Cooperative Control with General Linear Dynamics and Limited Communication: Centralized and Decentralized Event-Triggered Control Strategies

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Abstract—This paper presents event-triggered control techniques for the consensus problem with general linear dynamics. A novel consensus protocol is proposed, where each agent implements a model of the decoupled dynamics of its neighbors. We first provide a simple centralized condition to motivate the problem. Then, the focus is placed on designing decentralized consensus protocols. The decentralized approach proposed in this paper not only avoids the need for continuous communication between agents but also provides a decentralized and asynchronous method for transmission of information. This method gives more flexibility for scheduling information broadcasting compared to periodic and sampled-data implementations. Finally conditions are provided in order to guarantee positive inter-event times.

I. INTRODUCTION

An important problem in Multi-Agent Systems (MAS) is the design and implementation of decentralized algorithms for control and communication of autonomous vehicles [1]. It is well understood that each agent should be able to determine its own control laws independently and based only on local information. This has been an important research topic [2], [3], [4]. These papers consider agents with continuous-time dynamics and it is assumed that agents can have continuous access to the states of their neighbors. In many applications, continuous communication is not possible, and it becomes important to discern how frequently the agents should communicate in order to preserve the properties inherent in the corresponding control algorithms with continuous information exchange. The sampled-data approach is commonly used to estimate the sampling periods [5], [6], and [7]. An important drawback of periodic transmission is that it requires synchronization between the agents, that is, all agents need to transmit their information at the same time instants and, in some cases, it requires a conservative sampling period for worst case situations. In the present paper, in lieu of periodic approaches, we use an asynchronous communication scheme based on event-triggered control strategies and we consider agents that are described by general linear models.

Consensus problems where all agents are described by general linear models have been considered by different authors [8], [9], [10], [11], and [12]. The references [9] and [12] considered continuous time systems and continuous communication. Consensus and leader-following tracking involving agents with linear dynamics and switching topology was studied in [10]. The authors of [8] established conditions under which protocols will exist that achieve consensus. Such conditions are related to the communication network and to the stabilizability and detectability of the individual dynamics.

All the previous references concerning linear systems assumed that continuous communication between agents is possible. The work in [13] consider the consensus problem of agents with linear dynamics under communication constraints. Specifically, the authors consider the existence of continuous communication among agents for finite intervals of time and the total absence of communication among agents for other time intervals, and the minimum rate of continuous communication to no communication is given.

In addition to the consideration of agents with general linear dynamics, we also address the limited communication problem when studying consensus problems. In particular, event-triggered control strategies are developed in this paper. Different from periodic (or time-triggered) implementations, in the context of event-triggered control, information or measurements are not transmitted periodically in time but they are triggered by the occurrence of certain events. In event-triggered broadcasting [14], [15], [16], [17], [18], and [19], a subsystem sends its local state to the network only when it is necessary, that is, only when a measure of the local subsystem state error is above a specified threshold.

Event-triggered control strategies have been used for stabilization of multiple coupled subsystems as in [20] and [21]. Consensus problems have also been studied using these techniques [22], [23], [24], and [25]. Event-triggered control provides a more robust and efficient use of network bandwidth. Its implementation in MAS also provides a highly decentralized way to schedule transmission instants which does not require synchronization compared to periodic sampled-data approaches. Different authors have extended this approach, for instance, [26] studied event-triggered consensus for discrete-time single integrators. The authors of [27], [28] used event-triggered techniques for consensus problems involving a combination of discrete-time single and double integrators. The authors of [29] studied event-
triggered consensus of single integrators using nonlinear consensus protocols.

The present paper represents one of the first attempts to extend previous work on event-triggered control of MAS to the case of general linear dynamics. The contributions of this paper are as follows. First, we present a novel method for consensus with undirected graphs and with limited communication in which each agent implements models of the decoupled dynamics of each one of their neighbors; this approach offers better performance than the Zero-Order-Hold (ZOH) model, where updates from neighbors are kept constant, and the zero-input approach in [13]. Second, we design centralized and decentralized event thresholds that guarantee asymptotic convergence. The centralized approach requires knowledge of all states to trigger events while in the decentralized approach, each agent only needs to measure its own local state to schedule its own triggering time instants. Finally, an extension to the decentralized threshold design method is offered in order to guarantee positive inter-event times for every agent. Bounded consensus where the disagreement between any pair of states is bounded is achieved in this case. Bounds on the state disagreement are derived.

