transmission, orthogonal frequency and code division multiplexing (OFCDM), a combination of OFDM and CDMA can provide better performance than both traditional direct sequence CDMA (DS-CDMA) and OFDM schemes [10]. In an OFCDM system, symbols are spread in both frequency and time domain. Therefore OFCDM systems can deliver frequency diversity gain and can be adapted to applications with different transmission rates, which is preferred in the 4G downlink transmission systems [8, 10, 11]. A single-input single-output OFCDM (SISO-OFCDM) system with two-dimensional spreading in time and frequency domain has been studied in [10, 12, 13]. In these works, the authors focused on implementing hybrid detection technique in OFCDM systems. Minimum mean-square error (MMSE) combining technique is used to gain diversity in the frequency domain. By comparing the different detection techniques in [14], we notice that when no multi-code interference (MCI) is present, maximal ratio combining (MRC), equal-gain combining (EGC) and MMSE combining can achieve the full frequency/spatial diversity gains. However for systems with MCI, MMSE combining becomes superior to MRC and EGC.

As the demand to provide reliable transmission for high-data rate applications increases, multiple-input multiple-output (MIMO) techniques such as space–time trellis coding and space–time block coding (STBC) are introduced and widely studied [15–17]. To overcome the problem of fading through diversity, MIMO techniques implement multiple transmit antennas for the transmission of the same signal through independent channels to provide reliable wireless transmission. By adding spatial diversity, a MIMO system can provide more reliable transmission than a SISO system. In this paper, we explain how to combine MIMO techniques, specifically STBC, with OFCDM systems. The space–time scheme used in our study is the Alamouti scheme [18], which employs \( N = 2 \) and \( M \) antennas at the transmitter and receiver sides, respectively. We present the system structure, transmission and detection methods of MIMO-OFCDM systems. Also, we derive the bit-error rate (BER) expression for such MIMO-OFCDM systems when MRC, EGC and MMSE combining techniques are employed. In our simulations, the more general Ricean fading channel is used to model the MIMO channel.

In our study, the expressions for system BER are considered under the condition that no MCI is present. The accuracy of the BER expressions is verified when compared with the simulated ones for MIMO-OFCDM systems with different combining methods. These comparisons are carried over different channels and with different system parameters to explore the benefits of OFCDM-based systems. Both analytical and simulation results show the large diversity gains achieved when incorporating STBC with OFCDM. Also, we investigate the effect of correlation in frequency domain where we verify that MMSE frequency combining is robust to MCI.

The rest of the paper is organised as follows. In Section 2, we introduce the transmitter structure and the transmission scheme for the MIMO-OFCDM system. The combining and detection methods are explained in Section 3. In Section 4, we discuss the error probability for different gain combining techniques. Then, we present comparison of analytical results and simulation results in Section 5. In Section 6, we draw our conclusions.

## 2 System description

Combined with STBC technique, SISO-OFCDM systems introduced in [12] can be extended to MIMO-OFCDM systems to gain spatial diversity. The transmitter structure is shown in Fig. 1. The system considered here employs \( N = 2 \) transmit antennas and \( M \) receive antennas. At the beginning of each two frame durations, \( 2KN_B \) user data are modulated into binary phase-shift keying (BPSK) symbols, where \( K \) is the total number of code channels actually transmitted, and \( N_B \) is the group of data symbols to be spread with the same spreading code and sent to one transmit antenna. When quadrature phase-shift keying or higher order modulation is used, Grey coding should be employed. Among these modulated symbols, \( KN_B \) symbols are transmitted from the first transmit.
antenna, and the remaining \( KN_B \) symbols from the second transmit antenna. Before transmission from the corresponding antenna, the modulated symbols are then converted from a serial symbol stream into \( N_B \) parallel symbol streams. The spreading is provided by a spreading code generator, and each group of \( N_B \) symbols will be spread with a unique code. In the OFCDM system, let the spreading factor in frequency domain and time domain be \( N_F \) and \( N_T \), respectively, then the total length of the spreading code, \( N_S \), is given by

\[
N_S = N_T \times N_F
\]  

(1)

