Centralized Fusion Estimators for Multi-sensor Systems with Multiplicative Noises and Missing Measurements

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Abstract—This paper is concerned with the centralized fusion estimation problem for multi-sensor systems with multiplicative noises in state and measurement matrices and missing measurements. Based on the innovation analysis approach, the centralized fusion estimators including filter, predictor and smoother are developed in the least mean square sense. The steady-state estimators are also studied. A sufficient condition for the existence of the steady-state centralized fusion estimators is obtained. An illustrative example shows the effectiveness of the proposed algorithm.

Index Terms—Centralized fusion estimator, Multi-sensor system, Multiplicative noise, Missing measurement, Innovation analysis

I. INTRODUCTION

In recent years, the research on networked control and estimation has attracted much attention. Compared with traditional systems, networked systems have many advantages, such as low cost, inherent robustness, as well as high reliability. However, several new challenging issues have been posed due to limited communication capacity and unreliable communication networks, which are the results of network-induced delays, packet dropouts and missing measurements (or sensor failures).

For systems subject to random delayed measurements, some algorithms have been presented, such as a modification of the conventional minimum variance state estimator [1] and the least mean square filter [2]. For systems subject to packet dropouts, a steady-state $H_2$ filter is proposed by the LMI approach [3]. The optimal and steady-state linear estimators including filter, predictor and smoother are presented in the linear minimum variance sense by the innovation analysis approach [4], and the full- and reduced-order estimators are also investigated by the completion of square approach [5]. For the systems subject to missing measurements, the linear minimum variance filters are studied in [6-7], where the missing measurements are described by a Bernoulli distributed stochastic variable. In [8], a recursive estimator is presented based on a covariance information approach. Further, a four state Markov chain is used to describe the mixed uncertainties of sensor delays, packet dropouts and missing measurements. The minimum variance and adaptive filtering algorithms are proposed in [9-10]. Furthermore, the optimal linear estimators are investigated in [11, 16], where three Bernoulli distributed random variables are used to describe the mixed uncertainties. However, the above literatures do not consider the multiplicative noises in state or measurement matrices.

Systems subject to multiplicative noises have been gained a great deal of attention lately. This is mainly due to the fact that this kind of formulation has been found in many practice systems, such as image processing systems, communication systems and aerospace systems. Generally, there are two types of multiplicative noises of a system. One is the deterministic multiplicative noises; the other is the stochastic multiplicative noises. Some robust filtering algorithms for systems subject to deterministic multiplicative noise in both state and measurement matrices have been proposed [12]-[14]. For systems subject to stochastic state multiplicative noises, the synthesis problem of the filter and controller is discussed in [15]. Some polynomial filtering algorithms are presented in [26, 17]. The improved state estimator is proposed for linear stochastic systems subject to multiplicative noises affecting the control and feedback signals [18]. For systems subject to both deterministic and stochastic multiplicative noises, a robust finite-horizon Kalman filter is also designed via LMI approach [19]. Reference [20] designs the linear minimum mean square estimator for systems with both state and measurement multiplicative noises and Markov jumps on the parameters. However, the missing measurements are not taken into account in the abovementioned references. For systems with state multiplicative noises and missing measurements, some robust filtering algorithms are designed in [21-22]. The robust filter in [21] is also

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resilient minimum variance unbiased filter. In order to guarantee resilient operation, [21] introduces stochastic perturbations in the filtering gain matrix. But how to select the variances and coefficients of stochastic perturbations is of subjective. In [22], the measurement multiplicative noises are also considered.

In this paper, we explore the centralized fusion filter problem for the system with multiplicative noises in state and measurement matrices and missing measurements. The multiplicative noises are allowed to be stochastic white noises with known variance perturbations. The missing measurements are described by the Bernoulli distributed random variables. The centralized fusion estimators are designed in the least mean square sense by the innovation analysis approach. The forms of our centralized optimal linear estimators are simple. Furthermore, the steady-state estimators are also investigated. A sufficient condition for the existence of the steady-state centralized estimators is given.

