Robust Adaptive Control of Nonlinear Systems with Time-Varying Parameters and Its Application to Chua’s Circuit

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SUMMARY The problem of designing a robust adaptive control for nonlinear systems with uncertain time-varying parameters is addressed. The upper bound of uncertain parameters, considered even in control coefficients, are not required to be known. An adaptive tracking controller is presented and, using the Lyapunov theorem, the closed-loop stability and tracking error convergence is shown. In order to improve the performance of the method, a robust mechanism is incorporated into the adaptive controller yielding a robust adaptive algorithm. The proposed controller guarantees the boundedness of all closed-loop signals and robust convergence of tracking error in spite of time-varying parameter uncertainties with unknown bounds. The parametric uncertain systems under consideration describes a wide class of nonlinear circuits and systems. As an application, a novel parametric model is derived for nonlinear Chua’s circuit and then, the proposed method is used for its control. The effectiveness of the method is demonstrated by some simulation results.

key words: uncertain nonlinear systems, Chua’s circuit, robust adaptive control, tracking control, time-varying parameters

1. Introduction

Tracking control for uncertain nonlinear systems is one of the main topics in the field of control theory. In particular, considerable attention is paid to the nonlinear systems with unknown parameters. Depending on the assumptions made on parametric uncertainties and unknown disturbances, various control schemes have been reported in the last several years [1]–[8]. From a historical point of view, adaptive control algorithms were first developed for uncertain systems with unknown but constant parameters as in [9], [10]. Later, investigations into this field yielded various modifications to ensure robustness of such algorithms with respect to external disturbances [11], [12]. It is well known that the conventional adaptive methods may fail when the systems are affected by time-varying parameters with non-vanishing variations. Owing to this fact, design of tracking control for the case of time-varying parameters has become the focus of attention in recent years.

Many contributions have been made towards the underlying problem under various hypotheses. In [1], a class of single-input single-output (SISO) systems with known control gains, affected by a vector of time-varying parameters and bounded external disturbances, is considered and a robust adaptive control is established to ensure disturbance attenuation and boundedness of all closed-loop signals. In [7], the controller is developed for a larger class of SISO systems with unknown control coefficients whose bounds are known. For this case, unknown coefficients can be approximated by fuzzy models [13]. Developing control algorithms using adaptive backstepping approach [14], together with some nonlinear design tools [15], are also reported for parametric uncertain systems. For instance, a tracking control is established in [5] for SISO linear plants with unknown parameters that are smooth and bounded. A robust adaptive approach is addressed in [6] for a class of multiple-input multiple-output (MIMO) nonlinear systems with constant parametric uncertainties and smooth time-varying disturbances with known bounds. Design of an adaptive $H_\infty$ tracking control for MIMO systems with constant parametric uncertainties is considered in [3].

For the case of periodic time-varying parameters with known periodicity, an adaptive controller with periodic updating can be constructed for a class of SISO systems that achieves tracking error convergence [2]. Some other researchers are concerned with rejection of unknown but periodic disturbances for SISO systems with constant parameters (see, e.g., [4], [8] and the references therein).

On the other hand, nonlinear Chua’s circuit, as a benchmark to study such problems as stabilization [16], [17], tracking [18], chaos synchronization [19] and control [20]–[22], has received a great deal of attention. Although Chua’s circuit is implemented by simple electronic components, various assumptions on the nonlinearity and uncertainties of the system yield various control algorithms. Since circuit systems are mostly perturbed by uncertainties in the form of unknown parameters, adaptive-based schemes have been the focus of attention. In most of previous investigations, control coefficients are supposed to be known and constant whereas in practice, control gains may be inevitably affected by time-varying uncertainties.

