Comparison of Functionally Graded Material Plate with Metaland Ceramic Plateunder Transverse Loadfor Various Boundary Conditions

Manish Bhandari
Dept. of Mechanical Engg.
Jodhpur Institute of Engineering and Technology, Jodhpur

Ashirvad
Dept. of Mechanical Engg.
Jodhpur Institute of Engineering and Technology, Jodhpur

Kamlesh Purohit
Dept. of Mechanical Engg.
JNVU, Jodhpur

ABSTRACT
Functionally gradient materials (FGM) are one of the most widely used materials in various applications because of their adaptability to different situations by changing the material constituents as per the requirement. Most structural components used in the field of engineering can be classified as beams, plates, or shells for analysis purposes. In the present study the power law, sigmoid and exponential distribution is considered for the volume fraction distributions of the functionally graded plates. The work includes parametric studies performed by varying volume fraction distributions and boundary conditions. Also static analysis of functionally gradient material plate is carried out by sigmoid law and verified with the published results. The convergence study of the results is optimized by changing the mesh size and layer size. Power law and exponential law are applied for the same material and set of conditions.

Keywords
A. Functional composites B. Elastic properties C. Finite element analysis (FEA)

1. INTRODUCTION
The material property of the FGM can be tailored to accomplish the specific demands in various engineering utilizations to achieve the advantage of the properties of individual material. This is possible due to the material composition of the FGM changes sequentially in a preferred direction. The thermo-mechanical deformation of FGM structures have attracted the attention of many researchers in the past few years in different engineering applications which include design of aerospace structures, heat engine components and nuclear power plants etc. A huge amount of published literature has been observed for evaluation of thermo-mechanical behaviour of functionally gradient material plate using finite element techniques. It includes both linearity and non-linearity in various areas. A few of published literature highlights the importance of topic. A number of approaches have been employed to study the static bending problems of FGM plates. The assessment of thermo-mechanical deformation behaviour of functionally graded plate structures considerably depends on the plate model kinematics. Praveen and Reddy [1] reported that the response of the plates with material properties between those of the ceramic and metal is not intermediate to the responses of the ceramic and metal plates. Reddy [2] reported theoretical formulations and finite element analysis of the thermo mechanical, transient response of functionally graded cylinders and plates with nonlinearity. Cheng and Batra [3], developed a new solution in closed form for the functionally graded elliptic plate rigidly clamped at the edges. Reddy [4] formulated Navier's solutions in conjunction with Finite element models of rectangular plates based on the third-order shear deformation plate theory for functionally graded plates. Sankar[5] solved the thermo elastic equilibrium equations for a functionally graded beam in closed-form to obtain the axial stress distribution. Qian, Batra and Chen[6] analysed static deformations, free and forced vibrations of a thick rectangular functionally graded elastic plate by using a higher order shear and normal deformable plate theory and a meshless local Petrov–Galerkin (MLPG) method. Ferreira, Batra, Roque, Qian and Martins [7], presented the use of the collocation method with the radial basis functions to analyze several plate and beam problems with a third-order shear deformation plate theory (TSDT). Tahani1, Torabizadeh and Fereidoon [8], developed analytical method to analyse displacements and stresses in a functionally graded composite beam subjected to transverse load and the results obtained from this method were compared with the finite element solution done by ANSYS. Chi and Chung [9], [10] evaluated the numerical solutions directly from theoretical formulations and calculated by finite element method using MARC program. Besides, they compared the results of P-FGM, S-FGM and E-FGM. Wang and Qin [11] developed a meshless algorithm to simulate the static thermal stress distribution in two-dimensional (2D) FGMs. Shabana, Naotake and Noda [12] used the homogenization method (HM) based on the finite element method (FEM) to determine the full set of the macroscopic effective properties. Mahdavian[13], derived equilibrium and stability equations of a FGM rectangular plate under uniform in-plane compression. Zenkour and Mashat[14] determined the thermal buckling response of functionally graded plates using sinusoidal shear deformation plate theory (SPT). Alieldin, Alshorbagy and Shaat [15], proposed three transformation procedures of a laminated composite plate to an equivalent single-layer FG plate. Na and Kim[16] reported stress analysis of functionally graded composite plates composed of ceramic, functionally graded material and metal layers using finite element method. The 18-node solid element was selected for more accurate modelling. Vanam, Rajyalakshmi and Inala [17] analysed the static analysis of an isotropic rectangular plate with various boundary conditions and various types of load applications. Numerical analysis (finite element analysis, FEA) has been carried out by developing programming in mathematical software MATLAB and they compared results with that were obtained by finite element analysis software ANSYS. Raki, Reza and Kamanbedast [18], derived equilibrium and stability equations of a rectangular plate made of functionally graded material (FGM). Talha and Singh [19] reported formulations based on higher order shear deformation theory with a considerable amendment in the transverse displacement using finite element method (FEM).Srinivas and Prasad [20] focused on analysis of FGM flat plates under mechanical loading in order to understand the effect variation of material properties on structural response using ANSYS software. Srinivas and

Prasad [21] focused on analysis of FGM flat plates under thermal loading in order to understand the effect variation of material properties has on structural response.