II. PROBLEM FORMULATION

Consider the following multi-sensor discrete-time stochastic system with stochastic state and measurement multiplicative noises and missing measurements:

\[
\begin{align*}
\dot{x}(t+1) &= (A(t) + \sum_{l=1}^{N_s} \alpha_l(t) A_l(t))x(t) + B(t)w(t) + D(t)\nu(t) \\
\dot{y}_i(t) &= \gamma_i(t)(C_i(t) + \sum_{m=1}^{N_m} \beta_{im}(t)C_{im}(t))x(t) + \beta_{im}(t)\nu(t) + \beta_{im}(t)\nu(t), \\
&+ D_i(t)\nu(t), \quad i = 1, 2, \ldots, L
\end{align*}
\]

where \( x(t) \in \mathbb{R}^n \) is the state, \( y_i(t) \in \mathbb{R}^{n_i} \) is the measurement of the \( i \)-th sensor received by the estimator, \( w(t) \in \mathbb{R}^m, \dot{y}_i(t) \in \mathbb{R}^{n_i}, \alpha_l(t) \in \mathbb{R}^{n} \), \( l = 1, 2, \cdots, N_s \) and \( \beta_{im}(t) \in \mathbb{R}^{n_i}, \quad i = 1, 2, \cdots, N_{\nu} \) are white noises. \( \gamma_i(t), \dot{y}_i(t), \beta_{im}(t), \tau = 1, 2, \cdots, L \) are Bernoulli distributed random variables with the probabilities \( \text{Prob} [\dot{y}_i(t) = 1] = \pi_i(t) \) and \( \text{Prob} [\dot{y}_i(t) = 0] = 1 - \pi_i(t) \), where \( 0 \leq \pi_i(t) \leq 1 \), and are uncorrelated with other random variables. \( A(t), B(t), C_i(t), D_i(t), A_l(t), C_{im}(t) \) are known matrices with suitable dimensions. The superscript \( t \) denotes the \( \tau \)-th sensor and \( L \) is the number of sensors.

In this paper, the mathematical expectation \( E \) operates on \( \dot{y}_i(t) \) as well as \( w(t), \dot{v}_i(t), t = 1, 2, \cdots, L, \alpha_l(t), \beta_{im}(t), l = 1, 2, \cdots, n_s; \quad i = 1, 2, \cdots, n_{\nu} \).

**Assumption 1.** \( w(t) \) and \( \dot{v}_i(t) \) are correlated white noises with zero means and covariance matrices \( E[w(t)w^T(t)] = Q_{w}, \quad E[w(t)\dot{v}_i(t)] = S_{wi}, \quad \text{and} \quad E[\dot{v}_i(t)\dot{v}_j^T(t)] = Q_{\nu_{ij}}, \) with \( Q_{\nu_{ij}} = Q_{\nu_{jj}}, \) the superscript \( T \) denotes the transpose.

**Assumption 2.** \( \alpha_l(t), l = 1, 2, \cdots, n_s \) and \( \beta_{im}(t), i = 1, 2, \cdots, L \) are mutually uncorrelated scalar white noise sequences with zero means and variances \( Q_{\alpha_l} \) and \( Q_{\beta_{im}} \) and are uncorrelated with other random variables.

**Assumption 3.** The initial state \( x(0) \) is uncorrelated with \( w(t) \), \( \dot{v}_i(t) \), \( \dot{y}_i(t) \), \( \alpha_l(t) \) and \( \beta_{im}(t) \) and satisfies that \( E[x(0)\mu_k], E[(x(0) - \mu_k)(x(0) - \mu_k)^T] = p_0 \).

The system (1)-(2) with multiple sensors can be rewritten as the following form:

\[
\begin{align*}
x(t+1) &= (A(t) + \sum_{l=1}^{N_s} \alpha_l(t) A_l(t))x(t) + B(t)w(t) + D(t)\nu(t) \\
y(t) &= G(t)(C(t) + \sum_{m=1}^{N_m} \beta_{m}(t)C_{m}(t))x(t) + D_{m}(t)\nu(t) + D_{m}(t)\nu(t),
\end{align*}
\]

where

\[
\begin{align*}
\sum_{l=1}^{N_s} \alpha_l(t)A_l(t)
&= \sum_{l=1}^{N_s} \alpha_l(t)A_l(t), \\
\sum_{m=1}^{N_m} \beta_{m}(t)C_{m}(t)
&= \sum_{m=1}^{N_m} \beta_{m}(t)C_{m}(t),
\end{align*}
\]