The remainder of this paper is organized as follows. Section II provides a brief background on graph theory and describes the problem and the consensus protocol. Section III gives a result assuming continuous communication which will be used throughout the document. Section IV presents centralized conditions that guarantee asymptotic consensus. Decentralized thresholds are presented in Section V. These conditions guarantee asymptotic consensus but, in general, do not guarantee positive inter-event times. Section VI offers modified decentralized event-triggered conditions in order to exclude Zeno behavior. Section VII presents illustrative examples and Section VIII concludes the paper.

II. PRELIMINARIES

A. Graph Theory

Consider a graph $G = \{V, E\}$ consisting of a set of vertices or nodes $V = \{1, ..., N\}$ and a set of edges $E$. An edge between nodes $i$ and $j$ is represented by the pair $(i, j) \in E$. A graph $G$ is called undirected if $(i, j) \in E$ if and only if $(j, i) \in E$ and the nodes $i$ and $j$ are called adjacent. The adjacency matrix $A$ is defined by $a_{ij} = 1$ if the nodes $i$ and $j$ are adjacent and $a_{ij} = 0$ otherwise. If $(j, i) \in E$, then $j$ is said to be a neighbor of $i$. The set $N_i$ is called the set of neighbors of node $i$, and $N_i$ is its cardinality. A node $j$ is an element of $N_i$ if $(j, i) \in E$. A path from node $i$ to node $j$ is a sequence of distinct nodes that starts at $i$ and ends at $j$, such that every pair of consecutive nodes is adjacent. An undirected graph is connected if there is a path between every pair of distinct nodes. The Laplacian matrix $L$ of $G$ is defined as $L = D - A$ where $D$ represents the degree matrix which is a diagonal matrix with entries $d_{ii} = \sum_{j \in N_i} a_{ij}$. For undirected graphs, $L$ is symmetric and positive semi-definite. $L$ has zero row sums and, therefore, zero is an eigenvalue of $L$ with associated eigenvector $1_N$ (a vector with all entries equal to one), that is, $L1_N = 0$. If an undirected graph is connected then $L$ has exactly one eigenvalue equal to zero and all its non-zero eigenvalues are positive; they can be set in increasing order $\lambda_1(L) < \lambda_2(L) \leq \lambda_3(L) \leq ... \leq \lambda_N(L)$, with $\lambda_1(L) = 0$.

B. Problem Statement

Consider a group of $N$ agents with fixed communication graphs and fixed weights. Each agent can be described by the following:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1...N, \quad (1)$$

with

$$u_i(t) = cF \sum_{j \in N_i} (y_i(t) - y_j(t)), \quad i = 1...N, \quad (2)$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$, and the design parameters $F$ and $c$ are defined in (11) and (12), respectively. The variables $y_i \in \mathbb{R}^n$ represent a model of the $i^{th}$ agent’s state using the decoupled dynamics:

$$\dot{y}_i(t) = Ay_i(t), \quad i = 1...N. \quad (3)$$

Every agent in the network implements a model of itself $y_i(t)$ and also models of its neighbors $y_j(t)$. Local events for agent $i$ are defined as follows. When an event is triggered, agent $i$ will transmit its current state $x_i$ to its neighbors. Agent $i$ and its neighbors will all update their local models $y_i(t)$. Since agent $i$ and its neighbors use the same measurements to update the models, say, $x_i(t_{k_i})$ and the model dynamics (3) represent the decoupled dynamics where all agents use the same state matrix, then the model states $y_i(t)$ implemented by agent $i$ and by its neighbors are the same. The model update process is similar for all agents $i = 1...N$. The local control input (2) is decentralized since it only depends on local information, that is, on the model states of the local agent and its neighbors. This approach can be seen as a generalization of the sample-data approach where Zero-Order-Hold (ZOH) models are used as in [22], [23]. Further, the double integrator modeling in [24], where velocity is modeled as ZOH and position as a first-order-hold, represents a particular case of the framework shown in the present paper.

Note that the difference between the agent dynamics (1) and our proposed models (3) is given by the input term in (1) and this input tends to zero as the agents approach a consensus state.

