

## SYMBOLIC AND SUBSYMBOLIC COMPONENTS IN THE LOGIC OF BELIEF CHANGE

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The intention of this paper is to 'locate' symbolic and subsymbolic components in the theory of belief change. First I will consider the roles of these components within the framework of epistemic logic (Hintikka style) and then apply it on a wider context - i.e. to the theory of belief change (Gärdenfors style or AGM theory). In this way we shall obtain the basis of the logic of belief change.

For the needs of logical analysis of epistemic context as a starting position I take the first-order logic with two characteristics:

- (i) the role of quantifiers/ information independence
- (ii) universe of possible worlds semantics/ the set of presuppositions of epistemic states (wide sense)

It is important to (i) stress here that if we said for the quantified expression that a truth-making choice of one variable can be made independently of another variable, we actually are out of the first-order epistemic logic. However since such formulas can be reduced to first-order logic we think this should not pose greater problems. More precisely we do not intend to give the final word to the presentation of

dynamic epistemic logic but we intend to take a stand towards the 'main' problems of philosophy of first order modal logic. Thus the problem of partial quantifier ordering remains an important task for the building of such first order logic. We suppose that this would then be only a technical problem for the logicians.

For the second characteristic, (ii), it is important to stress that what some logicians called a set of possible worlds we call it a set of presuppositions of epistemic states. As a subset of this set we have all actual epistemic states of the cognizer which are directly tied to the epistemic input. Presuppositions are its complement and they are created either with the force of 'logical architecture' of the mind or with the memory inputs from higher (symbolic) levels but less conclusively.

Examples will make it more evident which role these two characteristics play. We start with language construction of knowledge that. Then we have two readings:

(1) b knows that there is an individual x such that  $S(x)$

and

(2) b knows of some individual x that  $S(x)$

which formally looks like

(3)  $K_b (\exists x) S(x)$

and respectively

(4)  $(\exists x) K_b S(x)$

Here the latter has roughly the force

(5) b knows who (what), say x, is such that  $S(x)$ ,

and this makes it harder for us to get the core in the analysis of epistemic notions (i.e. states). Namely, the grammatical construction what (who) appears. For now we can only mention that the problem of the relation of the verb + wh-construction to the verb + direct grammatical object construction will very likely appear as a problem.

For the explanation of the mechanism of these two readings we relativize the order of K and  $(\exists x)$  expressing it with two-dimensional representation (Hintikka):

(6)  $(\exists x)$   
 $K_b$   $S(x)$

Thus (6) can be logically written as

$$(7) \quad K_b((\exists x)/K_b) S(x).$$

For the clearer understanding of the above-mentioned here is another example (Carlson). Let us take a usual sentence-like epistemic state:

$$(8) \quad \text{Every writer likes some book of his almost as much as every critic dislikes a book he has reviewed.}$$

or more formally

$$(9) \quad (x)(\exists y) \quad (x \text{ is writer} \Rightarrow (y \text{ is a book} \ \& \ x \text{ has authored } y \ \& \ (z)(\exists u) \quad (\text{z is a critic} \Rightarrow (u \text{ is a book} \ \& \ z \text{ has reviewed } u \ \& \ x \text{ like } y \text{ almost as much as } z \text{ dislikes } u))))).$$

Epistemic state like (8) we can write down in natural language without destroying the framework of analysis. Our aim is to show the mechanism ( or logic) of 'self-translation' of the epistemic state. Thus we have the reading which completely reminds us of the traditional analysis of epistemic concepts of natural language:

$$(10) \quad \text{I know which book of his each writer likes almost as much as every critic dislikes some book he has reviewed.}$$

This sentence can be formalized:

$$(11) \quad (x)(\exists y) \quad K_I(z)(\exists u) \quad (x \text{ is writer} \Rightarrow (y \text{ is a book} \ \& \ x \text{ has authored } y \ \& \ K_I(z \text{ is a critic} \Rightarrow (u \text{ is a book} \ \& \ z \text{ has reviewed } u \ \& \ x \text{ like } y \text{ almost as much as } z \text{ dislikes } u))))).$$

In terms of possible world semantics (11) is represented as

$$(12) \quad (x)(\exists y) \quad (w)(z)(\exists u) \quad (x \text{ is writer at } w_0 \Rightarrow (y \text{ is a book at } w_0 \ \& \ x \text{ has authored } y \text{ at } w_0 \ \& \ ((w \text{ is an epistemic I - alternative to } w_0 \ \& \ z \text{ is a critic at } w) \Rightarrow (u \text{ is a book} \ \& \ z \text{ has reviewed } u \text{ at } w \ \& \ x \text{ like } y \text{ almost as much as } z \text{ dislikes } u \text{ at } w))))).$$

From the above example we see (or it should be seen) that we need variable-like epistemic operators within the formula that are bound by quantifier-like epistemic operators in the prefix. Example (10), although 'natural language'-like evidently shows two readings which coincide with the traditional difference of *de re/de dicto*. More precisely which and each are understood as *de re* with respect to know, whereas every and some may well be taken as *de dicto* with respect to know. First two quantifiers have knowledge within their domain while the two last quantifiers are in the domain of knowledge.

The main characteristic which we got refers to the information independence of quantifiers. Namely if (4) is formal representation of (5), which role is played by (3)? We have already said that we are not going to discuss the possibility of exportation/importation (i.e. translations of *de re/de dicto* and *vice versa*), but we try to get the 'working' mechanism of these two distinctions. More correct translation of (3) looks thus:

(13) b knows that something is S.