In the present study the power law, sigmoid and exponential distribution is considered for the volume fraction distributions of the functionally graded plates. The studies include comparison of the response of structural element specifically plates made of functionally graded materials, metal and ceramic. The work includes parametric studies performed by varying volume fraction distributions and boundary conditions. The finite element software ANSYS APDL-13 is used for the modelling and analysis purpose.

2. MATERIAL GRADIENT OF FGM PLATES

The effective material properties like Young's modulus, Poisson's ratio, coefficient of thermal expansion, thermal conductivity etc. on the upper and lower surfaces are different but are reassigned. However, the Young's modulus and Poisson's ratio of the plates vary continuously only in the thickness direction (z-axis) i.e. E = E(z), ν = ν(z). However, the Young's moduli in the thickness direction of the FGM plates vary with power-law functions (P-FGM), exponential functions (E-FGM), or with sigmoid functions (S-FGM). A mixture of the two materials composes the through-the-thickness characteristics. The FGM plate of thickness 'h' is modelled usually with one side of the material as ceramic and the other side as metal.

2.1 Power Load

The material properties of a P-FGM can be determined by the rule of mixture:

$$P(z) = (P_t - P_b)V_f + P_b \tag{1}$$

Material properties are dependent on the volume fraction V_f of P-FGM which obeys power law

$$V_f = (z/h+1/2)^n \tag{2}$$

where n is a parameter that dictates the material variation profile through the thickness known as is the volume fraction exponent. At bottom face, (z/h) = -1/2 and $V_f = 0$, hence P(z) = P_b and at top face, (z/h) =1/2 and so $V_f = 1$ hence P(z) = P_t where P denotes a generic material property like modulus, $P_t$and P_b denote the property of the top and bottom faces of the plate. At *n* = 0the plate is a fully ceramic plate while at *n* = ∞ the plate is fully metal.

2.2Sigmoid Law

In the case of adding an FGM of a single power-law function to the multi-layered composite, stress concentrations appear on one of the interfaces where the material is continuous but changes rapidly. Therefore, Chung and Chi [10] defined the volume fraction using two power-law functions to ensure smooth distribution of stresses among all the interfaces. The two power law functions are defined by:

$$g_1(z) = 1 - \frac{1}{2}\left(\frac{\frac{h}{2} - z}{\frac{h}{2}}\right)^p \quad \text{for } 0 \le z \le h/2,$$

$$g_2(z) = \frac{1}{2}\left(\frac{\frac{h}{2} + z}{\frac{h}{2}}\right)^p \quad \text{for } -h/2 \le z \le 0 \tag{3}$$

By using the rule of mixture, the Young's modulus of the S-FGM can be calculated by:

$$E(z) = g_1(z)E_1 + [1 - g_1(z)]E_2 \quad \text{for } 0 \le z \le h/2 \quad \text{and}$$

$$E(z) = g_2(z)E_1 + [1 - g_2(z)]E_2 \quad \text{for } -h/2 \le z \le 0$$

2.3 Sigmoid Law

Many researchers used the exponential function to describe the material properties of FGMs as follows:

$$E(z) = E_2 e^{\frac{1}{h}\ln\left(\frac{E_1}{E_2}\right)(z+h/2)} \tag{4}$$

3. FINITE ELEMENT MODELLING TECHNIQUE

The material properties of the FGM change throughout the thickness, the numerical model is to be broken up into various "layers" in order to capture the change in properties. These "layers" capture a finite portion of the thickness and are treated like isotropic materials. Material properties are calculated from the bottom surface using the various volume fraction distribution laws. The "layers" and their associated properties are then layered together to establish the through-the-thickness variation of material properties. Although the layered structure does not reflect the gradual change in material properties, a sufficient number of "layers" can reasonably approximate the material gradation. In this paper, the modelling and analysis of FGM plate is carried out using ANSYS software. ANSYS offers a number of elements to choose from for the modelling of gradient materials. The FGM characteristics under mechanical and thermal loads studied on a flat plate were modeled in 3-D.