**Lemma 1:** Let $\bar{L}$ be the symmetric Laplacian of an undirected and connected graph. Then, consensus is achieved if and only if

$$V = \xi^T \bar{L} \xi = 0, \quad (4)$$

where $\bar{L} = L \otimes Q$, $Q \in \mathbb{R}^{n \times n}$ is a symmetric positive definite matrix and $\otimes$ denotes the Kronecker product. Also, $\xi(t) = [\xi_1(t)^T \xi_2(t)^T ... \xi_N(t)^T]^T$ and $\xi_i(t) \in \mathbb{R}^n$ for any $t$. 

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Proof. Sufficiency: We can express \( V \) using the following:
\[
V = \sum_{i=1}^{N} \sum_{j \in N_i} T_i Q (i - j) : (5)
\]
Since the graph is undirected (5) can be written in the following form:
\[
V = \sum_{i=1}^{N} \sum_{j \in N_i} \frac{1}{2} (\xi_i^T Q \xi_i - \xi_i^T Q \xi_j - \xi_j^T Q \xi_i + \xi_j^T Q \xi_j) = \sum_{i=1}^{N} \sum_{j \in N_i} \frac{1}{2} (\xi_i - \xi_j)^T T (\xi_i - \xi_j).
\]
Since \( Q > 0 \) and the graph is connected it is clear that if \( V = 0 \) then \( \xi_i = \xi_j \) for \( i, j = 1 \ldots N \).

Necessity: Consider the following expression:
\[
\tilde{\xi} = (\mathcal{L} \otimes Q) \xi,
\]
where \( \tilde{\xi} \) represents an \( n \)-dimensional consensus state and is given by \( \xi = 1_N \otimes \varsigma \), where \( \varsigma = [s_1, s_2, \ldots, s_n]^T \). Then we have:
\[
\tilde{\xi} = (\mathcal{L} \otimes Q) 1_N \otimes \varsigma = \mathcal{L} 1_N \otimes Q \varsigma = 0.
\]
We can conclude that if consensus is achieved then \( V = \xi^T \tilde{\xi} = 0 \).

III. CONSENSUS WITH CONTINUOUS MEASUREMENTS

This section provides an important result that will be useful in subsequent sections. Let us assume in this section that continuous communication between agents is possible, then (2) is given by:
\[
u_i(t) = cP \sum_{j \in N_i} (x_i(t) - x_j(t)), \quad i = 1 \ldots N.
\]
Assume that the pair \((A, B)\) is controllable. Then, for \( \alpha > 0 \) there exists a symmetric and positive definite solution \( P \) to
\[
PA + A^TP - 2PB^TP + 2\alpha P < 0.
\]
Let
\[
F = -B^TP \quad c \geq 1/\lambda_2.
\]
We have the following result.

Theorem 1: Assume the pair \((A, B)\) is controllable and the communication graph is connected and undirected. Define \( F \) and \( c \) as in (11) and (12). Then the following symmetric matrix
\[
\tilde{\mathcal{L}} = \tilde{\mathcal{L}} A_c + A_c^T \tilde{\mathcal{L}}
\]
has only \( n \) eigenvalues equal to zero and the rest of its eigenvalues are negative. In addition, the eigenvectors associated with its \( n \) zero eigenvalues belong to the subspace spanned by the eigenvectors associated with the \( n \) zero eigenvalues of \( \tilde{\mathcal{L}} \), where \( \tilde{\mathcal{L}} = \mathcal{L} \otimes P, A_c = \tilde{A} + \tilde{B}, \tilde{A} = 1_N \otimes A, \tilde{B} = c\mathcal{L} \otimes BF \).

Proof. Since the communication graph is undirected and connected \( \tilde{\mathcal{L}} \) is symmetric and there exists a similarity transformation \( S \) such that \( \tilde{\mathcal{L}} D = S^{-1} \tilde{\mathcal{L}} S \) is diagonal with one eigenvalue equal to zero. Define \( T = S \otimes I_n \) then \( \tilde{\mathcal{L}} D = T^{-1} \tilde{\mathcal{L}} T = \mathcal{L} D \otimes P \) is block diagonal with \( n \) eigenvalues equal to zero.

