Quadrupedal Running at High Speed Over Uneven Terrain

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Abstract—High-speed legged locomotion is complicated by the challenge of uneven terrain because the system must respond to the fast-changing terrain elevation under each foot, and quickly secure a solid foothold after touchdown. This paper presents a leg stretch reflex and anti-slip retraction algorithm that are added to a previously presented controller to stabilize a high-speed trot over uneven terrain. Together with fuzzy control and a force redistribution algorithm, these control mechanisms stabilize a quadruped trot at 5.25 m/s. The quadruped can turn at 30 deg/s when running at 3.0 m/s, and can maneuver over uneven terrain with standard deviation of height variation of 3 cm at 4.0 m/s. This appears to be the first reported control of high-speed quadrupedal running over uneven terrain.

I. INTRODUCTION

Biological systems of varying sizes and travel speeds use legs to traverse all types of terrain. Wheeled vehicles require a clear continuous path, while legged systems only require discrete footholds to negotiate irregular or discontinuous terrain. Wheeled vehicles, however, presently hold a significant speed advantage over legged systems even over unprepared terrain. If legged vehicles are to reach their full potential as seen in the cheetah, as the fastest land animal, and the mountain goat for its speed over rough terrain, the feasibility of high speeds must be investigated.

This work attempts to control a quadruped trot over uneven terrain at significant running speeds. The trot was chosen because of its observed energy efficiency over a wide range of running speeds [1] and its widespread use in nature [2]. The trot, whose footfall pattern is shown in Fig. 1, is a symmetric gait during which the diagonal forelimb and hindlimb move in unison, ideally contacting and leaving the ground at the same time. The trot gait exhibits significant flight phases (approximately one-third of the stride period) during which the quadruped is largely uncontrollable.

As speed increases, the periods of flight lengthen and the animal must produce increasingly large corrective forces to reverse the body’s vertical momentum and stabilize the body during the remaining support time. These large forces are coupled to body motions through complex and dynamic relationships. When a diagonal leg pair is in contact with the ground, the pitch and roll moments produced by one leg can be opposed by moments from the other. When only one leg is in contact with the ground, tilt moments go unopposed and can produce significant tilt angles. The trot relies upon synchronous touchdown, which becomes more difficult at high speeds over irregular terrain because the system must quickly adjust to the fast-changing terrain elevation under each foot. A contact delay by one of the legs results in less time for body control, and could result in a catastrophic fall.

Another complexity of high-speed running is foot slippage. The fore-aft velocity of the foot closely matches the quadruped’s forward speed at the moment of touchdown, resulting in large planar ground reaction forces on the foot which cause the foot to slide on the ground. Foot slip ends when the foot velocity closely matches the terrain velocity relative to the body. The time spent sliding on the ground increases as speed increases, and reduces the support time available for body control.

Fukouka et. al [3] executed a dynamic quadruped walk over uneven terrain using a central pattern generator (CPG) and sensory feedback. The CPG produced coordinated leg motion that resulted in a sustainable trot over outdoor terrain. The BigDog project from Boston Dynamics [4] has also achieved dynamic motion over uneven terrain. Not much has been presented about its control system, but the mule-sized quadruped can trot at 0.8 m/s up and down significant inclines and over rough terrain. Neither of these systems run at speeds that require the large leg forces necessary for significant flight trajectories.

A controller has been previously developed to stabilize a trot up to 4.75 m/s [5]. The control system, which was designed to operate on flat terrain, incorporated an intelligent fuzzy controller and force redistribution in a hybrid strategy. This paper presents two additional control mechanisms, 1) a leg stretch reflex, and 2) an anti-slip retraction algorithm, that stabilize the quadruped trot at high speeds over uneven terrain. The system adapts to the terrain by using reflexes to adjust the motion of each leg immediately before and after touchdown.