4. CONVERGENCE STUDY

To ascertain the accuracy and proficiency of the present finite element formulation, two examples have been analysed for thermo-mechanical deformations of the FGM plates. It is concluded that the present finite element formulation gives satisfactory accuracy level. [22].

5. RESULTS AND ANALYSIS

The mechanical analysis is conducted for FGM made of combination of metal and ceramic. The metal and ceramic chosen are Aluminium and Zirconia respectively. The Young's modulus for Aluminium (E_m) is 70 GPa and for Zirconia (E_c) 151 GPa. The Poisson's ratio for both the materials is taken as 0.3. A square FGM plate of simply supported at all of its edges (SSSS) is considered here. The thickness of the plate (h) istaken 0.02m and the aspect ratio (a/b) is taken unity. The value of the udl (p_o) chosen was equal to 1Xl0^6 N/m^2.

The mechanical analysis is performed by applying uniformly distributed load (udl) and point load for various boundary conditions SSSS, CCCC, SCSC, CFCF, CCFF, CCSS, SSFF, SSSC, SSSF and SSCF. The abbreviation S, C and F stand for simply supported, clamped and free edges respectively. The boundary conditions imposed at a simply supported (S), a clamped (C) and a free (F) edge are:

Simply supported (S):

$$\sigma_{xx}=0;\ v=w=0;\ \text{on } x=0 \text{ and } a;$$
$$\sigma_{yy}=0;\ u=w=0;\ \text{on } y=0 \text{ and } b;$$

Clamped (C):

$$u=v=w=0;\ \text{on } x=0,\ a \text{ and } y=0,\ b;$$

Free (F):

$$\sigma_{xx}=\sigma_{yx}=\sigma_{zx}=0;\ \text{on } x=0 \text{ and } a;$$
$$\sigma_{yy}=\sigma_{xy}=\sigma_{zy}=0;\ \text{on } y=0,\ b$$

The analysis is performed for E-FGM and for various values of the volume fraction exponent (n) in P-FGM and S-FGM. The results are presented in terms of non-dimensional parameters i.e. non dimensional deflection ($\overline{uz}$), nondimensional tensile stress ($\overline{\sigma x}$) and shear strain (e_{xy}). The various non-dimensional parameters used are:

Non dimensional deflection:

$$\overline{uz} = (100\, E_m\, h^3 u_z) / (1-\nu^2)\, a^4 p_o$$

and,non dimensional stress:

$$\overline{\sigma x} = \sigma\, h^2 / p_o\, a^2$$

where 'u_z' is deflection, 'σ' is stress, 'ν' Poisson's ratio (0.3).

It is interesting to see the comparison of various parameters like non-dimensional deflection, non-dimensional tensile stress and shear strain for ceramic, metal and FGM's following power law, sigmoid and exponential distribution. The Figures 1, 2 and 3 show the comparison graphs for pure ceramic (n=0), pure metal (n=∞), P-FGM (n=2), S-FGM (n=2) and E-FGM.

Figure 1 shows the comparative bar charts of non-dimensional deflection u_z for various boundary conditions of a square plate under uniformly distributed load for P-FGM, S-FGM, E-FGM, metal and ceramic respectively. It can be observed that the isotropic ceramic plate has the lowest deflection for all the boundary conditions considered here, and the isotropic metallic has the largest deflection. This is due to the fact that the bending stiffness is the maximum for ceramic plate, while minimum for metallic plate. It is also found that the maximum deflection occurs for simply supported - free (SSFF) boundary conditions and minimum for clamped (CCCC) boundary condition for all the cases considered here. The non-dimensional deflection for S-FGM remains closer for various values of 'n' as compared to that of the P-FGM.

Figure 2 shows the variation of non-dimensional tensile stress (σ_x) for various boundary conditions of a square plate under uniformly distributed load for P-FGM, S-FGM, E-FGM, metal and ceramic respectively. It can be observed that the isotropic ceramic plate has the lowest tensile stress for all the boundary conditions considered here, and the isotropic metallic plate has the largest tensile stress. This is due to the fact that the bending stiffness is the maximum for ceramic plate, while minimum for metallic plate, and degrades continuously as *n* increases. It is also found that the maximum tensile stress occurs for clamped - free (CCFF) boundary conditions and minimum for simply supported – clamped (SCSC) boundary condition for all the cases considered here. The non-dimensional tensile stress for S-FGM remains closer for various values of 'n' as compared to that of the P-FGM.