\[
\begin{align*}
\gamma_i(t) &= \gamma_i(t)C_i(t) + \sum_{m=1}^{N_m} \beta_{im}(t)C_{im}(t)x(t) + \beta_{im}(t)\nu(t) + \beta_{im}(t)\nu(t), \\
\beta_{im}(t) &= \beta_{im}(t)I_{m}, \\
\nu(t) &= \nu(t)I_{m}, \\
C_{m}(t) &= \sum_{m=1}^{N_m} C_{m}(t)I_{m}, \\
\nu(t) &= \nu(t)I_{m},
\end{align*}
\]

\[
\begin{align*}
I(t) &= \text{diag}(I_{m}, \cdots, I_{m}), \\
\beta_{m}(t) &= \text{diag}(\beta_{m}(t), I_{m}, \cdots, I_{m}),
\end{align*}
\]

\[
D(t) = \text{diag}(D_{m}(t), \cdots, D_{m}(t))
\]

where \( \text{diag}(\bullet) \) denotes a diagonal matrix which diagonal elements are consist of \( \bullet \).

Further, we have the following noise statistical information

\[
\begin{align*}
Q_w &= E[w(t)w^T(t)] = Q_{w}, \\
S &= E[\nu(t)\nu^T(t)] = [S_{ij}], i, j = 1, 2, \cdots, L,
\end{align*}
\]

Our aims are to find the centralized fusion optimal linear filter \( \hat{x}(t|t) \), predictor \( \hat{x}(t+N|t) \), \( N > 0 \) and smoother \( \hat{x}(t+N|t) \), \( N > 0 \) for the state \( x(t) \) based on the received measurements. Next, we will investigate the centralized fusion optimal linear estimators based on the innovation analysis approach.
III. CENTRALIZED FUSION ESTIMATORS

A. One-step Predictor

Theorem 1. For system (3)-(4) with Assumptions 1-3, the recursive linear one-step predictor is given by

$$\hat{x}(t+1) = A(t)\hat{x}(t|t-1) + K(t)e(t)$$

(7)

$$e(t) = y(t) - F(t)C(t)\hat{x}(t|t-1)$$

(8)

$$K(t) = (A(t)P(t|t-1)C^T(t) + \Sigma(t))^{-1}$$

(9)

$$Q(t) = F(t)C(t)P(t|t-1)C^T(t)F(t) + \Xi \Xi$$

(10)

$$P(t|t) = [A(t) - K(t)F(t)]P(t|t-1)[A(t) - K(t)F(t)]^T$$

(11)

where $\hat{x}(t|t) = x(t+1)$ is the one-step prediction error. From (9) and (10), we have

$$E[x(t+1)e^T(t)] = A(t)E[x(t|t-1)x^T(t)]C^T(t)F^T$$

(12)

and

$$+B(t)E[w(t)v^T(t)]D^T(t)$$

(13)

where $x(t+1) = x(t+1) + \Delta x(t)$, $\Delta x(t)$ is the innovation sequence with covariance $Q(t)$, $K(t)$ is the filtering gain matrix, $P(t|t)$ is the filtering error covariance matrix, and $X(t)$ is the state covariance matrix of system (3)-(4). The initial values are $\hat{x}(0) = \mu_0$, $X(0) = P_0$, and $P(0) = P_0$.

Proof. From projection theory [23], we can easily obtain (7), where the filtering gain matrix is defined by

$$K(t) = E[x(t+1)e^T(t)]Q^{-1}(t)$$

(14)

Substituting (14) and $x(t+1) = x(t+1) - \Delta x(t)$ into $e(t) = y(t) - F(t)\hat{x}(t|t-1)$, we obtain the innovation equation (8). Substituting (4) into (8), the innovation $e(t)$ can be rewritten as

$$e(t) = [\Gamma(t) - F(t)C(t)x(t)] + \Gamma(t)\sum_{\rho}(t)x(t)$$

(15)

From the projection theory [23], it is clearly known that $v(t) \perp L(y(t+1),...,y(0))$, where symbol $\perp$ denotes orthogonality. Then we can obtain $v(t) = 0$.