The leg stretch reflex attempts to synchronize the touchdown of the diagonal leg pairs and reduce periods of single-leg support. The control system only knows the elevation of the terrain under each foot (no terrain preview), as is attainable by a proximity sensor on each foot. An anti-slip algorithm is implemented to actively retract the leg after...
touchdown, which causes the foot speed to decrease at a faster rate. A fixed desired foot deceleration is chosen for all forward body velocities which reduces the time spent sliding by approximately 50%. These control mechanisms increase the top speed of the quadruped to 5.25 m/s. The quadruped can turn at 30 deg/s when running at 3.0 m/s, and can maneuver over uneven terrain at 4.0 m/s.

The quadruped system and terrain model are described in the following section. This is followed by a brief description of the previously-presented quadruped control system. The leg stretch reflex and anti-slip retraction algorithm are then presented, followed by some results and a summary.

II. QUADRUPED AND TERRAIN MODEL

A simplified kinematic and dynamic model of the left fore (LF) quadruped leg is shown in Fig. 2. The leg model includes an articulated knee to more closely mimic the legs of cursorial quadrupeds in nature. Torque at the knee is produced by a series-elastic actuator sitting on the thigh (not shown). When the foot is on the ground, knee torque produces a force along the line between the foot and the hip. The angle of this line is called the virtual leg angle, $\theta_v$, and is measured with respect to the body normal. Energy is stored in the passive spring as the knee bends during the first half of stance and returned to the system as the leg lengthens. When the leg is in flight, the actuator can adjust the knee angle, $\theta_k$, which dictates leg length, $r$. Each leg also has two actuators at the shoulder/hip joint, one for abduction and adduction of the leg and another to swing (protract/retract) the leg. The resultant angles are $\theta_a$ and $\theta_s$, respectively. The backward knee configuration reduces the range of motion for the thigh during the flight period.

The quadruped weighs a total of 76 kg and stands 60 cm high with the knee springs in their nominal position. The shoulder separation is 35 cm and the shoulder-to-hip distance is 1.2 m. The thigh and shank are modeled as slim rods of length 35 cm with geometrically-centered masses of 2.0 kg and 1.0 kg respectively. The ab/ad axis is modeled as a cube centered at the shoulder with a mass of 2.0 kg. The legs combine to make up 26% of the complete system mass, each leg making up approximately 6.5%. This percentage corresponds to biological data [6] and results in realistic disturbances to the system during high-speed running.

The quadruped controller attempts to control body roll, pitch, and yaw, along with the forward velocity, lateral velocity, and height at the peak height of the aerial phase, referred to as the top of flight (TOF). The control algorithm is tested in RobotBuilder [7], a robot simulation environment built upon the DynaMechs [8] software package. System losses are modeled as damping in the compliant ground. Ground spring and damping coefficients are 75 kN/m and 2 kN/m/s respectively. Ground static and kinetic friction coefficients are 0.75 and 0.6 respectively, matching the properties of rubber on concrete. A more detailed description of the quadruped system can be found in [9].

A 2-dimensional (2D) sample of the uneven terrain with a scaled model of the quadruped is shown in Fig. 3. Figure 4 shows the actual quadruped and terrain in simulation. The terrain elevation only changes in the direction of the Earth’s $x$ axis. Therefore, the terrain height underneath foot $i$ is only a function of the foot’s $x$ position in the Earth’s coordinate system. Each floor panel is 60 cm wide and has infinite depth along the Earth’s $y$ axis. The elevation of each panel is randomly selected from a normal distribution centered at 0.0 cm with a standard deviation of 3 cm. The maximum panel elevation is 6.5 cm, which is greater than 10% of the nominal leg length.

III. QUADRUPED CONTROL

The quadruped controller incorporates principles of the SLIP (spring-loaded inverted pendulum) model and the idea that animals redistribute vertical impulses during stance to stabilize pitch. These concepts have been previously presented in further detail [5], [9], and are only summarized here.