Figure 3 shows the variation of strain (e_z) for various boundary conditions of a square plate under uniformly distributed load for P-FGM, S-FG, E-FGM, metal and ceramic respectively. It can be observed that the isotropic ceramic plate has the lowest strain (e_{xy}) for all the boundary conditions considered here, and the isotropic metallic has the largest strain (e_{xy}). The strain (e_{xy}) becomes higher with increasing *n*. This is due to the fact that the bending stiffness is the maximum for ceramic plate, while minimum for metallic plate, and degrades continuously as *n* increases. It is also found that the maximum strain (e_{xy}) occurs for simply supported - free (SSFF) boundary conditions and minimum for clamped (CCCC) boundary condition for all the cases considered here.

a) Non-Dimensional Deflection ($\overline{uz}$)

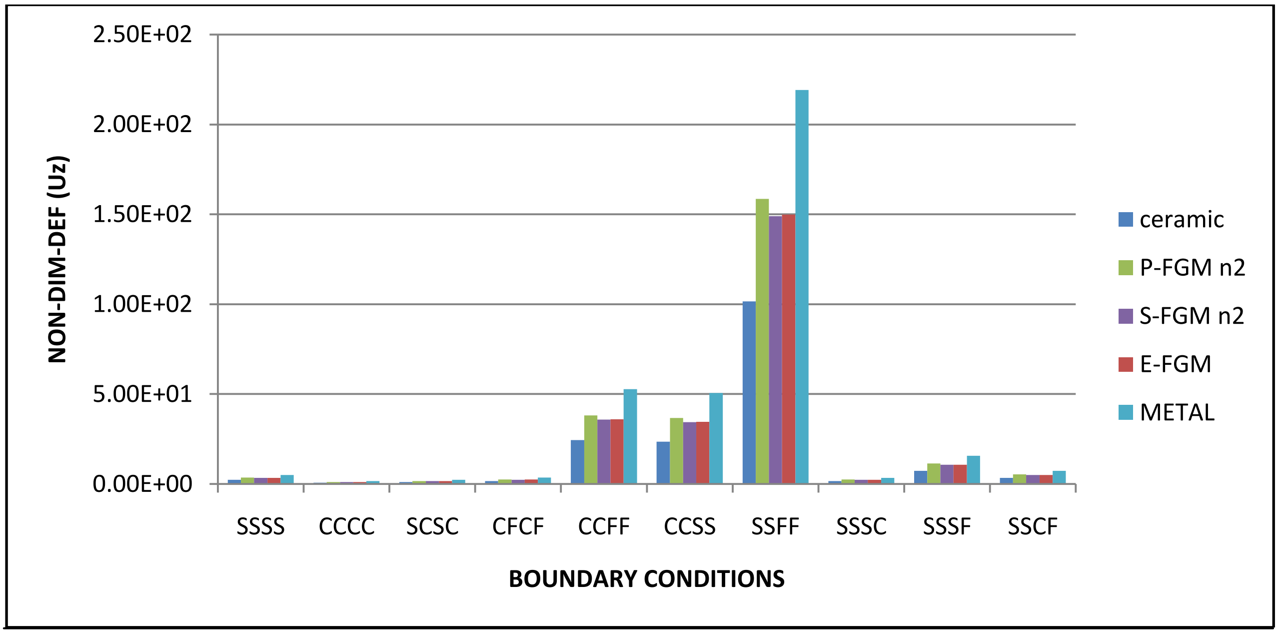

Figure 1: Non-dimensional deflection u_z for variousboundary conditions of a square plate under udl for various FGM's, ceramic and metal

b) Non-Dimensional Tensile Stress ($\overline{\sigma x}$)