Hence, a maximum of \( N_S \times N_B \) symbols can be transmitted at the same time when each group is assigned a unique spreading code. Since \( K \) is the total number of code channels, then \( K \leq N_S \). The overall spreading code \( C \) is a combination of the time-domain spreading code, \( C^T \), and frequency-domain spreading code, \( C^F \). Here, both \( C^T \) and \( C^F \) are orthogonal variable spreading factor (OVSF) codes. The generation and some attributes of OVSF codes can be found in [19]. Let \( C(k) \) represent the \( k \)th \((k = 0, 1, \ldots, K - 1)\) spreading code, \( C^T(a) \) represent the \( a \)th \((a = 0, 1, \ldots, N_T - 1)\) time-domain spreading code of length \( N_T \), \( C^F(b) \) represent the \( b \)th \((b = 0, 1, \ldots, N_F - 1)\) frequency-domain OVSF spreading code of length \( N_F \), denoted, respectively, as

\[
C(k) = \begin{bmatrix}
\psi_0(k) & \psi_1(k) & \cdots & \psi_{N_F - 1}(k) \\
\psi_{N_F}(k) & \psi_{N_F + 1}(k) & \cdots & \psi_{2N_F - 1}(k) \\
\cdots & \cdots & \cdots & \cdots \\
\psi_{(N_F - 1)N_F}(k) & \psi_{(N_F - 1)N_F + 1}(k) & \cdots & \psi_{N_F^2}(k)
\end{bmatrix}
\]  

(2)

\[
C^T(a) = \begin{bmatrix}
\psi_0^T(a) \\
\psi_1^T(a) \\
\vdots \\
\psi_{N_T - 1}^T(a)
\end{bmatrix}
\]  

(3)

\[
C^F(b) = \begin{bmatrix}
\psi_0^F(b) & \psi_1^F(b) & \cdots & \psi_{N_F - 1}^F(b)
\end{bmatrix}
\]  

(4)

When the code \( C(k) \) \((k = 0, 1, \ldots, K - 1)\) is generated by \( C^T(a) \) and \( C^F(b) \), then

\[
C(k) = [C^F(b)]^T C^T(a)
\]  

(5)

where \((\cdot)^T\) represents transpose operation.

In our study we focus on downlink transmission, where we assume perfect synchronisation. Also, we assume that each sub-carrier experiences flat fading, so the orthogonality of the OVSF code in time domain is preserved. However, the orthogonality in frequency domain is distorted because of different fading among different sub-carriers.

In an OFCDM system, multi-code interference arises when different users’ data are spread with different frequency-domain spreading codes and the same time-domain spreading code. To overcome this, the spreading code should be designed in such a way to take advantage of the orthogonality in time domain. Specifically, the spreading code generator should minimise the number of frequency-domain OVSF spreading codes used so that the level of MCI can be minimised. For example, the first \( N_T \) spreading codes are generated by multiplying \( C^F(0) \) with \( C^T(0), C^T(1), C^T(2), \ldots, C^T(N_F - 1) \), respectively, the second \( N_T \) spreading codes are generated by multiplying \( C^F(1) \) with \( C^T(0), C^T(1), C^T(2), \ldots, C^T(N_F - 1) \), respectively, and so forth. From this method, one can see that when the code \( C(k) \) is generated by \( C^T(a) \) and \( C^F(b) \), the relationship between \( k, a \) and \( b \) is given by

\[
a = k \mod N_T = k - N_T \left\lfloor \frac{k}{N_T} \right\rfloor
\]  

(6)

\[
b = \left\lfloor \frac{k}{N_T} \right\rfloor
\]  

(7)

where the operator \([x]\) represents the integer portion of \( x \) with \([x]\) \(\leq x\).