The SLIP model is a simple spring-mass system that produces behavior similar to that observed in six and four-legged trotters, and in two-legged runners and hoppers [10]. The motion of a spring-mass system during the support period is largely dictated by the angle of the leg at touchdown and the behavior of the passive spring. Understanding the SLIP model leads to heuristic control principles for forward,
vertical, lateral, and yaw motions. The body’s forward motion is controlled by varying the touchdown fore-aft angle of the legs. Altering the spring energy during stance is used to manipulate the maximum height during the subsequent flight phase, lateral motion is produced by placing both legs toward one side of the body, and turning is achieved by biasing the front legs to the outside of the turn and the hind legs to the inside. These motions (forward, vertical, lateral, and yaw) are controlled once per step by choosing appropriate leg touchdown angles and the amount of spring energy to add during stance.

The relationships between the touchdown angles and energy addition to the motions that these system inputs produce are complex and highly nonlinear. This type of problem is particularly suited to the benefits of intelligent control. The fuzzy controller stabilizes forward, vertical, lateral, and yaw motions without the need for a complex system model. The fuzzy controller is outfitted with an adaptive learning mechanism to continuously improve the tracking performance while the quadruped is running.

The SLIP model does not yield general principles for controlling the remaining two body motions, namely pitch and roll. The trot exhibits minimal pitch and roll oscillations, but these axes must be actively controlled because of the strong coupling between their motion and the large axial leg forces that naturally occur during a trot. Biomechanics studies have shown that pitch stability for the quadruped trot is achieved by redistributing the vertical impulses during stance between the fore and hind limbs [11]. The term “redistributing” suggests that the sum vertical impulse is not changed by the effort to control the pitch motion. This principle has been expanded to control both pitch and roll motions during stance without significantly changing the passive motion dynamics prescribed by the discrete fuzzy controller. Force redistribution is the process of altering the large leg forces that naturally occur during running to control selected body motions without significantly affecting others.

The combination of the fuzzy controller and force redistribution algorithm result in a hybrid controller that controls four of the six body motions (forward, vertical, lateral, and yaw) discretely once per step and controls the other two motions (pitch and roll) continuously during the support period. Again, these aspects of the control system have been previously presented in further detail [9], [5]. This paper focuses on two additional leg control mechanisms which dictate the motion of the leg immediately before and after touchdown, and allow the quadruped to maneuver over uneven terrain. The leg stretch reflex and anti-slip retraction algorithm are presented in the next section along with a description of the leg controller state machine.

IV. LEG CONTROL

The leg motions during a stride are organized into the phases of a discrete state machine as shown in Fig. 5. Each leg cycle constitutes a quadruped stride period and encompasses two steps which each have a top of flight (TOF) and bottom of flight (BOF). The Flight and Support labels in Fig. 5 refer to the status of the leg, and not the body. The BOF in the top of the diagram occurs while the described leg is protracting forward and the legs of the opposite diagonal are reversing the body’s vertical momentum while in contact with the ground.

Table I shows the actions taken during each state and the trigger to exit each state. The leg enters the SHORTEN phase from the THRUST phase immediately after liftoff. During the SHORTEN phase, the knee is actuated to shorten the leg to 35 cm, or $r_{pl}$, while the ab/ad actuators stop the transverse motion of the thigh. Also during this phase, each swing actuator further retracts the leg 10 deg from its fore-aft liftoff angle. This helps to eliminate toe stubbing during protraction by giving the leg more time to shorten before the foot moves forward underneath the body.

When the body reaches TOF, the leg enters the PROTRACT 1 phase. During this phase, the thigh is protracted forward by the swing axis and centered laterally with respect to the body by the ab/ad axis. Laterally centering puts the leg in a good position for any desired ab/ad touchdown position, which will be updated at the next TOF. The speed of the fore-aft protraction is dictated by the length of the previous support period, $T_s$. The body’s height decreases during this phase and the leg remains short to ensure ground clearance.

After BOF, the leg state transitions to PROTRACT 2 and the leg lengthens toward the terrain as the body begins to rise. A new protraction trajectory is computed to deliver the swing legs to their forward position near the time when the support legs lift off. A new spline computes a trajectory from the current position of the protracting leg to the desired position given a period of $T_s/2$. The thigh remains laterally centered.