The contribution of this paper is the derivation of a robust adaptive control algorithm for a class of uncertain systems with time-varying parameters even in control coefficients and perturbed by unknown time-varying disturbances. The parameters are supposed to be bounded but unlike the most of previous works, the bound of uncertainty is not known a priori. In fact, a wide class of nonlinear circuits and systems can be described in this form. First, an adaptive controller is designed that guarantees asymptotic track-
ing error convergence. Theoretically, the designed adaptive controller ensures closed-loop stability but in practice, some limitations, especially on achieving small sampling times, may cause unacceptable performance and even instability. In order to eliminate this drawback, a robust mechanism is incorporated into the adaptive controller yielding a robust adaptive control law. The proposed control algorithm guarantees that all the closed-loop signals are bounded and the tracking error is robustly converged to an arbitrarily small bound $\epsilon > 0$, chosen by the designer. Simplicity of implementation and robustness property are two benefits of the method with respect to previous investigations. The proposed method is then applied to uncertain nonlinear Chua’s circuit. It is shown that in practical implementation the nonlinear system (1) can be described by

$$\dot{x} = f_0(x) + \varphi(x)a(t) + b(t)u$$

$$x(t_0) = x_0$$

(2)

For notational consistency, without loss of generality, suppose the initial time $t_0 = 0$. Moreover, the following assumptions are made regarding this system.

A1) The parameter vector $a(t)$ belongs to a bounded closed set $\Omega_a := \{a(t) : \|a(t)\| \leq \alpha\}$ where $\alpha > 0$ is an unknown constant parameter.

A2) The control direction is known a priori; that is, $b_i(t)$ is either positive or negative and nonsingular for all $t$, $i = 1, 2, \ldots, n$ [2],[13], and there is a known lower bound $b_0 > 0$ such that $|b_i(t)| \geq b_0$.

Remark1: Compared with [2] and [13] which develop adaptive control laws for single input systems, this paper presents a robust adaptive controller for a class of MIMO systems. Moreover, the assumptions of periodicity of parameters in [2] and existence of a known upper bound for control coefficients in [13] are relaxed here.

A smooth state trajectory is given as $x_d = [x_{d1}, x_{d2}, \ldots, x_{dn}]^T$. Defining the tracking error as $e := x - x_d = [e_1, e_2, \ldots, e_n]^T$ and using (2), the error dynamic can be expressed by

$$\dot{e} = M(x)\varphi(x)a(t) + b(t)u$$

(3)

where $M(x) = f_0(x) - \dot{x}_d$.

Consider a class of uncertain nonlinear systems transformed to or originally described by (2). The objective is to design a robust adaptive control that ensures the boundedness of all closed-loop signals and convergence of tracking error $e$ in spite of parametric uncertainties.

3. Robust Adaptive Control Design

In order to show the main idea, the design procedure of an adaptive controller is first presented for the case the nonlinear system (2) represents a SISO plant. Then, the method is extended for the general class of nonlinear systems with time-varying parameters described by (2) and finally, the proposed robust adaptive control algorithm is presented.

3.1 Design Procedure for the Case of SISO Systems

Theorem1: For uncertain nonlinear system (2) with $x \in R$ and $u \in R$, let the assumptions A1–A2 hold. Consider the control law

$$u = -\lambda \frac{M^2(x)e}{|M(x)e| + \delta_1e^{-\sigma_1t}} + \delta^2 \frac{\varphi(x)\theta^T(x)e}{\|\varphi^T(x)e\|\theta + \delta_2e^{-\sigma_2t}}$$

(4)

where $K > 0$ is the state feedback gain and $\lambda$ denotes the
The control law (4) with adaptation rule

\[ V = \frac{1}{2} \sigma^2 + \frac{1}{2} \tilde{\alpha}^2 \]

where \( \alpha = \alpha - \tilde{\alpha} \) denotes the estimated error. Differentiating (6) along (3) and substituting the control law (4) yields

\[
\dot{V}(e, \tilde{\alpha}) = M(x)e + x(\alpha \tilde{e}(t)e - \frac{\lambda b(t)}{b_0} (K e^2 + \frac{M^2(x)e^2}{M(x)} + \delta_1 e^{-\sigma_1^2 t}) + \tilde{\alpha}^2 (e^2 + \frac{1}{2} \tilde{\alpha})
\]