Given the spreading factor in frequency domain, \( N_F \), then \( N_F \) sub-carriers are dedicated to transmit the same data symbol. In order to minimise the correlation among these \( N_F \) sub-carriers, one should ensure that the sub-carriers are separated enough using frequency interleaving. The method of interleaving is explained as follows. In a group of \( N_B \) symbols spread with the same spreading code, the first symbol is spread into the zeroth, \( N_B \)th, \((2N_B)\)th, and \((N_B - 1)\)th sub-carriers, the second symbol is spread into the first, \((N_B + 1)\)th, \((2N_B + 1)\)th, and \([(N_B - 1)N_B + 1]\)th sub-carriers and so forth. Finally, the \( N_B \)th symbol is spread into the \((N_B - 1)\)th, \((2N_B - 1)\)th, \ldots, and \((L - 1)\)th sub-carriers, where \( L = N_F N_B \) is the total number of sub-carriers employed in the OFCDM system.

To explain how the two-dimensional spreading is done, let us take the spreading code assigned to the \( j \)th \((j = 1, 2, \ldots, N_B)\) symbol in the \( k \)th \((k = 0, 1, \ldots, K - 1)\) group as an example. This assigned spreading code, noted as \( C(k) \), is presented in Fig. 2. The first \( N_T \) chips of \( C(k) \), \( \{\psi_0(k), \psi_1(k), \ldots, \psi_{N_T - 1}(k)\} \) are assigned to the \((j - 1)\)th sub-carrier; the second \( N_T \) chips of \( C(k) \), \( \{\psi_{N_T}(k), \psi_{N_T + 1}(k), \ldots, \psi_{2N_T - 1}(k)\} \) are assigned to the \((j - 1 + N_B)\)th sub-carrier, and so forth. Finally, the last \( N_T \) chips of \( C(k) \) are assigned to the \((j - 1 + (N_B - 1)N_B)\)th sub-carrier.

As shown in Fig. 2, a signal transmitted during one time slot on one sub-carrier in OFCDM system is called an OFCDM symbol. After two-dimensional spreading and frequency interleaving for a group of \( N_B \) modulated symbols, an OFCDM frame consisting of \( L \times N_T \) OFCDM symbols is constructed. Assigned with different spreading codes, in total, \( K \) OFCDM frames are multiplexed to build a superframe, and this step is done using a frame multiplexer, shown in Fig. 3.
Let the indices $l_0, l_1, \ldots, l_{N_F-1}$ represent the $N_F$ sub-carriers carrying a user data spread by the $k$th spreading code $C(k)$ after frequency interleaving, and $C(k)$ is given by (5). Within the OFCDM superframe transmitted from the $n$th ($n=1, 2$) transmit antenna, the transmitted signal on the $l_f$th ($f=0, 1, \ldots, N_F-1$) sub-carrier during the $t$th ($t=0, 1, \ldots, N_T-1$) OFCDM symbol duration is a summation of $K$ OFCDM symbols and is given by

$$S_n(l_f, t) = \frac{E_s}{2N_S} \sum_{k=0}^{K-1} d_{n,k} c(k)$$

where $E_s$ is the transmitted signal energy for each OFCDM symbol, normalised by the number of transmit antennas, $d_{n,k}$ is the user data spread by $C(k)$ on the $l_f$th sub-carrier transmitted from the $n$th antenna during the first frame duration and $c(k)$ is the $\delta$th chip of $C(k)$. Here, $\delta$ is the corresponding chip index of the spreading code, given by

$$\delta = \left\lfloor \frac{l_f}{N_B} \right\rfloor N_T + t$$

(9)

Let $S_1$ and $S_2$ denote, respectively, the two superframes transmitted from antenna 1 and 2. Then as shown in Fig. 4, the OFCDM superframes $S_1, S_2, -S_1^\ast$ and $S_2^\ast$ are transmitted according to the Alamouti scheme through OFDM transmission system. During $T_1$, the first frame duration, $S_1$ is transmitted from the first transmit antenna and $S_2$ from the second antenna. In the second frame period, $-S_1^\ast$ and $S_2^\ast$ are transmitted with the antenna order reversed. Here we assume that the OFCDM symbols transmitted on the same sub-carrier experience the same fading within a pair of OFCDM superframes duration. That is, fading is fixed for the duration of two frames and changes independently from one frame to another.