At TOF, new desired touchdown virtual leg and ab/ad angles are computed by the fuzzy controller. The leg enters the LENGTHEN state and lengthens to its touchdown length. During this phase, the leg length is now controlled by the leg stretch reflex to synchronize the touchdown of the diagonal leg pair. The swing and ab/ad angles are actuated to their desired touchdown positions.

After touchdown, the anti-slip retraction algorithm actively retracts the thigh under the body to reduce slip time during the RETRACTION phase. After sliding ends, the spring extends and stores energy as the knee bends during the COMPRESS phase of stance. The COMPRESS state ends when the body reaches its bottom of flight, signifying that the springs are maximally loaded. The leg then enters the THRUST phase and the spring returns energy to the body in
the form of forward and vertical kinetic energy. During both
the COMPRESSION and THRUST phase, the force redistribution
algorithm corrects the body pitch and roll without signifi-
cantly affecting forward, lateral, vertical, and yaw motions
dictated by the leg touchdown angles. The leg stays in the
THRUST phase until the foot breaks contact with the ground
or the spring energy is fully depleted. The process then
repeats from the SHORTEN phase.

The leg stretch reflex and anti-slip retraction algorithm that
synchronize diagonal pair foot touchdowns and achieve solid
footholds during the LENGTHEN and RETRACT phases are
described below.

### A. Leg Stretch Reflex

The leg stretch reflex receives information about the terrain
elevation directly under each foot. These sensory inputs,
along with the body position and attitude from inertial
sensors, and the axis information from joint sensors, allow
the leg stretch reflex to control the foot height with respect
to the terrain.

During the PROTRACT 2 phase, the leg stretch reflex
algorithm servo controls the foot from its height above the
terrain with a leg length of \( r_{p1} \). The desired leg length is computed from the desired foot height.
As the terrain level changes, the desired leg length changes
to maintain a foot height of \( 3 \) \( cm \). The given desired foot
height, \( h_{f,d} \), the desired leg length, \( r_d \), is computed based
upon the inertial and joint sensors by

\[
r_d = \frac{h_h - h_{f,d} - h_g}{\cos(\theta_e + \beta)\cos(\theta_a - \gamma)},
\]

where

- \( h_h \) = the hip height,
- \( h_g \) = the terrain elevation under the foot,
- \( \theta_e + \beta \) = the virtual leg angle, and
- \( \theta_a - \gamma \) = the ab/ad leg angle.

The hip height is computed from the body height and tilt
angles (pitch \( \beta \) and roll \( \gamma \)), and the virtual and ab/ad leg
angles are computed with respect to the vertical.

In the LENGTHEN phase, the desired foot height follows
a cubic spline trajectory down to zero in 60 \( ms \). This lightly
initiates contact with the terrain, only if the trajectory can be
followed. The maximum achievable leg length is 61.67 \( cm \)
because of the kinematic configuration of the leg. If the
height and attitude of the body result in a desired leg length
greater than this maximum value, the offending leg takes on a
desired length of this maximum and the minimum achievable
foot height is computed from algebraic manipulation of
Eq. 1. The desired leg length for the opposite diagonal leg is
then computed so that its foot height matches the minimum
foot height achievable by the offending leg. This ensures that
both feet maintain the same height above the terrain.

Figure 6 shows the foot height and leg length of a diagonal
pair through the touchdown process. The selected data shows
the most complex scenario of the terrain rising underneath
one foot while falling underneath the other. The data begins
during the support phase, noted by the high on the dash-
line. Immediately after liftoff, the desired leg lengths
follow a cubic spline trajectory to 30 \( cm \), and the actual leg
lengths follow. After bottom of flight (BOF), which occurs
at the 0.28 \( s \) mark, the legs enter the PROTRACT 2 phase
and the desired foot height becomes 3 \( cm \). As the terrain
elevation under the left fore (LF) foot rises, the desired foot
position rises to maintain 3 \( cm \) of separation. The desired