As a result, one can obtain

\[
\dot{V}(e, \tilde{\alpha}) \leq |M(x)e| + \alpha ||\varphi^T(x)e|| - K e^2 - \frac{M^2(x)e^2}{M(x)e} + \delta_1 e^{-\sigma_1^2 t} - \frac{\tilde{\alpha} e^2}{2} ||\varphi^T(x)e|| + (\alpha ||\varphi^T(x)e|| - K e^2 - \frac{M^2(x)e^2}{M(x)e} + \delta_1 e^{-\sigma_1^2 t} - \frac{\tilde{\alpha} e^2}{2} ||\varphi^T(x)e||)
\]

By the adaptation law (5), \( \dot{V}(e, \tilde{\alpha}) \) is bounded as

\[ \dot{V}(e, \tilde{\alpha}) \leq -K e^2 + \delta_1 e^{-\sigma_1^2 t} + \delta_2 e^{-\sigma_2^2 t} \]

Integrating the inequality (7) from \( t = 0 \) to \( t = T \) yields

\[ K \int_0^T \|e(t)\|^2 dt + V(e(T), \tilde{\alpha}(T)) \leq V(e(0), \tilde{\alpha}(0)) + \frac{\delta_1}{\sigma_1^2} (1 - e^{-\sigma_1^2 T}) + \frac{\delta_2}{\sigma_2^2} (1 - e^{-\sigma_2^2 T}) \]

for all \( 0 \leq T < \infty \). This implies that \( e(t) \) is square-integrable.

On the other hand, by the inequality (7), it can be concluded that \( \dot{V} \leq -K \|e\|^2 + \delta_1 + \delta_2 \). Choosing \( K > \frac{\delta_1 + \delta_2}{\varepsilon} \), for any small \( \varepsilon > 0 \), there exists a \( \kappa > 0 \) such that \( \dot{V} \leq -\kappa \|e\|^2 < 0 \) for all \( \|e\| > \zeta \). As a result, the tracking error is uniformly ultimately bounded (UUB) [15], and all the closed-loop signals are also bounded. Error dynamics (3) and the boundedness of all variables imply that \( \dot{e}(t) \) is also bounded. Consequently, by Barbalat’s lemma [15], one can conclude that \( \lim_{t \to \infty} e(t) = 0 \), i.e., the tracking error \( e \) is asymptotically driven to zero.

**Remark 2:** The exponential terms in control law (4) provide smoothness of the law and do not violate asymptotic convergence property of tracking error. Clearly, the smaller values of \( \delta_i, i = 1, 2 \), give less smoothness to the control law.

### 3.2 Design Procedure for the Case of MIMO Systems

In the previous subsection, an adaptive controller was presented for SISO systems with time-varying parameters that guarantees the convergence of tracking error despite the perturbations. Now, the objective is to construct a robust adaptive controller for the case the nonlinear system (2) represents a MIMO plant.

**Theorem 2:** Consider the parametric uncertain nonlinear system (2). Suppose assumptions A1–A2 are satisfied. Choosing \( K = K^T > 0 \) as a gain matrix, the adaptive control law

\[
u = -\frac{1}{b_0} \left( \lambda K e + \frac{\lambda M(x)M^T(x)e}{M(x)e} + \delta_1 e^{-\sigma_1^2 t} + \frac{\lambda e^T \varphi(x)\varphi^T(x) e}{\|\varphi^T(x)e\|^2 \|\varphi^T(x)e\| + \frac{1}{\gamma} \tilde{\alpha}} \right)
\]

with adaptation law (5) and \( \Lambda = \text{diag} \{\lambda_1, \lambda_2, \ldots, \lambda_n\} \) where

\[ \lambda_i = \begin{cases} 1 & \text{if } b_i(t) > 0 \\ -1 & \text{if } b_i(t) < 0 \end{cases} \quad i = 1, 2, \ldots, n.
\]

ensures the boundedness of all closed-loop signals and that the tracking error \( e \) is asymptotically driven to zero.