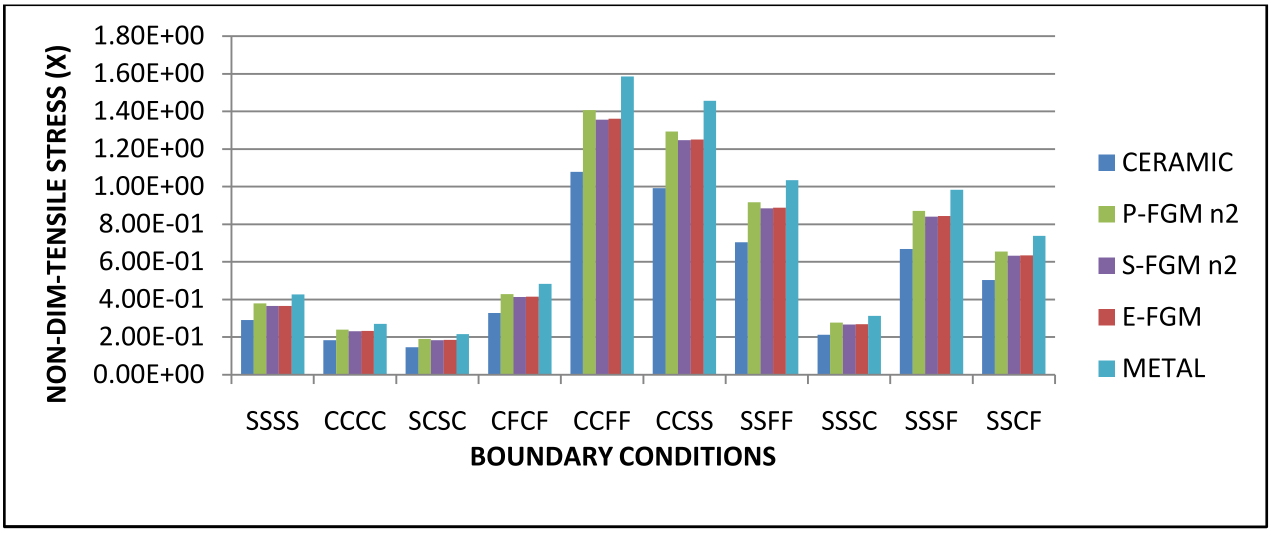

Figure 2: Non-dimensional tensile stress (σ_x) for various boundary conditions of a square plate under udl for various FGM's, ceramic and metal

c) Shear Strain (e_{xy})

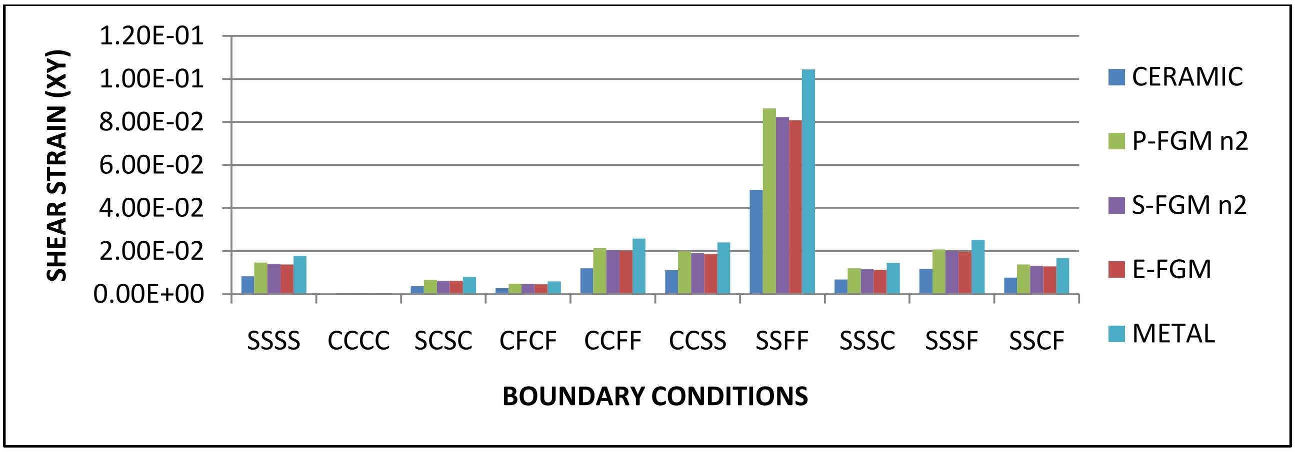

Figure 3: Shear strain (e_{xy}) for various boundary conditions of a square plate under udl for various FGM's, ceramic and metal

6. CONCLUSION AND FUTURE SCOPE

Mechanical deformation of functionally graded ceramic-metal plates under various boundary conditions is analyzed. Convergence and validation studies have been carried out to inculcate the accuracy of the present formulation [22]. The results show a good agreement with those available in the literature. It is observed that

(a) The bending response of the functionally graded plate is intermediate to those of the metal and the ceramic plate. This behaviour is found to be true irrespective of boundary conditions.

(b) The bending response for S-FGM remains closer for various values of 'n' as compared to that of the P-FGM.

The bending response of E-FGM is nearer to the behaviour of P-FGM. The work can be extended for variation in load, loading pattern and other ceramic metal combinations. Also thermal environment may be imposed in addition to the mechanical loading.

7. ACKNOWLEDGMENTS

Our thanks to the people who have helped us towards development of this paper directly or indirectly.

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