Substituting (14) and $v(t) = 0$ into $e(t) = y(t) - F(t)\hat{x}(t|t-1)$, we obtain the innovation equation (8). Substituting (4) into (8), the innovation $e(t)$ can be rewritten as

$$e(t) = [\Gamma(t) - F(t)C(t)x(t)] + \Gamma(t)\sum_{\rho}(t)x(t)$$

(16)

Substituting (4) into (8), the innovation $e(t)$ can be rewritten as

$$e(t) = [\Gamma(t) - F(t)C(t)x(t)] + \Gamma(t)\sum_{\rho}(t)x(t)$$

(17)

where $\hat{x}(t+1) = x(t+1) - \Delta x(t)$, $\Delta x(t)$ is the innovation sequence with covariance $Q(t)$, $K(t)$ is the filtering gain matrix, $P(t|t)$ is the filtering error covariance matrix, and $X(t)$ is the state covariance matrix of system (3)-(4). The initial values are $\hat{x}(0) = \mu_0$, $X(0) = P_0$, and $P(0) = P_0$.

Proof. From projection theory [23], we can easily obtain (7), where the filtering gain matrix is defined by

$$K(t) = E[x(t+1)e^T(t)]Q^{-1}(t)$$

(18)

Taking projection on both sides of (4) onto the linear space spanned by $(y(t-1),...,y(0))$, we have
Using \( \hat{x}(t \mid t-1) \perp w(t) \), \( \hat{x}(t \mid t-1) \perp v(t) \), \( \mathbb{E}[F(t)-\bar{F}]=0 \), \( \mathbb{E}[\Sigma(t)]=0 \), \( \mathbb{E}[\sigma(t)]=0 \), the one-step prediction error variance matrix is obtained as

\[
P(t+1 \mid t) = \mathbb{E}[\tilde{x}(t+1 \mid t)\tilde{x}^T(t+1 \mid t)]
\]

\[
= [A(t) - K(t)\bar{P}(t)C(t)]P(t \mid t-1)[A(t) - K(t)\bar{P}(t)C(t)]^T \\
+ K(t)[\bar{P}(t)]C(t)\mathbb{E}[x(t)x^T(t)]C^T(t)[\bar{P}(t)]K^T(t) \\
- K(t)\bar{P}(t)\mathbb{E}\left[\sum_{\rho}\rho(t)x(t)x^T(t)\sum_{\rho}'(t)\right]K^T(t) \\
+ E\left[\sum_{\rho}(t)x(t)x^T(t)\sum_{\rho}'(t)\right] - K(t)D(t)Q(t)D^T(t)K^T(t) \\
+ B(t)Q(t)B^T(t) - B(t)SD^T(t)K^T(t) - K(t)D(t)S^T(t)B^T(t)
\]

(19)

Using the definition in Theorem 1 and rearranging (19), we have (12).

Remark 1: The proposed optimal linear one-step predictor (prior filter) can also solve the special cases of the optimal linear prior filter for systems with single sensor, i.e., \( L = 1 \) and without multiplicative noises, i.e., \( \alpha(t) = 0 \), \( \beta(t) = 0 \), for systems with missing measurements and without the multiplicative noises in the measurement matrix, i.e., \( \beta^{(ij)}(t) \) (the prior filter in [21]). Furthermore, our prior filter has a simple form and avoids selecting the variances and coefficients of stochastic perturbations of [21]. Furthermore, we remove the assumptions of independent noises.

Next, we will solve the optimal linear N-step (\( N > 1 \)) predictor, filter and smoother based on Theorem 1.

B. N-step Predictor

Theorem 2. For system (3)-(4) with Assumptions 1-3, the N-step (\( N > 1 \)) predictor is given by

\[
\hat{x}(t \mid t+N) = A(t \mid t-1)\hat{x}(t \mid t-1)
\]

(20)

\[
P(t \mid t+N) = A(t \mid t-1)P(t \mid t-1)A^T(t \mid t-1) + \sum_{i=1}^{N}Q_iA_i(t \mid t-1)X(t \mid t-1)A_i^T(t \mid t-1)
\]

(21)

where the initial values \( \hat{x}(t \mid t) \), \( P(t \mid t) \) and \( X(t \mid t) \) are computed by Theorem 1.