The receiver structure is shown in Fig. 5. After experiencing channel fading and noise distortion, the signal is picked up by $M$ receive antennas. Let us use $R_{m,1}, R_{m,2}$ to represent the received OFCDM superframes for the two
consecutive frame duration at the receiver side shown in Fig. 4. $R_{m,1}(t, f)$ and $R_{m,2}(t, f)$ represent the received OFCDM signal at the $m$th $(m = 1, 2, \ldots, M)$ receive antenna on the $j$th sub-carrier during the $t$th frame duration, and variance $E[\hat{s}_c^2]$, where $\hat{s}_c$ is the transmitted spreading code. As explained before, the orthogonality in

\begin{equation}
R_{m,1}(t, f) = \sqrt{E_s \frac{E}{2N}} \sum_{k=0}^{K-1} (b_{m,1}^{1} |d_{1,k}^{1}| + b_{m,2}^{2} |d_{2,k}^{2}|) \delta_{k}(f) + \eta_{m,1}(t, f)
\end{equation}

\begin{equation}
R_{m,2}(t, f) = \sqrt{E_s \frac{E}{2N}} \sum_{k=0}^{K-1} (-b_{m,1}^{1} |d_{1,k}^{1}| + b_{m,2}^{2} |d_{2,k}^{2}|) \delta_{k}(f) + \eta_{m,2}(t, f)
\end{equation}

where $b_{m}^{n}$ ($m = 1, 2, \ldots, M$, $n = 1, 2$) is the fading coefficient on the $j$th sub-carrier from the $n$th $n = 1, 2$ transmit antenna to the $m$th receive antenna. $\eta_{m,1}(t, f)$ and $\eta_{m,2}(t, f)$ are the noise terms on the $j$th sub-carrier during the $t$th OFCDM symbol duration at the $m$th receive antenna. $\eta_{m,1}(t, f)$ and $\eta_{m,2}(t, f)$ are additive white Gaussian noise (AWGN) samples with zero mean and variance $N_0/2$ per dimension.

3 Detection algorithm

In this section, we propose the detection algorithm to recover the transmitted data symbols. The algorithm extracts receive diversity based on the optimal decision for the Alamouti scheme with multiple receive antennas.

When the transmitted signals are picked up by receive antennas, they are first combined to decouple the data symbols. Let $Y_1(t, f)$ and $Y_2(t, f)$ be the transformed received signals after combining at the $j$th sub-carrier within the $t$th OFCDM symbol duration. Then $Y_1(t, f)$ and $Y_2(t, f)$ are given by

\begin{equation}
\hat{Y}_1(t, f) = \sum_{m=1}^{M} [(b_{m,1}^{1})^* R_{m,1}(t, f) + b_{m,2}^{2} R_{m,2}(t, f)]
\end{equation}

\begin{equation}
\hat{Y}_2(t, f) = \sum_{m=1}^{M} [((b_{m,1}^{1})^* R_{m,1}(t, f) - b_{m,2}^{2} R_{m,2}(t, f)]
\end{equation}

where $b_{m}^{n}$ is the overall equivalent channel on the $j$th sub-carrier, defined as

\begin{equation}
b_{m}^{n} = \sum_{n=1}^{M} (|b_{m,1}^{1}|^2 + |b_{m,2}^{2}|^2)
\end{equation}

and the equivalent noise after combining becomes

\begin{equation}
\eta_{1}^{eq}(t, f) = \sum_{m=1}^{M} [(b_{m,1}^{1})^* \eta_{m,1}(t, f) + b_{m,2}^{2} \eta_{m,2}(t, f)]
\end{equation}

\begin{equation}
\eta_{2}^{eq}(t, f) = \sum_{m=1}^{M} [((b_{m,1}^{1})^* \eta_{m,1}(t, f) - b_{m,2}^{2} \eta_{m,2}(t, f)]
\end{equation}

$\eta_{1}^{eq}(t, f)$ and $\eta_{2}^{eq}(t, f)$ are both Gaussian with zero mean and variance

\begin{equation}
E[|\eta_{1}^{eq}(t, f)|^2] = E[|\eta_{2}^{eq}(t, f)|^2] = b_{m}^{n}
\end{equation}

where we considered the variance of each $\eta_{m,1}(t, f)$ and $\eta_{m,2}(t, f)$ to be 1/2 per dimension.