<table>
<thead>
<tr>
<th>PHASE</th>
<th>DESCRIPTION</th>
<th>EXIT TRIGGER</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHORTEN</td>
<td>- Begin shortening the virtual leg length to ( r_{p1} ).</td>
<td>Body reaches TOF</td>
</tr>
<tr>
<td></td>
<td>- Retract swing axis another 10 ( deg ).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Stop the ab/ad movement.</td>
<td></td>
</tr>
<tr>
<td>PROTRACT 1</td>
<td>- Finish shortening the virtual leg length to ( r_{p1} ).</td>
<td>Body reaches BOF</td>
</tr>
<tr>
<td></td>
<td>- Start thigh protraction.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Center the ab/ad axis with respect to the body.</td>
<td></td>
</tr>
<tr>
<td>PROTRACT 2</td>
<td>- Servo the foot to 3 ( cm ) above the terrain elevation.</td>
<td>Body reaches TOF</td>
</tr>
<tr>
<td></td>
<td>- Finish thigh protraction with period of ( T_s/2 ).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Maintain centered ab/ad axis.</td>
<td></td>
</tr>
<tr>
<td>LENGTHEN</td>
<td>- Servo the foot to 0 ( cm ) above the terrain elevation in 60 ( ms ).</td>
<td>Foot touchdown</td>
</tr>
<tr>
<td></td>
<td>- Servo the virtual leg angle to its updated touchdown value.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Servo the virtual ab/ad angle to its updated touchdown value.</td>
<td></td>
</tr>
<tr>
<td>RETRACT</td>
<td>- Retract the leg.</td>
<td>Sliding stops</td>
</tr>
<tr>
<td>COMPRESSION</td>
<td>- Redistribute the leg forces.</td>
<td>Body reaches BOF</td>
</tr>
<tr>
<td>THRUST</td>
<td>- Redistribute the leg forces.</td>
<td>No contact is detected, or spring energy is depleted</td>
</tr>
</tbody>
</table>

**TABLE I**

**TABLE OF LEG CONTROLLER STATES.**
foot position of the right hind (RH) leg decreases as the terrain lowers beneath it. The body reaches TOF at the 0.45 s mark, causing the legs to enter the LENGTHEN phase and the desired foot positions decrease toward the terrain. Ideally, each foot will touch down 60 ms later, but the LF foot contacts the terrain after 45 ms and the RH foot touches down 57 ms later. The beginning of the support phase is marked by the rising edge of the stance plot in Fig. 6. The touchdowns are not synchronized, and both are earlier than expected. The data presented shows the maximum error observed using the leg stretch reflex.

There are two reasons for the touchdown timing errors. First, the elevation of the terrain is known only for the current position of the foot. The actuation delay to adjust for a fast-changing terrain is on the order of milliseconds and results in the separation between the contact times. Second, both touchdowns being early is the result of swing axis motions to correct for velocity error. At TOF, a new touchdown virtual leg angle is computed by the fuzzy controller and the swing actuator quickly drives the leg toward its goal.

### B. Anti-Slip Retraction

Immediately after touchdown, a fixed desired foot deceleration of \(-125 \, \text{m/s}^2\) is commanded for both support legs. This deceleration rate is chosen for all forward body velocities, which results in the reduced slip times shown in Table II. The original slip times result from touchdown with no anti-slip retraction (the trot could not be stabilized at 5.0 m/s without the anti-slip retraction).

The foot speed deceleration is caused by a torque applied to the swing axis. Given a desired planar foot speed that ramps toward zero, the swing axis rate to achieve this speed can be computed using kinematic analysis. The desired and actual foot speeds and swing axis speeds, and the resulting swing torque during a 5.0 m/s step are shown in Fig. 7. At the beginning of the stance phase, which is represented by the rising edge of the dash-dot line, the foot rate matches the body’s forward velocity. The desired foot velocity ramps to zero immediately after contact, and the corresponding desired swing rate is computed. Large torques are required to achieve this swing rate, and the actual foot speed closely matches the desired speed. The foot stops sliding when the foot speed approaches 0 m/s, which is approximately 40 ms after the stance phase begins.