Let us now consider the following:
\[
T^{-1} \tilde{\mathcal{L}} T = T^{-1} \left( \hat{\mathcal{L}} A_c + A_c^T \hat{\mathcal{L}} \right) T
\]
\[
= T^{-1} \hat{\mathcal{L}} T T^{-1} A_c T + T^{-1} A_c^T T T^{-1} \hat{\mathcal{L}} T
\]
\[
= \tilde{\mathcal{L}} D (I_N \otimes P + P \tilde{\mathcal{L}} D \otimes BF) + (I_N \otimes P + P \tilde{\mathcal{L}} D \otimes BF)^T T \tilde{\mathcal{L}} D.
\]
The term \( I_N \otimes A + c\mathcal{L} D \otimes BF \) is of the form
\[
\begin{bmatrix}
A & 0 \\
0 & U
\end{bmatrix}
\]
In our case since \( \mathcal{L} D \) is diagonal then \( U \) is block diagonal. Furthermore, each block is given by \( U_i = A + c\lambda_i BF, \quad i = 1 \ldots N \). Then we have that (14) is given by the block diagonal matrix:
\[
diag \{0, \mathcal{L}_2, \mathcal{L}_3, \ldots, \mathcal{L}_N \}, \quad \mathcal{L}_1 = \lambda_i (PA + AT - 2c\lambda_i PB^TP) \quad \text{for} \quad i = 1 \ldots N.
\]
Since \( c \geq 1/\lambda_2 \) and \( P \) is the solution of (10) we can conclude that \( \lambda_i (PA + AT - 2c\lambda_i PB^TP) < 0 \), for \( i = 1 \ldots N \). We can see that \( \tilde{\mathcal{L}} \) has \( n \) zero eigenvalues and the rest of its eigenvalues are negative.

Consider the following:
\[
\hat{\mathcal{L}} A_c \rho = \mathcal{L} \otimes P (I_N \otimes A + c\mathcal{L} \otimes BF) \rho, \quad (15)
\]
where \( \rho \) is an eigenvector of \( \hat{\mathcal{L}} \) given by \( \rho = 1_N \otimes \varsigma \) associated with a zero eigenvalue of \( \hat{\mathcal{L}} \) where \( \varsigma = [s_1, s_2, \ldots, s_n]^T \). Then we have:
\[
\hat{\mathcal{L}} A_c \rho = \left( \mathcal{L} \otimes PA + c\mathcal{L}^2 \otimes PBF \right) \rho = L_1 N \otimes PA \varsigma + c\mathcal{L}^2 1_N \otimes PBF \varsigma (16)
\]
\[
= 0.
\]
Similarly, \( A_c^T \hat{\mathcal{L}} \rho = 0 \). Then it is clear that \( \rho \) is an eigenvector of \( \mathcal{L} \) associated with a zero eigenvalue.

Remark. Note that the candidate Lyapunov function \( V = x^T \hat{\mathcal{L}} x \) has derivative along the trajectories of (1) with inputs (9) given by \( V = x^T \hat{\mathcal{L}} x \). It can be seen that \( V \) is negative when the overall system is in disagreement and is equal to zero only when the corresponding states are in total agreement. Different from consensus with single integrators, where the agents converge to a constant value, here it is only required that the difference between states of agents tend to zero, regardless of the particular response of the systems.

As with many consensus algorithms, an estimate of the second smallest eigenvalue of the Laplacian matrix is required; this is the only global information needed by the agents. Algorithms for distributed estimation of the second eigenvalue of the Laplacian have been presented in [30], [31], and [32]. Readers are referred to these papers for details.

IV. CENTRALIZED EVENT TRIGGERED CONSENSUS

A centralized and synchronous condition for event-triggered consensus is given.

Theorem 2: Assume the pair \((A, B)\) is controllable and the communication graph is connected and undirected. Define \( F \) and \( c \) as in (11) and (12). Then the agents (1) with
inputs (2) achieve consensus asymptotically if the events are triggered when
\[ \|e\| > -\frac{\sigma x^T \hat{L} x}{2} \] (17)
where \( 0 < \sigma < 1 \), \( e_i(t) = y_i(t) - x_i(t) \), \( x(t) = [x_1(t)^T \ldots x_N(t)^T]^T \), \( y(t) = [y_1(t)^T \ldots y_N(t)^T]^T \), and \( e(t) = [e_1(t)^T \ldots e_N(t)^T]^T \).

