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SOME RANDOM APPROXIMATIONS AND RANDOM FIXED POINT THEOREMS FOR 1-SET-CONTRACTIVE RANDOM OPERATORS

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ABSTRACT. In this paper, we will prove that the random version of Fan's Theorem (Math. Z. **112** (1969), 234–240) is true for 1-set-contractive random operator $f : \Omega \times B_R \to X$, where B_R is a weakly compact separable closed ball in a Banach space X and Ω is a measurable space. This class of 1-set-contractive random operator includes condensing random operators, semicontractive random operators, *LANE* random operators, nonexpansive random operators and others. As applications of our theorems, some random fixed point theorems of non-self-maps are proved under various well-known boundary conditions.

1. INTRODUCTION AND PRELIMINARIES

Since Bharucha-Reid [1] proved the stochastic version of the well-known Schauder's fixed point theorem, random fixed point theory and applications have been developed rapidly in recent years (see, e.g., [5, 6, 8, 9, 13, 15, 16]). In this paper, we will consider a stochastic version of a very interesting theorem of Fan [4, Theorem 2] which is stated as follows:

Let K be a nonempty compact convex set in a normed linear space X. For any continuous map f from K into X, there exists a point $u \in K$ such that

$$\|u - f(u)\| = d(f(u), K).$$

This theorem has been of great importance in Nonlinear Analysis, Approximation Theory, Game Theory and Minimax Theorems. Various aspects of the above theorem have been studied by Lin [7, 8], Papageorgiou [13], Sehgal and Singh [15], Sehgal and Waters [16], and others. Recently, Lin [8, Theorem 1] proved that Ky Fan's Theorem is true for a continuous condensing random operator $f : \Omega \times B_R \to X$, where B_R is a separable closed ball in a Banach space X and Ω is a measurable space. The purpose of the present paper is to extend Lin's result to more general 1-set-contractive random operators in a Banach space. We will also prove that the random version of the above theorem is true for a semicontractive (or *LANE*) random operator $f : \Omega \times B_R \to X$, where B_R is a separable closed ball in a uniformal convex Banach space. As applications of our theorems, some stochastic fixed point theorems are derived under various well-known boundary conditions.

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Throughout this paper, (Ω, Σ) denotes a measurable space with Σ a sigma algebra of subsets of Ω. Let X be a Banach space. A map $F : \Omega \to X$ is said to be measurable (respectively, weakly measurable) if $F^{-1}(B) = \{\omega \in \Omega : F(\omega) \in B\} \in \Sigma$ for each closed (respectively, open) subset B of X. Let D be a nonempty subset of X. Then a map $f : \Omega \times D \to X$ is called a random operator if for each fixed $x \in D$, the map $f(\cdot, x) : \Omega \to x$ is measurable. A measurable map $\varphi : \Omega \to D$ is called a random fixed point of f if $f(\omega, \varphi(\omega)) = \varphi(\omega)$ for each $\omega \in \Omega$.

A map $f : D \to X$ is called nonexpansive if $\|f(x) - f(y)\| \le \|x - y\|$ for x, $y \in D$; f is called completely continuous if it maps weakly convergent sequences into strongly convergent sequences. We recall that a map $f : D \to X$ is said to be demiclosed at $y \in X$ [3] if, for any sequence $\{x_n\}$ in D, the conditions $x_n \to x \in D$ weakly and $f(x_n) \to y$ strongly imply $f(x) = y$. A map $f : D \to X$ is called a k-set-contractive [14] ($k \ge 0$) if f is continuous and, for each bounded subset B of D, $\alpha(f(B)) \le k\alpha(B)$, where $\alpha(\cdot)$ denotes Kuratowski's measure of noncompactness; $f : D \to X$ is said to be condensing if f is continuous and, for each bounded subset B of D with $\alpha(B) > 0$, $\alpha(f(B)) < \alpha(B)$. A map $f : D \to X$ is said to be semicontractive [3, 14] if there exists a map V of $D \times D$ into X such that $f(x) = V(x, x)$ for $x \in D$, while:

(a) For each fixed x in D, $V(\cdot, x)$ is nonexpansive from D to X.

(b) For each fixed x in D, $V(x, \cdot)$ is completely continuous from D to X, uniformly for x in bounded subset of D.