The large swing axis torques required for anti-slip retraction also affect other body motions, particularly forward velocity. Figure 8 shows the body’s forward velocity as it changes over several steps. The desired velocity is 5.0 m/s at TOF, and the support period is noted by the raised period of the dash-dot line. Immediately after each touchdown, the body velocity increases as the legs are actively retracted by the swing axes torques. At the first touchdown, which occurs at the 0.1 s mark, the body’s forward velocity is approximately 4.9 m/s. The active retraction of the legs causes the acceleration to 5.0 m/s during the 40 ms of sliding. After foot slip ends, the body velocity follows the expected trajectory due to the decelerating and accelerating fore-aft leg forces that are observed during animal locomotion. These are the natural dynamics that the force redistribution algorithm attempts to maintain. The 0.1 m/s increase in velocity after touchdown is the only perturbation of concern and it is quite small. The fuzzy controller learns to account for this repeating effect and achieves good velocity control at TOF.

### V. RESULTS

Figure 9 shows continuous body states as the quadruped runs over uneven terrain. The bottom subplot shows the elevation of the terrain under the body’s center of mass. The top and middle subplots show the continuous velocity and yaw data. As desired, the continuous velocity data displays a deceleration and acceleration during each support phase.

At the 10 s mark of Fig. 9, the desired velocity decreases from 3.0 m/s to 2 m/s, and then to 4.0 m/s at the 15 s mark. The velocity is commanded back to 3.0 m/s at the 20 s mark. The results show no significant problems accelerating and decelerating over uneven terrain. The desired yaw

### Table II

<table>
<thead>
<tr>
<th>Body velocity (m/s)</th>
<th>0.0</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original slip time (ms)</td>
<td>0</td>
<td>31</td>
<td>32</td>
<td>35</td>
<td>39</td>
<td>44</td>
</tr>
<tr>
<td>Reduced slip time (ms)</td>
<td>0</td>
<td>7</td>
<td>19</td>
<td>27</td>
<td>34</td>
<td>42</td>
</tr>
</tbody>
</table>
The body’s motion is along the Earth’s surface; the foot trajectory that will reduce the time spent sliding after touchdown. The anti-slip algorithm defines a time mark. The controller achieves good control of the TOF velocity over uneven terrain.

angle changes to 90 deg and back to 0 deg at the 25 s and 35 s marks respectively. When the body yaw angle is 90 deg, the body’s motion is along the Earth’s y axis. Because the terrain elevation only changes in the Earth’s z direction, no changes in elevation occur between the 29 s and 36 s marks. Again, no significant problems arise when controlling the heading over uneven terrain. The peak running speed over the uneven terrain is 4.0 m/s.

VI. CONCLUSIONS AND FUTURE WORK

Two additional control mechanisms which improve the performance of the quadruped trot controller have been presented. The leg stretch reflex synchronizes diagonal leg pair touchdown to begin each support phase. Synchronous contact reduces periods of single-leg contact when the pitch and roll moments produced by the stance leg cannot be opposed by another leg. The anti-slip algorithm defines a foot trajectory that will reduce the time spent sliding after touchdown. The slippage time is reduced from 70 ms to 24 ms when anti-slip retraction is used at 4.0 m/s.

Together with fuzzy control and the force redistribution algorithm, the control mechanisms presented here stabilize a quadruped trot at 5.25 m/s. The quadruped can turn at 30 deg/s when running at 3.0 m/s, and can maneuver over uneven terrain with standard deviation of height variation of 3 cm at 4.0 m/s. This appears to be the first reported control of high-speed quadrupedal running over uneven terrain.

The ultimate goal of this work is to develop a control system for an experimental system that can negotiate uneven terrain at high speeds. This process is aided by a dynamic simulation of the robotic system, which is useful for the design of the hardware and control system. Control algorithms can be easily modified in simulation to identify useful control principles or to verify the viability of a control method. The controller presented here stabilizes a high-speed trot over uneven terrain given biomimetic mass distributions on a quadruped system and realistic friction coefficients for the terrain.

In the future, some of the computational complexities of the algorithm can be simplified to run on a real-time system with limited processing power. Future work will also measure the robustness of the algorithm to imperfect sensing and actuation constraints. The algorithm will also be tested on inclines and declines.

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REFERENCES