Proof. Consider the candidate Lyapunov function \( V = x^T \hat{L} x \) and evaluate the derivative along the trajectories of systems (1) with inputs (2).
\[
\dot{V} = x^T \hat{L} (\hat{A} x + \hat{B} y) + (\hat{A} x + \hat{B} y)^T \hat{L} x = x^T (\hat{L} A + \hat{A}^T \hat{L}) x + x^T \hat{L} B (x + e) + (x + e)^T \hat{B}^T \hat{L} x = x^T (\hat{L} A_c + \hat{A}^T \hat{L}) x + 2 x^T \hat{L} B e \\
\leq x^T \hat{L} x + 2 \left\| x^T \hat{L} B \right\| \|e\| \] (18)
We have that by applying the threshold (17) the error is bounded by: \( \|e\| \leq \frac{\sigma x^T \hat{L} x}{2\|x^T \hat{L} B\|} \). Also note that \( x^T \hat{L} x \leq 0 \). Then we have that:
\[ \dot{V} \leq (1 - \sigma) x^T \hat{L} x \] (19)
then by Lemma 1 and Theorem 1 the agents achieve consensus asymptotically.

Remark. In the event-triggered case there exists a difference in the control inputs from that of the continuous communication case. In (17) we limit how large the error grows by updating the models and making the error equal to zero when the error grows large compared to the current disagreement.

V. DECENTRALIZED EVENT TRIGGERED CONSENSUS

In this section we derive decentralized thresholds that depend only on local information and can be measured and applied in a decentralized way. Instead of \( c \), the new coupling factor \( c_1 \) is now used in the inputs (2).

Theorem 3: Assume the pair \((A, B)\) is controllable and the communication graph is connected and undirected. Define \( F \) in (11) and \( c_1 = c + c_2 \) where \( c \geq 1/\lambda_2 \) and \( c_2 > 0 \). Then agents (1) with inputs (2) and coupling strength \( c_1 \) achieve consensus asymptotically if the events are triggered when
\[ \delta_i > \sigma z_i^T \Theta_i z_i, \] (20)
where
\[
\delta_i = 2(c_2 - c)N_i z_i^T P B B^T P e_i + 2e_i N_i^2 (1 + b_i) + \frac{2e_i}{b_i} + c N_i (N - 1) \left( b_i + \frac{3}{c_2} \right) e_i^T P B B^T P e_i, \\
0 < \sigma < 1, \ z_i = \sum_{j \in N_i} (y_i - y_j), \ \text{and} \ \\
\Theta_i = (2c_2 - b_i N_i (c_2 - c)) P B B^T P. \] (21)

The parameter \( b_i \) is given by \( 0 < b_i < \frac{2c_2}{N_i (c_2 - c)} \) if \( c_2 > c \), and \( b_i > 0 \) otherwise.

Proof. Proof is provided in [33].

Remark. Note that the variables used to compute (20) - (22), which define the events at node \( i \), are available locally. Concerning global information we only need an estimate of the second eigenvalue of the Laplacian, as it was mentioned earlier.

Remark. In the decentralized approach we have that threshold (20) guarantees asymptotic convergence but it does not necessarily guarantee positive inter-event times. The matrix \( \Theta_i \) which is used for computing the threshold is positive semi-definite in general and positive definite only for particular cases of \( B \). In the less restrictive case, \( \Theta_i \) being positive semi-definite implies that \( z_i^T \Theta_i z_i = 0 \) for some \( z_i \neq 0 \) and continuous communication may be unavoidable. A solution to this problem is proposed in the next section.

VI. DECENTRALIZED EVENT TRIGGERED CONSENSUS WITH GUARANTEED INTER-EVENT TIMES

In this section we consider a threshold that incorporates a small positive constant \( \eta \) to guarantee positive inter-event times. Asymptotic convergence is not obtained but state disagreement can be bounded. The selection of \( \eta \) provides flexibility in the design of the event-triggered protocol by offering a tradeoff between reduction of communication and size of disagreement bounds.