We say that a continuous map f of D into X is *LANE* [11] (locally almost nonexpansive) if given $x \in D$ and $\epsilon > 0$, there exists a weak neighborhood N_x of x in D (depending also on ϵ) such that $\|f(u) - f(v)\| \le \|u - v\| + \epsilon$ for u, $v \in N_x$.

A random operator $f : \Omega \times D \to X$ is said to be continuous (1-set-contractive, condensing, nonexpansive, semicontractive, *LANE*, completely continuous, etc.) if the map $f(\omega, \cdot) : D \to X$ is so, for each $\omega \in \Omega$. For $R > 0$, let

$$B_R = \{x \in X : \|x\| \le R\}, \qquad S_R = \{x \in X : \|x\| = R\}.$$

2. Main results

In order to prove our main theorems we need the following lemma which will play a crucial role in this paper.

Lemma 2.1. *Let X be a Banach space and $T : B_R \to X$ be a 1-set-contractive map such that $I - T$ is demiclosed at 0, where I is the identity map on X. Then $I - B$ is also demiclosed at 0, where $B \equiv hT : B_R \to B_R$, and $h : X \to B_R$ is a map defined by*

$$h(x) = \begin{cases} x & \text{if } \|x\| \le R, \\ \dfrac{Rx}{\|x\|} & \text{if } \|x\| \ge R. \end{cases}$$

Proof. Let $\{x_n\}$ be any sequence in B_R such that

$$x_n \to u \in B_R \text{ weakly}, \qquad x_n - B(x_n) \to 0 \quad \text{as } n \to \infty. \tag{2.1}$$

We have to distinguish two possible cases:

Case 1. If there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\|T(x_{n_k})\| \le R$ for all k, then $B(x_{n_k}) = T(x_{n_k})$ by the definition of h. Hence, $x_{n_k} - T(x_{n_k}) = x_{n_k} - B(x_{n_k}) \to 0$ as $k \to \infty$. It follows from the demiclosedness at 0 of $I - T$ that $u - T(u) = 0$. Therefore, $B(u) = h(T(u)) = h(u) = u$ by the definition of h.

Case 2. Otherwise, there exists an integer N such that, for $n \geq N$, $\|T(x_n)\| > R$, then $B(x_n) = RT(x_n)/\|T(x_n)\|$ by the definition of h. Since T is a 1-set-contractive map and $\{x_n\}$ is bounded, $\{\|T(x_n)\|\}$ is bounded. Hence there exist a convergent subsequence $\{\|T(x_{n_i})\|\}$ of $\{\|T(x_n)\|\}$ and a real number $r > 0$ such that

$$\|T(x_{n_i})\| \to r \quad \text{as } i \to \infty. \tag{2.2}$$

From $\|T(x_{n_i})\| > R$ $(i = 1, 2, 3, \cdots)$, we have $r \geq R$. Therefore, by (2.1) and (2.2), we have

$$\begin{aligned} x_{n_i} - \frac{R}{r}T(x_{n_i}) &= \left(x_{n_i} - \frac{RT(x_{n_i})}{\|T(x_{n_i})\|}\right) + \left(\frac{RT(x_{n_i})}{\|T(x_{n_i})\|} - \frac{R}{r}T(x_{n_i})\right) \\ &= (x_{n_i} - B(x_{n_i})) + \left(\frac{1}{\|T(x_{n_i})\|} - \frac{1}{r}\right) RT(x_{n_i}) \\ &\to 0 \text{ as } i \to \infty. \end{aligned} \tag{2.3}$$

If $r = R$, then $x_{n_i} - T(x_{n_i}) \to 0$ $(i \to \infty)$, by (2.3). It follows from the demiclosedness at 0 of $I - T$ that $u = T(u)$. Therefore, $B(u) = h(T(u)) = h(u) = u$ by the definition of h. If $r > R$, then $(R/r)T$ is a (R/r)-set-contractive map with $R/r < 1$. Let $y_{n_i} \equiv x_{n_i} - (R/r)T(x_{n_i})$. Then $\alpha(\{y_{n_i}\}) = 0$ and $x_{n_i} = y_{n_i} + (R/r)T(x_{n_i})$. It follows that $\alpha(\{x_{n_i}\}) = 0$. Hence there exists a subsequence $\{x_{n_{i_j}}\}$ of $\{x_{n_i}\}$ such that $\{x_{n_{i_j}}\}$ converges to u. This and the continuity of T imply that $y_{n_{i_j}} \to u - (R/r)T(u) = 0$, i.e., $u = (R/r)T(u)$. From (2.2), we have $\|T(u)\| = r > R$. Therefore $u = RT(u)/\|T(u)\| = B(u)$ by the definition of h.