**Proof:** Take a Lyapunov function as

\[ V(e, \tilde{\alpha}) = \frac{1}{2} e^T e + \frac{1}{2} \tilde{\alpha}^2 \]

The time derivative of \( V(e, \tilde{\alpha}) \) along the error trajectory (3) is

\[ \dot{V}(e, \tilde{\alpha}) = M^T(x)e + e^T \varphi(x)\varphi^T(x) e - e^T K e - \frac{e^T M(x)M^T(x)e}{M(x)e} + \delta_1 e^{-\sigma_1^2 t} + \frac{\lambda e^T \varphi(x)\varphi^T(x) e}{\|\varphi^T(x)e\|^2 \|\varphi^T(x)e\| + \frac{1}{\gamma} \tilde{\alpha}} \]

and consequently,

\[
\dot{V}(e, \tilde{\alpha}) \leq |M^T(x)e| + \alpha ||\varphi^T(x)e|| - e^T Ke - \frac{e^T M(x)M^T(x)e}{M(x)e} + \tilde{\alpha}^2 \frac{e^T \varphi(x)\varphi^T(x)e}{\|\varphi^T(x)e\|^2 \|\varphi^T(x)e\| + \frac{1}{\gamma} \tilde{\alpha}} - \frac{1}{\gamma} \tilde{\alpha}
\]
Similar to proof of theorem 1, taking into account the adaptation law (5) implies that
\[ \dot{V}(e, \hat{a}) \leq -e^T Ke + \delta_1 e^{-\sigma_1 t} + \delta_2 e^{-\sigma_2 t}. \]
Also, it can be concluded that
\[ \dot{V}(e, \hat{a}) \leq -\lambda_K \|e\|^2 + \delta_1 + \delta_2 \]
where \(\lambda_K\) denotes the minimum eigenvalue of \(K\). The remaining steps are straightforward following the proof of theorem 1. 

Along the preceding lines, the control problem was formulated and it was shown by theorem 2 that the adaptive controller (8) yields asymptotic convergence of tracking error, that is \(\lim_{t \to \infty} e(t) = 0\). Nevertheless, limitations on choosing small sampling times and imperfect implementation of adaptation mechanism (5) may cause the estimated value \(\hat{a}\) increase without bound. On the other hand, \(\hat{a}\) has direct impact on control law (8) and may cause instability. Hence, an effective modification is needed to alleviate this practical drawback. To this end, a robust but not adaptive controller is first presented by the following lemma and then the main adaptive control algorithm is proposed.

**Lemma:** Consider the uncertain nonlinear system (2) satisfying assumption A2. If the norm of parameter vector \(\alpha(t)\) is bounded by a known positive scalar \(\alpha_m\), the robust control law
\[ u = -\frac{1}{b_0} \left( \Lambda Ke + \frac{\Lambda M(x)M^T(x)e}{|M^T(x)e|} + \delta_1 e^{-\sigma_1 t} \right) + \alpha^2 \frac{N_{\rho}(x)\varphi^T(x)e}{\|\varphi(x)e\| \alpha + \delta_2 e^{-\sigma_2 t}} \]
with \(\alpha > \alpha_m\), guarantees the closed-loop stability and the convergence of tracking error to zero.