Theorem 4: Assume the pair \((A, B)\) is controllable and the communication graph is connected and undirected. Define \( F \) in (11) and \( c_1 = c + c_2 \) where \( c \geq 1/\lambda_2 \) and \( c_2 > 0 \). Then agents (1) with inputs (2) and coupling strength \( c_1 \) achieve bounded consensus where the difference between any two states is bounded by
\[ \lim_{t \to \infty} \|x_i(t) - x_j(t)\|^2 \leq \frac{N \eta}{\beta_{\max} (\beta)} \] (23)
for \( i, j = 1, \ldots, N \), if the events are triggered when
\[ \delta_i > \sigma z_i^T \Theta_i z_i + \eta, \] (24)
where \( \delta_i \) and \( \Theta_i \) are given by (21) and (22), respectively. Additionally \( 0 < \sigma < 1, \ \eta > 0, \ z_i = \sum_{j \in N_i} (y_i - y_j), \ \beta = \frac{\beta_{\max} (\beta)}{\lambda_{\max} (\beta)}, \ 0 < \beta < c_2 < c_1 \) if \( c_2 > c \), and \( b_i > 0 \) otherwise. Furthermore, the agents do not exhibit Zeno behavior and the inter-event times \( t_{k+1} - t_k \) for every agent \( i = 1, \ldots, N \) are bounded by the positive times \( \tau_i \), that is
\[ 0 < \tau_i \leq t_{k+1} - t_k \] (25)
where
\[ \tau_i = \frac{\ln \left( \frac{n}{\beta^2} + g^2 \right)^{1/2} - g + 1}{\|A\|} \] (26)
and the parameters \( k_2 \) and \( g \) are given by:

\[
k_2 = \frac{2cN_i^2(1 + b_i) + \frac{c_i}{b_i}N_i + cN_i(N - 1)(b_i + \frac{3}{b_i})}{\|PBB^TP\|} \left( \frac{z_{i,\text{max}}\|cBF\|}{\|A\|} \right)^2
\]

\[
g = \frac{2cN_i^2(1 + b_i) + \frac{c_i}{b_i}N_i + cN_i(N - 1)(b_i + \frac{3}{b_i})}{2\|cBF\|} \frac{\|z_{i,\text{max}}\|\|cBF\|}{\|A\|}
\]

(27)

and \( z_{i,\text{max}} \) represents a bound on \( z_i(t) \), that is, \( \|z_i(t)\| \leq z_{i,\text{max}} \).

Remark. The results provided in this paper hold for agents described by general linear dynamics. The common cases of single integrators and double integrators are particular cases covered by this framework. The single integrator is modeled as a ZOH and the double integrator is modeled similar to [24], that is, velocity as a ZOH and position as a first-order-hold model.

VII. EXAMPLE

Consider a decentralized model-based implementation of four second order agents with unstable linear dynamics given by:

\[
A = \begin{bmatrix} 0.5 & -1 \\ 0.6 & 0.1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \quad P = \begin{bmatrix} 0.7624 & -0.1788 \\ -0.1788 & 0.2257 \end{bmatrix}
\]

where the matrix \( P \) is obtained by solving (10). The parameter \( \eta \) is selected to be \( \eta = 0.5 \). The nonzero elements of the undirected adjacency matrix are \( a_{12} = a_{23} = a_{34} = 1 \) (the corresponding symmetric elements are also equal to one). Figure 1 shows the response of the agents and the models. Figure 2 shows that the disagreement between agents converge to a bounded region; it also shows the communication time instants for every agent.

Traditional event-triggered strategies use a ZOH implementation, i.e. the updates remain constant until new measurements are obtained. Fig. 3 shows the response of the same example with the same parameters and the only difference is that the models using the decoupled dynamics are replaced by ZOH devices. Fig. 4 shows the disagreement trajectories and update instants. When comparing these two cases it is clear that the model-based implementation offers better performance in terms of reducing inter-agent communication. For instance, the total number of updates by all agents during the simulation time shown in Fig. 2 was 163. The simulation time shown in Fig. 4 is the same but the total number of updates is of a magnitude greater and equal to 1623. The improved performance by the model implementation is easily explained by realizing that the difference between the response of the model and the response of the agents becomes smaller as the agents progress into a consensus trajectory, regardless of the particular response of the systems. This means that the models are able to obtain a more accurate estimate of the states of the agents as time progresses. On the other hand, the ZOH devices are not able to follow a time-varying trajectory without updating at faster rates. When the free response of the systems is unstable the update intervals of the ZOH tend to zero which clearly makes this implementation unsuitable for event-triggered consensus with general linear dynamics.

VIII. CONCLUSIONS

Event-based consensus protocols for linear systems and discontinuous communication have been studied. Centralized
and decentralized events have been designed in order for each agent to broadcast measurement updates only when it is necessary, that is, when a function of discrepancy between real and model states is greater than a specified threshold. The decentralized event triggered technique allows each agent to transmit information based on its own decisions and synchronization of updates is not required as in periodic approaches. The use of models and event-based techniques provides a formal framework that reduces communication and provides freedom to each agent in order to determine its own broadcasting instants.

**REFERENCES**