This example is like (2), but more like inner presupposition. More precisely, we cannot always with certainty claim the dependence of b's knowledge of the actual epistemic inputs. We can bring out 'valid' arguments for the claim that apart from the actual epistemic states, we deal with different aspects of presuppositions such as: existence (as in the example (13)), pragmatics, strong and weak logical (or semantical) presuppositions, etc. But we do not need this thesis for now. What is more important is that direct object construction, that is, *de re* reading leans on the actual epistemic inputs. *De dicto* reading and the verb + *wh*-construction are mainly tied to the symbolic level and also entail elements of complement of the set of actual epistemic states.

Let us see further the problems of epistemic logic.

The relevant problems are best illustrated by the analysis of perceptual states. In such logic we differentiate two kinds of crossidentification of objects: The *first* method of crossidentification is called perceptual (perspective) which can formally be represented as:

(14)  $(\exists x) (s \text{ knows } b=x)$  i.e.  
 $(\exists x) (s \text{ knows that } x \text{ exists})$

Perspective crossidentification is found in situations when the percipient can find himself in the spatial disorder in relation to his own body and it also includes mistakes in the recognition, naming or commanding of different parts of the body, etc. While the *other* method is descriptive (public) crossidentification, which formally looks like the following:

(15)                    (Ex) (s knows  $x = b$ )

Thus if we have the case where the form of epistemic input is not essential (these are the presuppositions) then let us take that it is language. For logic it is enough that it is symbolic so we further assume certain parallelism between language and thought. Logic represents or it tends to formally represent the mind (definitely in the normative sense where the relationship of declarative and normative is not an unsolvable problem).

Here we can put two questions. The first question is: If we cannot avoid the flow of information (although we have presuppositions), how do we process the information and in which shape the information comes?

The second question concerns the logical analysis, i.e., if the epistemic logic presupposes language (and this makes the main problem for the introduction of information in the logical analysis of epistemic concepts), what is then the connection between this language and information at some subsymbolic level?

Let us look at the first possible alternative of answers to these two questions where we shall introduce a concept: scheme.

We shall differentiate schemes by presenting their common characteristics (Gärdenfors):

- They represent objects, situations, actions
- They have variables. There are ways of scheme change as adaptation to the situation. (So one scheme can have another scheme as part of an instance of the variable).
- They help when there is lack of information about the environment.
- They are capable of filling in the missing information.

The answer to our question is then the following: There are two levels of analysis: symbolic and subsymbolic. Scheme represents a certain bridge between symbolic and subsymbolic levels. Symbolic level emerges from the subsymbolic in one and the same system (Gärdenfors).

The second possible answer to our questions is the following: Relying on Marr's (that is Vaina's) modular analysis of perception we can show that Hintikka's two ways of crossidentification correspond to two modules of high level of perception. In other words, we find that perspective world line corresponds to what Vaina calls 'where' (subject centered) module and public with 'what' (object centered) module. Thus schemes as high level of perception remain important for the analysis, but the answer

about the connection of symbolic and subsymbolic levels goes in another direction. Namely, it seems that epistemic logic as a system of conceptual analysis relies on what neuroscientists and psychologists call schemes. In other words, it is not directly determined from the object language although it supports it. With this we have no obligation to 'avoid' object language and he can claim that Hintikka uses it only instrumentally - as a starting point. Furthermore, he is not obliged to the claim that natural language from which he started the analysis of epistemic concepts has clear distinction as contrasting sets of two ways of crossidentification have. This is supported by translations of (3) and (4), i.e. the difference between (5) and (6). The same is with the insufficient transparency of verb + direct object construction versus verb + wh-construction.

Let us conclude with this part of the analysis. The fact is that we start with knowledge that and we then differentiate the following:

perspective (contextual)	descriptive
crossidentification,	crossidentification;
knowledge + direct object constr.,	knowledge + wh. constr.;
<i>de re</i> ,	<i>de dicto</i> ;
'where' module,	'what' module;
putting a name to a face,	putting face to a name;

The right side is responsible for the complement of the set of actual epistemic states. Left side supports all the demands of the theory of belief change. The only condition left is the introduction of dynamics. Following Fuhrmann we can do this in the following way:

$$(16) \quad \text{def. } [+ ] \quad [+a]b \quad =\text{def. } K_a (a \Rightarrow b)$$

which represents the definition of expansion. For contraction and revision we have definitions:

$$(17) \quad \text{HI: } [-a]b =\text{def. } K_a b \ \& \ [-a](\neg a \Rightarrow b)$$

$$(18) \quad \text{LI: } [*a]b =\text{def. } [-\neg a][+a]b$$

Further there is no problem for a logician to postulate the set of wff as axiomatic base, formation and transformation rules.

A somewhat more difficult task could be testing the presented information independence of quantifier. As a consequence we should not have problems of quantification of intentional contexts such as : problem of logical omniscience, problems about existential generalization rule, problems about universal instantiation rule and problem of identity substitution.

In the end we can put one more question. What these presuppositions represent? The answer we wanted to avoid is that presuppositions represent the opaque collection of the result of the epistemic input work on actual epistemic states. If this was not avoided, then we have a question: What happens if I am convinced that the participant in the conversation 'does not speak the 'truth" and he is of the same opinion? The answer has to be left for some other occasion.

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***Slavko Brkić: SIMBOLIČKE I SUBSIMBOLIČKE KOMPONENTE U LOGICI  
PROMJENE VJEROVANJA***

*Sažetak:*

Autor istražuje mjesto simboličkih i subsimboličkih komponenata u teoriji promjene vjerovanja.

Prvo se razmatra uloga tih komponenata u okviru epistemičke logike (Hintikin način razrade). Dobiveni rezultati se, u drugom koraku, primjenjuju na teoriju promjene vjerovanja (Alchourron-Gärdenfors-Makinsinova teorija).

Na taj način autor određuje temelje logike promjene vjerovanja.