Proof. From (3), we have

\[
x(t \mid t+N) = (A(t \mid t-1) + \sum_{\rho}(t \mid t-1))x(t \mid t-1) \\
+ B(t \mid t-1)w(t \mid t-1)
\]

(22)

Taking projection of both sides of (22) onto the linear space spanned by \( \{\psi^{(i)}(t), \ldots, \psi^{(N)}(0)\} \) yields (20).

Subtracting (20) from (22) gives the prediction error equation

\[
\tilde{x}(t \mid t+N) = A(t \mid t-1)\tilde{x}(t \mid t-1)
\]

(23)

Taking into consideration that \( \tilde{x}(t \mid t-1) \perp w(t \mid t-1) \), \( \mathbb{E}[\sum_{\rho}(t \mid t-1)] = 0 \), (21) follows from \( P(t \mid t) = \mathbb{E}[\tilde{x}(t \mid t)\tilde{x}^T(t \mid t)] \) by using (23). This proof is completed. □

C. Filter And Smoother

Theorem 3. For system (3)-(4) with Assumptions 1-3, the filter and fixed-lag N-step \( (N \geq 0) \) smoother is given by

\[
\tilde{x}(t \mid t+N) = \tilde{x}(t \mid t+N-1) + M(t \mid t+N)\epsilon(t \mid t+N)
\]

(24)

where the filtering and smoothing gain matrices \( M(t \mid t+N) \) are given by

\[
M(t \mid t+N) = K(t \mid t-1)C^T(t \mid t+N)P(t \mid t-1)C(t \mid t+N)
\]

(25)

\[
\epsilon(t \mid t) = \epsilon(t \mid t-1) + \frac{K(t \mid t-1)C(t \mid t+N)}{C^T(t \mid t+N)P(t \mid t-1)C(t \mid t+N)}\epsilon(t \mid t-1)
\]

(26)

and the estimation error covariance matrix is computed by

\[
P(t \mid t+N) = P(t \mid t-1)
\]

(27)

where the initial values \( \tilde{x}(t \mid t-1) \) and \( \Omega(t) = P(t \mid t-1) \) are computed by Theorem 1.

Proof. Based on the projection theory [23], we have (24), where the smoothing gain matrix \( M(t \mid t+N) \) is defined by

\[
M(t \mid t+N) = \mathbb{E}[x(t)\epsilon^T(t \mid t+N)]Q^{-1}(t \mid t+N)
\]

(28)

From (15), the innovation is given by

\[
\epsilon(t \mid t) = \mathbb{E}[\epsilon(t \mid t)] = \mathbb{E}[F(t \mid t) - \bar{F}][C(t \mid t+N)x(t \mid t+N) \\
+ \mathbb{E}[\epsilon(t \mid t)] = \mathbb{E}[\epsilon(t \mid t)][C(t \mid t+N)x(t \mid t+N)]
\]

(29)

Using

\[
\mathbb{E}[F(t \mid t)-\bar{F}] = 0 \quad \text{and} \quad x(t) \perp \nu(t \mid t), N \geq 0 \text{ we have}
\]

\[
\mathbb{E}[x(t)\epsilon^T(t \mid t+N)] = \Omega(t \mid t+N)C^T(t \mid t+N)\Gamma
\]

(30)

where \( \Omega(t \mid t+N) = \mathbb{E}[x(t)\epsilon^T(t \mid t+N \mid t+N-1)] \). By replacing \( t \) of (18) with \( t+N-1 \), we have
\[ \dot{x}(t+N | t+N-1) = [A(t+N-1) - K(t+N-1)\bar{F}]x(t+N-1) + C(t+N-1)\tilde{x}(t+N-1 | t+N-2) + B(t+N-1)w(t+N-1) - K(t+N-1)D(t+N-1)v(t+N-1) + \sum_{\alpha} (t+N-1) - K(t+N-1)(\Gamma(t+N-1) - \bar{F}) \times C(t+N-1)\dot{x}(t+N-1) \]  

Noting that \( x(t) \perp w(t+N-1) \), \( x(t) \perp v(t+N-1) \), \( N > 0 \) and \( \mathbb{E}[\Gamma(t+N-1) - \bar{F}] = 0 \), \( \mathbb{E}[\sum_{\alpha}(t+N-1)] = 0 \), (31) follows from (31). Furthermore, substituting (30)into (28) yields (25).