**Proof:** Choose a Lyapunov function candidate as
\[ V(e) = \frac{1}{2} e^T e. \]
Substituting \(\alpha\) instead of \(\hat{a}\) in (9) and noting that \(\hat{a} = 0\), one can easily obtain
\[ \dot{V}(e) \leq -e^T Ke + \delta_1 e^{-\sigma_1 t} + \delta_2 e^{-\sigma_2 t}. \]
Following the proofs of theorems 1 and 2 completes the proof. \(\square\)

**Theorem 3:** Consider the parametric uncertain nonlinear system (2). Let the assumptions A1-A2 hold. The control law (8) with
\[ \dot{\hat{a}} = \eta \|\varphi(x)e\|, \quad \hat{a}(t_0) > 0 \]
(11)
where
\[ \eta = \begin{cases} \gamma & \text{if } \|e\| > \epsilon \\ 0 & \text{otherwise} \end{cases} \]
\((\gamma > 0\) is the adaptation gain) guarantees that all the signals and states of the closed loop system are bounded and tracking error \(e\) is robustly converged to a (small) prescribed bound \(\epsilon > 0\).

**Proof:** In fact, control law (8) with adaptation mechanism (11) consists of two sub-controllers. The first is control law (8) with adaptation mechanism (5) and the second is control (8) without any adaptation law. At first, without loss of generality, assume \(\|e\| > \epsilon\). Consequently, the first sub-controller is activated and the proof of theorem 1 ensures asymptotic convergence of tracking error \(e\). As a result, there exists a finite time \(t_e > 0\) such that the norm of tracking error \(e\) meets the prescribed error bound \(\|e\| = \epsilon\) and the second sub-controller is activated. At this time, if \(\hat{a} \geq \alpha\) (\(\alpha\) is unknown) is satisfied, the second sub-controller acts as the robust controller introduced by the lemma and guarantees asymptotic convergence of tracking error \(e\) to zero. Otherwise, \(\|e\|\) either remains below the prescribed bound \(\epsilon\), that completes the proof, or exceeds the bound \(\epsilon\) and the first sub-controller is switched on again. In this case, it is shown that there exists a time \(t_{off}\) such that the first sub-controller is switched off for all \(t > t_{off}\) and consequently, the second sub-controller ensures robust tracking. For this purpose, note that the adaptation law (11) ensures that the estimated value \(\hat{a}\) can eventually get to unknown bound \(\alpha\). So the first sub-controller is switched off with an \(\hat{a} \geq \alpha\). At this time, say \(t_{off}\), the second sub-controller act as the robust controller of the lemma. This completes the proof. \(\square\)

4. Application to Chua’s Circuit
Nonlinear Chua’s circuit, due to its simplicity and universality, has become a benchmark to study the behavior and control of nonlinear systems. In spite of all existing methods [16]–[18], [20]–[22], a significant attention is paid towards control of nonlinear Chua’s circuit, depicted in Fig. 1.

The dynamic equations of Chua’s circuit with control inputs are given as [23]
\[ \begin{align*}
\dot{v}_1 &= \frac{1}{C_1} \left( \frac{1}{R} (v_2 - v_1) - g(v_1) \right) + \frac{1}{C_1} u_1 \\
\dot{v}_2 &= \frac{1}{C_2} \left( \frac{1}{R} (v_1 - v_2) + i_L \right) + \frac{1}{C_2} u_2 \\
\dot{i}_L &= -\frac{1}{L} v_2 + \frac{1}{L} u_3
\end{align*} \]
(12)
(13)
(14)
where \(v_1\) and \(v_2\) denote the voltages across the capacitors \(C_1\) and \(C_2\) respectively, \(i_L\) is the current through the inductor \(L\) and \(g(v_1)\) is the current through the nonlinear resistor \(g\).