This completes the proof of Lemma 2.1. □

We shall also need the following random fixed point theorem which is Theorem 2.1 in [9]:

Lemma 2.2. *Let D be a nonempty weakly compact convex subset of a separable Banach space X, and $f : \Omega \times D \to D$ be a 1-set-contractive random operator such that $I - f(\omega, \cdot)$ is demiclosed, for each $\omega \in \Omega$. Then f has a random fixed point.*

Remark 2.1. From the proof of Lin's theorem above, we can easily see that we only need the hypothesis $I - f(\omega, \cdot)$ to be demiclosed at 0 for each $\omega \in \Omega$ instead of $I - f(\omega, \cdot)$ demiclosed for each $\omega \in \Omega$, and Lin's result also holds, if we assume that D is separable instead of X is separable.

Theorem 2.1. *Let B_R be a weakly compact separable subset of X and $f : \Omega \times B_R \to X$ be a 1-set-contractive random operator such that $I - f(\omega, \cdot)$ is demiclosed at 0, for each $\omega \in \Omega$, where I is the identity map on X. Then there exists a measurable map $\varphi : \Omega \to B_R$ such that for each $\omega \in \Omega$*

$$\|\varphi(\omega) - f(\omega, \varphi(\omega))\| = d(f(\omega, \varphi(\omega)), B_R). \tag{2.4}$$

Moreover, to this φ, for each $\omega \in \Omega$, if $\|f(\omega, \varphi(\omega))\| > R$, then $\varphi(\omega) \in S_R$ and

$$\varphi(\omega) = \frac{Rf(\omega, \varphi(\omega))}{\|f(\omega, \varphi(\omega))\|}, \qquad \|f(\omega, \varphi(\omega)) - \varphi(\omega)\| = \|f(\omega, \varphi(\omega))\| - R > 0; \tag{2.5}$$

if $\|f(\omega, \varphi(\omega))\| \leq R$, then $\varphi(\omega) = f(\omega, \varphi(\omega))$.

Proof. Let h be the same as in Lemma 2.1. From Nussbaum [10, Corollary 1], h is a 1-set-contractive map. Define $F : \Omega \times B_R \to B_R$ by $F(\omega, x) = h(f(\omega, x))$. It is easy to see that F is a 1-set-contractive random operator. For arbitrary but fixed

$\omega \in \Omega$, $I - F(\omega, \cdot)$ is demiclosed at 0 by Lemma 2.1. From Lemma 2.2, F has a random fixed point $\varphi : \Omega \to B_R$, i.e., φ is a measurable map and $F(\omega, \varphi(\omega)) = \varphi(\omega)$ for each $\omega \in \Omega$. Now we will prove that this measurable map φ satisfies the desired property. For each $\omega \in \Omega$, we consider the following two cases:

(i) If $\|f(\omega, \varphi(\omega))\| \leq R$, then $F(\omega, \varphi(\omega)) = f(\omega, \varphi(\omega))$ by the definition of h. Thus, we have $\|\varphi(\omega) - f(\omega, \varphi(\omega))\| = \|F(\omega, \varphi(\omega)) - f(\omega, \varphi(\omega))\| = 0$.

(ii) If $\|f(\omega, \varphi(\omega))\| > R$, then $F(\omega, \varphi(\omega)) = Rf(\omega, \varphi(\omega))/\|f(\omega, \varphi(\omega))\|$ by the definition of h. Hence, we have

$$\begin{aligned}\|\varphi(\omega) - f(\omega, \varphi(\omega))\| &= \|F(\omega, \varphi(\omega)) - f(\omega, \varphi(\omega))\| \\ &= \left\| \frac{Rf(\omega, \varphi(\omega))}{\|f(\omega, \varphi(\omega))\|} - f(\omega, \varphi(\omega)) \right\| \\ &= \|f(\omega, \varphi(\omega))\| - R.\end{aligned}$$

Thus, for any $x \in B_R$, we have

$$\begin{aligned}\|\varphi(\omega) - f(\omega, \varphi(\omega))\| &\leq \|f(\omega, \varphi(\omega))\| - R \\ &\leq \|f(\omega, \varphi(\omega))\| - \|x\| \\ &\leq \|f(\omega, \varphi(\omega)) - x\|.\end{aligned}$$

Therefore, (2.4) holds.