Next, the smoothing error equation can be obtained from (24) as

\[ \dot{x}(t | t+N) = \dot{x}(t | t+N-1) - M(t | t+N)x(t+N) . \]  

Further, (32) can be rewritten as

\[ \dot{x}(t | t+N) + M(t | t+N)x(t+N) = \dot{x}(t | t+N-1) . \]  

Using \( \dot{x}(t | t+N) \perp \epsilon(t+N) \), (27) follows from \( P(t | t+N) = \mathbb{E}[\dot{x}(t | t+N)^2(t+N)] \) by (33). This proof is completed. □

Remark 2: Theorems 1-3 give the centralized fusion estimators for system (3)-(4) which contain the probabilities \( \pi^{(i)} \), \( i = 1, 2, \ldots, L \) of the Bernoulli distributed parameters \( \gamma^{(i)}(t) \), and the variances of the stochastic parameters \( \alpha_k(t), k = 1, 2, \ldots, n_a \) and \( \beta_l^{(i)}(t), l = 1, 2, \ldots, n_b \). When \( \pi^{(i)} = 1 \) and \( \alpha_k(t) = 0 \), \( \beta_l^{(i)}(t) = 0 \), it can be easily seen that the proposed estimators are reduced to the standard Kalman estimators.

IV. CENTRALIZED FUSION STEADY-STATE ESTIMATORS

In the preceding section, we have obtained the centralized linear estimators including filter, predictor and smoother. In this section, we will investigate the stability problems of the optimal linear estimators. From Theorems 1-3, it is easily known that multi-step predictor, filter and smoother are computed based on the one-step predictor. So, we only need to analyze the stability of the one-step predictor in Theorem 1.

We shall consider the corresponding time-invariant system (3)-(4), where \( A, A_k, k = 1, 2, \ldots, n_a, B, C^{(i)}, C^{(i)}_l, l = 1, 2, \ldots, n_b^{(i)}, i = 1, 2, \ldots, L \) and \( D^{(i)} \) are constant matrices.

Theorem 4. For the corresponding time-invariant system (3)-(4), if \( \rho(A \otimes A + \sum_{k=1}^{n_a} Q_{a_{k}} \otimes A_{k}) < 1 \), and \( (A, F C) \) is a detectable pair and \( (A - \bar{S}R^{-1}FC, \bar{Q}_{a}) \), where \( \bar{Q}, \bar{Q}^T = \bar{Q} - \bar{S}R^{-1}\bar{S}^T \), and \( \bar{Q} \) and \( \bar{R} \) are defined in the latter, is a stabilizable pair, the solutions \( X(t) \) and \( P(t+1 | t) \) with any initial values \( X(0) \) and \( P(0 | 0) \geq 0 \) converge, respectively, to the unique positive semi-definite solutions \( X \) and \( \Sigma \) to the following algebraic Lyapunov and Riccati equations:

\[ X = AXA^T + \bar{Q} \]  

\[ \Sigma = [A - K \bar{F}C]\Sigma[A - K \bar{F}C]^T + \bar{Q} - K \bar{S}^T - \bar{S}K^T + K \bar{R}K^T \]  

where \( \bar{R} = \Xi - (CXC^T) + \sum_{a=1}^{n_a} F_a \otimes (C_a X C_a^T) + R \), \( \bar{S} = \sum_{a=1}^{n_a} Q_a A_{a} A_{a}^T + Q \), \( \rho(\bullet) \) is the spectrum radius of matrix \( \bullet \) and \( \otimes \) is the Kronecker product.

Moreover, we have that \( K = \lim_{t \to \infty} K(t) \), \( \Sigma = \lim_{t \to \infty} P(t | t-1) \) and \( Q_e = \lim_{t \to \infty} Q_e(t) \). Furthermore, the centralized fusion steady-state filter

\[ \dot{x}(t+1) = (A - K \bar{F}C)\dot{x}(t) + Ky(t) \]  

is asymptotically stable.

Proof. Form \( \rho(\bullet) < 1 \), we have \( X = \lim X(t) \) [24].