Note that the transmitted symbol pair have the same spreading code. As explained before, the orthogonality in

Figure 5 Receiver structure of MIMO-OFCDM system
time domain is preserved, and therefore time-domain despreading is carried out before frequency-domain despreading. Now, the received signal transmitted from the \( n \)th transmit antenna during \( T_1 \) on the \( l_j \)th sub-carrier using the \( n \)th time-domain spreading code, and after time-domain despreading is given by

\[
r_n(l_j) = \frac{1}{N_T} \sum_{r=0}^{N_T-1} \left[ \hat{Y}_{n,r}(l_j, r) \times c_J^T(s) \right]
= \sqrt{\frac{E_s}{2N_S}} h_{n,l_j}^T d_{n,l_j} \mathcal{F}_{l_j/N_0} \left( b + \bar{\eta}_n(l_j) \right) \tag{18}
\]

Since the same data symbol is transmitted on the subcarriers with indices \( l_0, l_1, \ldots, l_{N_T-1} \), the outputs of time-domain despreader over these \( N_T \) sub-carriers are then combined, which is also called frequency-domain despreading. Here, a suitable combining method is employed to extract frequency-domain diversity over these \( N_T \) sub-carriers of interest, and the output of the frequency-domain despreader can then be written as

\[
y_{n,k} = \sum_{j=0}^{N_T-1} r_n(l_j) \mathcal{F}_{l_j/N_0} \left( b + \bar{\eta}_n(l_j) \right) \tag{19}
\]

where \( \bar{\eta}_n(l_j) \) is the equivalent combining weight of the \( l_j \)th sub-carrier. For some commonly used gain combining techniques, \( \bar{\eta}_n(l_j) \) is given by

\[
\bar{\eta}_n(l_j) = (\delta_{l_j}^{\text{eq}})^* = \delta_{l_j}^{\text{eq}} \tag{20}
\]

for MRC \([20]\]

\[
\bar{\eta}_n(l_j) = \frac{(\delta_{l_j}^{\text{eq}})^*}{|\delta_{l_j}^{\text{eq}}|} = 1 \tag{21}
\]

for EGC \([20, 21]\) and

\[
\bar{\eta}_n(l_j) = \left( \frac{(\delta_{l_j}^{\text{eq}})^*}{1 + K}(\delta_{l_j}^{\text{eq}})^2 + (E_s/2N_TN_0) \right)^{-1} \tag{22}
\]

for MMSE combining \([14]\) where here we generalised the optimisation function defined for the SISO-OFCDM system in \([12]\) to a MIMO-OFCDM system, with \( K \) being the number of interfering codes, given by the number of users transmitted on the same sub-carriers and using the same time-domain spreading code but different frequency-domain spreading codes. In a MIMO-OFCDM system with no MCI, the combining weight of the MMSE becomes

\[
\bar{\eta}_n(l_j) = \left( \frac{(\delta_{l_j}^{\text{eq}})^*}{(\delta_{l_j}^{\text{eq}})^2 + (E_s/2N_TN_0)} \right)^{-1} \tag{23}
\]

This output is normalised to form the decision vector, which will be used in MCI cancellation if needed. The decision variable is then given by

\[
z_{n,k} = \frac{1}{\sqrt{E_s/2N_S}} \sum_{j=0}^{N_T-1} \frac{y_{n,k}}{b_j^{\text{eq}} \bar{\eta}_n(l_j)} \tag{24}
\]

Finally, a hard decision based on minimum distance criteria is made to recover the corresponding user data

\[
\hat{d}_{n,k} = \arg\min_{\{i\}} |z_{n,k} - i|^2 \tag{25}
\]

where \( \{i\} \) is the set of all possible transmitted data symbols.

## 4 Performance analysis

In this section, we study the performance of MIMO-OFCDM systems without MCI, In that, we obtain a semi-analytical expression for the BER considering BPSK transmission. The average BER of the system will then be evaluated using Monte Carlo approach.