\[ \begin{align*}
\dot{x}_1 &= \frac{1}{C_{10}} \left( \frac{1}{R_0} (x_2 - x_1) - g_0(x_1) \right) \\
&\quad + \frac{1}{C_{10}} \left( -R_0 C_1 - C_1 R + \delta C_1 R \right) (x_2 - x_1) \\
&\quad + \frac{1}{C_{10} R_0 C_1} g_0(x_1) - \frac{1}{C_1} u_1 \\
&\quad + \frac{1}{C_{10} R_0 C_1} \left( -R_0 C_1 - C_1 R + \delta C_1 R \right) (x_2 - x_1) \\
&\quad + \frac{1}{C_{10} R_0 C_1} g_0(x_1) - \frac{1}{C_1} u_1
\end{align*} \]
(15)
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circuit as a parametric uncertain system of the form (2), we

\[ \dot{x}_2 = \frac{1}{C_{20}} \left( \frac{1}{R_0} (x_1 - x_2) + x_3 \right) \]
\[ + \frac{-RC_2 - C_2 \delta R + \delta C_2 \delta R}{C_{20}R_0C_2R} (x_1 - x_2) \]
\[ - \frac{\partial C_2}{C_{20}C_2} x_3 + \frac{1}{C_2} u_2 \]
\[ \dot{x}_3 = -\frac{1}{L_0} x_2 + \frac{\delta L}{L_0L} x_2 + \frac{1}{L} u_3 \] (16)

In practice, the values of circuit’s components, e.g. resis-
tors, capacitors and inductors, are dependent on ambient
circumstances such as temperature. Due to this fact,
each electrical component \( E \) can be expressed as a no-
minal value \( E_0 \) plus an uncertain time-varying part \( \delta E \), that is \( E = E_0 + \delta E \). Particularly for Chua’s circuit, we consider \( C_1 = C_{10} + \delta C_1, C_2 = C_{20} + \delta C_2, L = L_0 + \delta L, R = R_0 + \delta R \) and \( g = g_0 + \delta g \) where \( C_{10}, C_{20}, L_0, R_0, g_0 \), represent the nominal values and \( \delta C_1, \delta C_2, \delta L, \delta R, \delta g \) denote the perturbations.

By the above considerations and defining \( x = [x_1, x_2, x_3]^T \) as the state vector and \( u = [u_1, u_2, u_3]^T \) as the control input, Chua’s circuit can be de-
scribed by a set of uncertain differential equations as (15)–
(17). In order to describe the dynamics of nonlinear Chua’s
circuit as a parametric uncertain system of the form (2), we
take the parameter vectors
\[ a(t) := \left[ \begin{array}{c} \frac{\partial C_1}{C_1} \frac{\partial R}{R} \frac{\partial C_1 \partial R}{C_1} \frac{\partial C_2 \partial R}{C_2} \frac{\partial L}{L} \frac{\partial \delta R}{\delta R} \frac{\partial \delta g}{\delta g} \end{array} \right]^T \] (18)
\[ b(t) := \left[ \begin{array}{c} \frac{1}{C_1} \frac{1}{C_2} \frac{1}{T} \end{array} \right]^T \] (19)
\[ \varphi(x) = \left[ \begin{array}{c} \frac{1}{C_{10}} (x_2 - x_1) - g_0(x_1) - \frac{1}{C_{10}R_0} (x_2 - x_1) \frac{1}{C_{10}R_0} (x_2 - x_1) \\
0 & 0 & 0 \\
0 & 0 & 0 \\
-\frac{1}{C_{20}} \frac{1}{R_0} (x_1 - x_2) + x_3 & \frac{1}{C_{20}R_0} (x_1 - x_2) & 0 \\
0 & 0 & 0 \\
0 & \frac{1}{L_0} x_2 & 0 \end{array} \right] \] (20)

that imply the regressor matrix \( \varphi(x) \) as (20) and the known
vector \( f_0(x) \) as
\[ f_0(x) = \left[ \begin{array}{c} \frac{1}{C_{10}} \frac{1}{R_0} (x_2 - x_1) - g_0(x_1) \\
\frac{1}{C_{20}} \frac{1}{R_0} (x_1 - x_2) + x_3 \\
\frac{1}{L_0} x_2 \end{array} \right]^T \] (21)

Therefore, the nonlinear Chua’s circuit is known with (18)–
(21) as a parametric uncertain nonlinear system of the
form (2). Taking a desired smooth state trajectory \( x_d = [x_{d1}, x_{d2}, x_{d3}]^T \), the robust adaptive controller proposed by
theorem 3 is developed for control of Chua’s circuit.