For each $\omega \in \Omega$, if $\|f(\omega, \varphi(\omega))\| > R$ with $\varphi(\omega) \in B_R$, then $f(\omega, \varphi(\omega)) \notin B_R$ and by (ii), (2.5) holds. If $\|\varphi(\omega)\| < R$, then there exists $a \in (0, 1)$ such that $u \equiv a\varphi(\omega) + (1 - a)f(\omega, \varphi(\omega)) \in B_R$. It follows that

$$\begin{aligned}d(f(\omega, \varphi(\omega)), B_R) &\leq \|f(\omega, \varphi(\omega)) - u\| \\ &= a\|f(\omega, \varphi(\omega)) - \varphi(\omega)\| \\ &< \|f(\omega, \varphi(\omega)) - \varphi(\omega)\| \\ &= d(f(\omega, \varphi(\omega)), B_R).\end{aligned}$$

We get a contradiction. Hence $\varphi(\omega) \in S_R$. If $\|f(\omega, \varphi(\omega))\| \leq R$ with $\varphi(\omega) \in B_R$, then by (i) we have $\varphi(\omega) = f(\omega, \varphi(\omega))$.

This completes the proof of the theorem. □

Theorem 2.2. *Let B_R be a separable subset of a uniformly convex Banach space X and $f : \Omega \times B_R \to X$ be a semicontractive random operator. Then the conclusions of Theorem 2.1 hold.*

Proof. From Petryshyn [14, Lemma 3.2 and p. 338], $f : \Omega \times B_R \to X$ is 1-set-contractive random operator. By Browder [3, Theorem 3], $I - f(\omega, \cdot)$ is demiclosed, for each $\omega \in \Omega$. Hence, Theorem 2.2 follows from Theorem 2.1. □

Corollary 2.1. *Let B_R be a separable subset of a uniformly convex Banach space X, $g : \Omega \times B_R \to X$ a nonexpansive random operator and $h : \Omega \times B_R \to X$ a completely continuous random operator. If $f \equiv g + h$, then the conclusions of Theorem 2.1 hold.*

Proof. Since $f : \Omega \times B_R \to X$ is a semicontractive random operator under the representation $V(\omega, u, v) = g(\omega, u) + h(\omega, v)$, this corollary just follows from Theorem 2.2. □

Theorem 2.3. *Let B_R be a separable subset of a uniformly convex Banach space X, $g : \Omega \times B_R \to X$ a LANE random operator, and $h : \Omega \times B_R \to X$ a completely* *continuous random operator. If $f \equiv g + h$, then the conclusions of Theorem 2.1 hold.*

Proof. From [14, Remark 3.7], $f : \Omega \times B_R \to X$ is also a *LANE* random operator. By [11], f is a 1-set-contractive random operator and $I - f(\omega, \cdot)$ is demiclosed, for each $\omega \in \Omega$. Therefore, Theorem 2.3 follows from Theorem 2.1. □

The following is another random approximation theorem of Ky Fan type. Before we state the theorem, we recall a definition. A Banach space X is said to satisfy Opial's condition [12] if the following holds: If $\{x_n\}$ converges weakly to x_0, and $x \neq x_0$, then $\liminf \|x_n - x\| > \liminf \|x_n - x_0\|$. Banach spaces satisfying Opial's condition include Hilbert spaces and l^p $(1 \le p < \infty)$ spaces.

Theorem 2.4. *Let B_R be a weakly compact separable closed ball of a Banach space X, $g : \Omega \times B_R \to X$ a nonexpansive random operator, and $h : \Omega \times B_R \to X$ a completely continuous random operator. If X satisfies Opial's condition and $f \equiv g + h$, then the conclusions of Theorem 2.1 hold.*

Proof. Since f is 1-set-contractive and $I - f(\omega, \cdot)$ is demiclosed, for each $\omega \in \Omega$, by Opial [12] if X satisfies Opial's condition, Theorem 2.4 follows from Theorem 2.1. □

3. Applications to random fixed point theorems

Theorem 3.1. *Suppose that X, B_R and f are the same as in Theorem 2.1. Then f has a random fixed point if f satisfies one of the following conditions*:

(B_1) *For each $\omega \in \Omega$, each $x \in S_R$ with $\|f(\omega, x)\| > R$, there exists y, depending on ω and x, in $I_{B_R}(x) = \{x + c(z - x) : \text{some } z \in B_R,\ c > 0\}$ such that $\|y - f(\omega, x)\| < \|x - f(\omega, x)\|$.*

(B_2) *f is weakly inward, i.e., for each $\omega \in \Omega$, $f(\omega, x) \in \overline{I_{B_R}(x)}$ for $x \in S_R$.*

(B_3) *$x \neq \lambda f(\omega, x)$ for each $\omega \in \Omega$ and $x \in S_R$ with $\|f(\omega, x)\| > R$ and $0 < \lambda < 1$.*

(B_4) *$\|f(\omega, x) - x\| \neq \|f(\omega, x)\| - R$, for each $\omega \in \Omega$ and $x \in S_R$ with $\|f(\omega, x)\| > R$.*

(B_5) *For each $\omega \in \Omega$ and $x \in S_R$, with $\|f(\omega, x)\| > R$, there exists $\alpha \in (1, \infty)$ such that*

$$\|f(\omega, x)\|^\alpha - R^\alpha \le \|f(\omega, x) - x\|^\alpha.$$

(B_6) *For each $\omega \in \Omega$ and $x \in S_R$, with $\|f(\omega, x)\| > R$, there exists $\beta \in (0, 1)$ such that*

$$\|f(\omega, x)\|^\beta - R^\beta \ge \|f(\omega, x) - x\|^\beta.$$

Proof. By Theorem 2.1, there exists a measurable map $\varphi : \Omega \to B_R$ such that, for each $\omega \in \Omega$, (2.4) holds, and for each $\omega \in \Omega$, if $\|f(\omega, \varphi(\omega))\| > R$, then $\|\varphi(\omega)\| = R$ and (2.5) holds; if $\|f(\omega, \varphi(\omega))\| \le R$, then $\varphi(\omega) = f(\omega, \varphi(\omega))$. We will prove that φ is the desired random fixed point of f. Toward this end, we consider the following two cases:

(a) If there exists $\omega \in \Omega$, such that $\|\varphi(\omega)\| < R$, then there exists $\lambda \in (0, 1)$ such that $\|\lambda\varphi(\omega) + (1 - \lambda) f(\omega, \varphi(\omega))\| < R$. Therefore

$$\begin{aligned}\|\varphi(\omega) - f(\omega, \varphi(\omega))\| &= d(f(\omega, \varphi(\omega)), B_R)\\ &\le \|f(\omega, \varphi(\omega)) - [\lambda\varphi(\omega) + (1 - \lambda) f(\omega, \varphi(\omega))]\| = \lambda\|\varphi(\omega) - f(\omega, \varphi(\omega))\|,\end{aligned}$$

it follows from $0 < \lambda < 1$ that $\varphi(\omega) = f(\omega, \varphi(\omega))$.

(b) If there exists $\omega \in \Omega$, such that $\|\varphi(\omega)\| = R$, we will show that $\varphi(\omega) = f(\omega, \varphi(\omega))$. By (2.4), we need only show that $f(\omega, \varphi(\omega)) \in B_R$.

(1) If $\|f(\omega, \varphi(\omega))\| > R$ and f satisfies (B_1), then there exists y in $I_{B_R}(\varphi(\omega))$ such that

$$\|y - f(\omega, \varphi(\omega))\| < \|\varphi(\omega) - f(\omega, \varphi(\omega))\|.$$

Since $y \in I_{B_R}(\varphi(\omega))$, there exists $z \in B_R$, $c > 0$ such that $y = \varphi(\omega) + c(z - \varphi(\omega))$. Since $y \notin B_R$ (otherwise it contradicts the choice of φ), we can assume $c > 1$. Then

$$z = \frac{y}{c} + \left(1 - \frac{1}{c}\right)\varphi(\omega) = (1-\beta)y + \beta\varphi(\omega),$$

where $\beta = 1 - 1/c$, $0 < \beta < 1$. Therefore

$$\begin{aligned}\|z - f(\omega, \varphi(\omega))\| &\leq (1-\beta)\|y - f(\omega, \varphi(\omega))\| + \beta\|\varphi(\omega) - f(\omega, \varphi(\omega))\| \\ &< (1-\beta)\|\varphi(\omega) - f(\omega, \varphi(\omega))\| + \beta\|\varphi(\omega) - f(\omega, \varphi(\omega))\| \\ &= \|\varphi(\omega) - f(\omega, \varphi(\omega))\|,\end{aligned}$$

which contradict the choice of φ. Hence $f(\omega, \varphi(\omega)) \in B_R$.