Let \( e_n \) represent the error vector corresponding to the data symbols sent to the \( n \)th transmit antenna, then \( e_n \) is given by

\[
e_n = z_{n,k} - d_{n,k} = \sum_{j=0}^{N_T-1} r_n(l_j) \mathcal{F}_{l_j/N_0} \left( b + \bar{\eta}_n(l_j) \right) / \sqrt{E_s/2N_S} \sum_{j=0}^{N_T-1} \delta_{j}^{\text{eq}} \bar{\eta}_n(l_j) - d_{n,k} \tag{26}
\]

where \( \delta_{j}^{\text{eq}} \) is defined in (14). Conditioned on the channel, \( \sum_{j=0}^{N_T-1} \bar{\eta}_n(l_j) \bar{\eta}_n(l_j) \) is also Gaussian with zero mean and variance

\[
\sigma^2 = \frac{1}{N_T} \sum_{j=0}^{N_T-1} \delta_{j}^{\text{eq}} |\bar{\eta}_n(l_j) |^2 \tag{27}
\]

Let \( \rho \) be the ratio of the total transmitted signal power to the background noise power. Since the additive noise terms \( \eta_{m,1}(l_j, r) \) and \( \eta_{m,2}(l_j, r) \) are complex AWGN with variance 0.5 per dimension, then \( \rho = E_S \), and the error vector, \( e_n \), becomes

\[
e_n = \frac{\sum_{j=0}^{N_T-1} \bar{\eta}_n(l_j) \bar{\eta}_n(l_j) \rho}{\sqrt{\rho/2N_S} \sum_{j=0}^{N_T-1} \delta_{j}^{\text{eq}} |\bar{\eta}_n(l_j) |^2} \tag{28}
\]

where \( \bar{\eta}_n(l_j) \) is the equivalent noise of the \( l_j \)th sub-carrier at the receiver side during \( T_n \) \((n = 1, 2)\). Since the additive noise terms \( \eta_{m,1}(l_j, r) \) and \( \eta_{m,2}(l_j, r) \) are complex AWGN with 0.5 per dimension, it is easy to show that the
equivalent noise, $\overline{\eta}_n^\text{eq}(f)$, is complex Gaussian noise with zero mean and variance \( (1/2N_f) \sum_{n=0}^{M} (|h_f^\text{eq}|^2 + |\eta_f^\text{eq}|^2) = h_f^\text{eq}/2N_T \) per dimension. Conditioned on the channel, \( \sum_{f=0}^{N_f-1} \overline{\eta}_n^\text{eq}(f)w_{\text{eq}}(f) \) is also Gaussian with zero mean and variance
\[
\sigma_n^2 = \frac{1}{N_T} \sum_{f=0}^{N_f-1} \beta_f^\text{eq}|w_{\text{eq}}(f)|^2
\]

(29)

### 4.1 MRC

With MRC, since the equivalent channel coefficient $h_f^\text{eq}$ is a real number
\[
w_f^\text{eq} = (h_f^\text{eq})^* = h_f^\text{eq}
\]

(30)

The error vector $e_n$ becomes
\[
e_n = \frac{\sum_{f=0}^{N_f-1} \overline{\eta}_n^\text{eq}(f)w_{\text{eq}}(f)}{\sqrt{(1/2N_f) \sum_{f=0}^{N_f-1} h_f^\text{eq}|w_{\text{eq}}(f)|^2}} = \frac{\sum_{f=0}^{N_f-1} \overline{\eta}_n^\text{eq}(f)h_f^\text{eq}}{\sqrt{(1/2N_f) \sum_{f=0}^{N_f-1} (h_f^\text{eq})^2}}
\]

(31)

and
\[
\sigma_n^2 = \frac{1}{N_T} \sum_{f=0}^{N_f-1} h_f^\text{eq}|w_{\text{eq}}(f)|^2 = \frac{1}{N_T} \sum_{f=0}^{N_f-1} (h_f^\text{eq})^3
\]

(32)