To illustrate the performance of the proposed tracking
controller, consider the nonlinear Chua’s circuit shown in
Fig. 1 with nominal values \( C_{10} = 1, C_{20} = 0.5, L_0 = 1,\)
\( R_0 = 5, g_0(v_1) = -v_1 + 0.02v_1^3 \) [18]. Moreover, the uncer-
tain parts are taken as \( \delta C_1 = 0.1 + 0.1 \cos(t/2), \delta C_2 = 0.1,\)
δL = 0.15, δR = sin(τ/2) and δg = 0.2sin(τ). Note that in [18] no perturbation is considered for nonlinearity g. Let the desired reference trajectory be a vector of rather complex sinusoidal functions as \( x_d = [0.5\sin(τ) + \cos(2τ), 0.5\sin(τ) + 0.5\cos(3τ), -\sin(2τ) - 0.5\cos(τ)]^T \). It should be mentioned that the sinusoidal trajectories can efficiently evaluate the performance of tracking control [5],[13], particularly in chaos control [18],[24].

Taking the gain matrix \( K = 2I_{3\times3} \), the error bound \( \epsilon = 0.05 \) and the adaptation gain \( \gamma = 1 \), the robust adaptive control of theorem 3 is constructed as

\[
\begin{align*}
    u &= -1.5 \left( 2e + \frac{M(x)M^T(x)e}{|M^T(x)e| + 3e^{-0.1\tau}} \right) \\
    &\quad + \hat{\alpha}^2 \frac{\varphi(x)\varphi^T(x)e}{||\varphi^T(x)e||} (\hat{\alpha} + 0.3e^{-0.1\tau})
\end{align*}
\]

with

\[
\dot{\hat{\alpha}} = \eta||\varphi^T(x)e||
\]

where

\[
\eta = \begin{cases} 
1 & \text{if } ||e|| > 0.05 \\ 
0 & \text{otherwise}
\end{cases}
\]

Assuming the initial conditions to be \( v_1(0) = 0.5, v_2(0) = 0, i_L(0) = 0 \) and \( \hat{\alpha}(0) = 0.1 \), some simulation results are shown in the sequel. Figure 2 shows the effectiveness of the control algorithm to make the outputs track the reference trajectories. The control inputs are illustrated in Fig. 3. The smoothness of such control signals facilitates practical implementation. Their magnitudes, however, depend on the element values and the size of uncertain time-varying parameters. Note that most of the existing adaptive approaches do not concern with time-varying perturbations with unknown large magnitudes. As depicted in Fig. 4, the
estimated parameter value \( \hat{a} \) eventually gets to a fixed value, as proved by theorem 3. The convergence of tracking error is shown in Fig. 5. The phase-plane trajectories \( v_1 - \hat{v}_1 \) and \( v_2 - \hat{v}_2 \) for Chua’s circuit are plotted in Figs. 6 and 7. As the simulation results show, accurate tracking performance is achieved for the uncertain model of Chua’s circuit by using the developed robust adaptive controller.

5. Conclusions

Tracking control problem for uncertain systems with time-varying parameters is addressed. A robust adaptive algorithm is constructed to ensure convergence of tracking error in spite of time-varying parameters with unknown bounds. The proposed control scheme is applied to uncertain nonlinear Chua’s circuit. Compared with previous investigations, two practical aspects of the problem are focused. These include time variance nature of electronic components and unknown control coefficients. Due to the various fields of application of the proposed method, its extension to broader classes of MIMO nonlinear systems is under investigation by the authors.

References


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