(2) If f satisfies condition (B_2), then f satisfies (B_1).

(3) If$\|f(\omega, \varphi(\omega))\| > R$ and f satisfies (B_3), then by (2.5) we have $\varphi(\omega) = \lambda_0 f(\omega, \varphi(\omega))$, where $\lambda_0 = R/\|f(\omega, \varphi(\omega))\|$, $0 < \lambda_0 < 1$, which contradicts condition (B_3). Therefore, $f(\omega, \varphi(\omega)) \in B_R$.

(4) If $\|f(\omega, \varphi(\omega))\| > R$ and f satisfies (B_4), then by (2.5) we have $\|f(\omega, \varphi(\omega)) - \varphi(\omega)\| = \|f(\omega, \varphi(\omega))\| - R$. This is a contradiction to the condition (B_4). Therefore, $f(\omega, \varphi(\omega)) \in B_R$.

(5) If $\|f(\omega, \varphi(\omega))\| > R$ and f satisfies (B_5), then there exists $\alpha \in (1, \infty)$ such that

$$\|f(\omega, \varphi(\omega))\|^\alpha - R^\alpha \leq \|f(\omega, \varphi(\omega)) - \varphi(\omega)\|^\alpha.$$

Let $\lambda = R/\|f(\omega, \varphi(\omega))\|$; then $0 < \lambda < 1$ and

$$\frac{(\|f(\omega, \varphi(\omega))\| - R)^\alpha}{\|f(\omega, \varphi(\omega))\|^\alpha} = (1-\lambda)^\alpha < 1 - \lambda^\alpha \leq \frac{\|f(\omega, \varphi(\omega)) - \varphi(\omega)\|^\alpha}{\|f(\omega, \varphi(\omega))\|^\alpha}.$$

Hence $\|f(\omega, \varphi(\omega)) - \varphi(\omega)\| > \|f(\omega, \varphi(\omega))\| - R$, we get a contradiction to (2.5). Therefore, $f(\omega, \varphi(\omega)) \in B_R$.

(6) If $\|f(\omega, \varphi(\omega))\| > R$ and f satisfies (B_6), then similar to the proof of (5), we can prove that $f(\omega, \varphi(\omega)) \in B_R$.

In sum, we have shown that the measurable map $\varphi : \Omega \to B_R$ satisfies $\varphi(\omega) = f(\omega, \varphi(\omega))$ for each $\omega \in \Omega$, i.e., φ is a random fixed point of f. □

Using similar methods, we can show the following theorems and corollary, and we omit their proof.

Theorem 3.2. *Let B_R be a separable subset of a uniformly convex Banach space X. If $f : \Omega \times B_R \to X$ is a semicontractive random operator and satisfies any one of the six conditions of Theorem 3.1, then f has a random fixed point.*

Corollary 3.1. *Let B_R be a separable subset of a uniformly convex Banach space X, $g : \Omega \times B_R \to X$ a nonexpansive random operator, and $h : \Omega \times B_R \to X$ a completely continuous random operator. If $f \equiv g + h$ satisfies any one of the six conditions of Theorem 3.1, then f has a random fixed point.*

Theorem 3.3. *Let B_R be a separable subset of a uniformly convex Banach space X, $g : \Omega \times B_R \to X$ a LANE random operator, and $h : \Omega \times B_R \to X$ a completely continuous random operator. If $f \equiv g + h$ satisfies any one of the six conditions of Theorem 3.1, then f has a random fixed point.*

Theorem 3.4. *Let B_R be a weakly compact separable closed ball of a Banach space X, $g : \Omega \times B_R \to X$ a nonexpansive random operator, and $h : \Omega \times B_R \to X$ a completely continuous random operator. If X satisfies Opial's condition and $f \equiv g + h$ satisfies any one of the six conditions of Theorem 3.1, then f has a random fixed point.*

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