Then, from the assumptions of detectability and stabilizability, a similar proof as in [25] can be applied to show \( X(t) \) and \( P(t | t-1) \) of equations (11) and (12) with any initial conditions \( X(0) \) and \( P(0 | 0) \geq 0 \) converge to the unique positive semi-definite solutions \( X \) and \( \Sigma \) of (34)-(35), and \( (A - K \bar{F}C) \) is a stable matrix, which implies the stability of the steady-state filter (36). This proof is completed. □

Remark 3: In Theorem 4, the existence of the steady-state one-step predictor means that of the steady-state filter, multi-step predictor and smoother.

V. SIMULATION RESEARCH

Consider the stochastic system with two sensors

\[ A = \begin{bmatrix} 0.8 & 0 \\ 0.9 & 0.2 \end{bmatrix}, A_k = \begin{bmatrix} 0.1 & 0.05 \\ 0.2 & 0.5 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, \]

\[ C^{(i)} = [0.5 \ 1], C^{(2)} = [1 \ 1], D^{(1)} = 0.5, D^{(2)} = 0.2, \]

\[ C^{(i)}_1 = [0.1 \ 0.1], C^{(2)}_1 = [0.2 \ 0.1], Q_{a_1} = 1, Q_{a_2} = 0.1, \]

\[ Q^{(i)}_R = 0.2, Q^{(2)}_R = 0.05, n_a = n_{\rho^{(i)}} = n_{\rho^{(2)}} = 1 \]  

where the measurement noises \( w^{(i)}(t), i = 1, 2 \) are correlated with the process noise \( w(t) \), satisfying relation \( w^{(i)}(t) = c^{(i)} w(t) + \zeta^{(i)}(t) \) where \( c^{(1)} = 0.6 \) and \( c^{(2)} = 0.4 \) are the correlated coefficients. \( \zeta^{(i)}(t), i = 1, 2 \) are independent Gaussian white noises with zero means and variances \( Q^{(i)}_{\zeta} = 1 \) and \( Q^{(2)}_{\zeta} = 2 \), respectively, and are
independent of the process noise $w(t)$. The probabilities of Bernoulli distributed random variables $\gamma^{(i)}(t)$ are taken as $\pi^{(1)} = 0.8$ and $\pi^{(2)} = 0.7$. Our aims are to find the centralized filter $\hat{x}(t | t)$, one-step predictor $\hat{x}(t+1 | t)$ and one-step smoother $\hat{x}(t | t+1)$.

Figure 1. Centralized optimal linear filter.

Figure 2. Comparison of the variances of centralized filter, predictor and smoother.

Figure 3. Comparison of the variances of all local filters and centralized fusion filter.

Figure 4. Comparison of the variances of all local predictors and centralized fusion predictor.
Applying Theorems 1-3, we can obtain the centralized linear estimators including filter, predictor and smoother and corresponding estimation errors variances matrices. Fig. 1 shows the tracking performance of centralized filter, where the solid curves denote the true values and the dotted curves denote the filtering values. Fig. 2 shows the comparison of estimation error variances for the centralized linear estimators. It is clear that the smoother performs better accuracy than the filter and the filter performs better accuracy than the predictor. The comparison of estimation error variances of centralized estimators and local estimators are given in Fig.3-Fig.5. From Fig.3-Fig.5, we can see that the accuracy of the centralized estimators is better than that of all local estimators based on single sensor. Fig. 6 shows the comparison of estimation accuracy of our filter and standard Kalman filter over an average of 200 runs of Monte Carlo method. The standard Kalman filter does not consider the multiplicative noises in the state matrix and measurement matrix and missing measurements, i.e., $\alpha(t) = 0$, $\beta_{i}^c(t) = 0$ and $\pi^{(i)} = 1$. From Fig. 6, we see that our filter outperforms the standard Kalman filter.

VI. CONCLUSION

For the discrete-time stochastic system with multiple sensors subject to state and measurement multiplicative noises and missing measurements, the centralized fusion optimal linear estimators including filter, predictor and smoother have been developed via an innovation analysis approach. Further, the centralized fusion steady-state estimators have also been investigated. A sufficient condition for the existence of the steady-state estimators has been obtained.

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