When BPSK modulation is used, the probability of bit error conditioned on the channel is given by
\[
\text{BER}_{\text{b}} = P(\text{Re}(e_n) > 1)
\]

\[
= P(\text{Re}\left(\frac{\sum_{f=0}^{N_f-1} \overline{\eta}_n^\text{eq}(f)w_{\text{eq}}(f)}{\sqrt{(1/2N_f) \sum_{f=0}^{N_f-1} h_f^\text{eq}|w_{\text{eq}}(f)|^2}}\right) > 1)
\]

\[
= P(\text{Re}\left(\frac{\sum_{f=0}^{N_f-1} \overline{\eta}_n^\text{eq}(f)h_f^\text{eq}}{\sqrt{(1/2N_f) \sum_{f=0}^{N_f-1} (h_f^\text{eq})^2}}\right) > \frac{\rho}{\sqrt{2N_S}} \sum_{f=0}^{N_f-1} (h_f^\text{eq})^3)
\]

\[
= Q\left(\frac{\sqrt{(1/2N_f) \sum_{f=0}^{N_f-1} (h_f^\text{eq})^2}}{\sqrt{2N_S} \sum_{f=0}^{N_f-1} (h_f^\text{eq})^3}\right)
\]

(33)

### 4.2 EGC

With EGC, the combining weight on the $f$th equivalent channel is given by
\[
w_f^\text{eq} = \frac{(h_f^\text{eq})^*}{|h_f^\text{eq}|} = 1
\]

(34)

Then, the error vector $e_n$ in (26) can be simplified to
\[
e_n = \frac{\sum_{f=0}^{N_f-1} \overline{\eta}_n^\text{eq}(f)}{\sqrt{(\rho/2N_S) \sum_{f=0}^{N_f-1} (h_f^\text{eq})^2}}
\]

(35)

Therefore the probability of bit error conditioned on the channel is given by
\[
P_{\text{e}\text{b}} = P(\text{Re}(e_n) > 1)
\]

\[
= P\left(\text{Re}\left(\frac{\sum_{f=0}^{N_f-1} \overline{\eta}_n^\text{eq}(f)}{\sqrt{(\rho/2N_S) \sum_{f=0}^{N_f-1} (h_f^\text{eq})^2}}\right) > 1\right)
\]

\[
= Q\left(\frac{\sqrt{(\rho/2N_S) \sum_{f=0}^{N_f-1} (h_f^\text{eq})^2}}{\sqrt{\sum_{f=0}^{N_f-1} (h_f^\text{eq})^2}}\right)
\]

(37)

### 4.3 MMSE combining

When the MMSE is used for frequency combining
\[
e_n = \frac{\sum_{f=0}^{N_f-1} \overline{\eta}_n^\text{eq}(f)h_f^\text{eq}/(h_f^\text{eq})^2 + (\rho/2N_f)^{-1}}{\sqrt{(\rho/2N_S) \sum_{f=0}^{N_f-1} [(h_f^\text{eq})^2/(h_f^\text{eq})^2 + (\rho/2N_f)^{-1}]}}
\]

(38)

\[
\sigma_n^2 = \frac{1}{N_T} \sum_{f=0}^{N_f-1} (h_f^\text{eq})^3
\]

(39)

and
\[
P_{\text{e}\text{b}} = P(\text{Re}(e_n) > 1)
\]

\[
= P\left(\text{Re}\left(\frac{\sum_{f=0}^{N_f-1} \overline{\eta}_n^\text{eq}(f)h_f^\text{eq}/(h_f^\text{eq})^2 + (\rho/2N_f)^{-1}}{\sqrt{(\rho/2N_S) \sum_{f=0}^{N_f-1} [(h_f^\text{eq})^2/(h_f^\text{eq})^2 + (\rho/2N_f)^{-1}]}}\right) > 1\right)
\]

\[
= Q\left(\frac{\sqrt{2\sum_{f=0}^{N_f-1} (h_f^\text{eq})^2}}{\sqrt{\sum_{f=0}^{N_f-1} [h_f^\text{eq}/(\Delta^{-1} + 2 + \Delta)]}}\right)
\]

(40)

where
\[
\Delta = \left(\frac{\rho}{2N_f}\right)^{-1} (h_f^\text{eq})^{-2}
\]

(41)

### 5 Simulation results

Here we assess the performance of the MIMO-OFCDM system described earlier over Ricean fading channels, where both simulation and analytical results are presented.
Channel state information (CSI) is assumed to be perfectly known at the receiver side, and CSI is fixed for the duration of two consecutive OFCDM frames. Different system parameters are used when investigating the performance with MRC, EGC and MMSE combining. The effect of sub-carrier correlation because of imperfect frequency interleaving is considered.

Fig. 6 presents the performance of MIMO-OFCDM systems with different number of receive antennas over Rayleigh fading channels using MRC. The performance of a SISO-OFCDM system introduced in [10] with $N_F = 8$ is shown as a reference. From this figure we can see that the slope of the curves increases as the spatial diversity of the system increases. The MIMO-OFCDM systems with 1, 2 and 3 receive antennas are shown.

![Figure 6](image1.png)

**Figure 6** BER for MIMO-OFCDM system using MRC over Rayleigh fading channel: $N_r = 8$, $N_T = 8$, $K = 8$

![Figure 7](image2.png)

**Figure 7** Effect of MCI on the BER for MIMO-OFCDM system over Rayleigh fading channel: $M = 3$, $N_r = 16$, $N_T = 8$, $K = 32$
antennas achieve overall diversity orders of 16, 32 and 48 respectively, whereas the SISO-OFCDM system achieves a diversity order of 8. At the same time, we can see the accuracy of our analytical results when compared with the simulated ones.

In Fig. 7 we compare MRC, EGC and the MMSE-based gain combining used in MIMO-OFCDM systems over Rayleigh fading channels when MCI is present in the system. The considered system is running at 25% of full load and $N_F = 16$. That is, every four spreading codes are interfering with each other, and hence $K_c$ equals 3. From these results, and compared to the performance of the MRC and EGC, the MMSE combiner is more robust to MCI.

We present the BER performance of two MIMO-OFCDM systems over Ricean fading channels in Figs. 8 and 9.

**Figure 8** BER for MIMO-OFCDM system using EGC over Ricean fading channel: $M = 3$, $N_F = 8$, $N_T = 8$, $K = 8$

**Figure 9** BER for MIMO-OFCDM system using MMSE combining over Ricean fading channel: $M = 1$, $N_F = 8$, $N_T = 8$, $K = 8
and 9, where three receive antennas with EGC and one receive antenna with MMSE combining are used in the two systems, respectively. The parameter, $\kappa$, known as the Ricean factor, represents the ratio of the dominant component (line-of-sight) to the total power of scattered waves. From these two figures we see that as $\kappa$ increases, the system performance changes from a Rayleigh fading channel towards a Gaussian channel.

In Fig. 10 we study the performance degradation caused by correlation among sub-carriers of interest for both SISO and MIMO-OFCDM systems when employing EGC. The correlated Rayleigh fading channels with given correlation coefficient are generated according to the scheme introduced in [22]. From the figure we can see that for SISO and MIMO-OFCDM systems, the impact on the overall diversity order is the same when sub-carriers of interest (i.e. assigned to the same user data symbol) are correlated. This is because the correlation is only present among sub-carriers from the same transmit antenna to the same receive antenna. From this example we can see that BER of MIMO-OFCDM systems can be easily obtained once the fading channel model is defined.

6 Conclusion

We presented the transmission and detection methods for MIMO-OFCDM downlink transmission in Ricean fading channels. It was shown that by employing STBC, MIMO-OFCDM systems provide significant performance improvement relative to SISO-OFCDM. Semi-analytical expressions for the BER of MRC, EGC and MMSE combining techniques have been obtained. It was shown that in the absence of multi-code interference, MRC, EGC and MMSE combining achieve the full frequency/spatial diversity gain. However, when the system suffers from multi-code interference, only the MMSE combiner can be more tolerant to interference with much higher gain than both MRC and EGC. Furthermore, we have shown that when frequency correlation exists among sub-carriers, both the SISO and MIMO-OFCDM systems experience same degree of diversity loss.

7 References


