OPTIMIZATION MODEL AND SIMULATION FOR IMPROVING AMBULANCE SERVICE SYSTEM

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Abstract

This paper presents a brief survey of operations research works for ambulance service design, focusing on two mainstreams, i.e., facility location and simulation methods. Then we show some application examples of those methods using actual ambulance dispatch data.

1 Introduction

Ambulance service has been an important topic of operations research for a long time due to its crucial function in social services. Not a few operations research workers have been attracted to this topic and developed various models and techniques to make the system much better. Although an extensive literature has accumulated on the topic, we focus on facility location models and discrete event simulation models, because they are frequently applied to the problem by many researchers and can be considered as mainstreams of the research.

Facility location models consider locating facilities, which are ambulance stations in our case, to optimize a given goodness measure of location under some constraints. Since uncertainty in call demand is involved in, it is a central theme of the problem how to deal with the uncertainty. A good survey of the topic is found in, e.g., [1, 2, 3, 4]. Following them, ambulance location problems are classified into two large groups: covering model and median model, and each group contains several varieties.

- Covering model seeks the location which covers as much demand as possible under a given coverage standard.
  - Maximum expected covering location problem (MEXCLP) [5] is an early model which incorporates a queueing theory into facility location problem to deal with the busy probability of ambulance, and maximizes the expected covered demand under given number of ambulances.
  - Maximal availability location problem (MALP) [6] includes a reliability level, which is the probability of covering demand, into constraints and maximizes total covered demand. Q-MALP [7] is an extension combining MALP and queueing theory.
- Median model minimizes the total traveling distance of ambulances. Capacitated median model is often used, which put a constraint on the maximum number of dispatches by an ambulance.

Recent works of facility location analysis on ambulance service systems show various extensions. Some examples are covering by two types of servers [8], dynamic relocation [9], minimizing response time [10] maximizing patient survival probability [11], and so on.

Another branch of the research, discrete event simulation models, may originate in well-known hypercube model [12]. It is a spatial queueing model as well as a Markov chain model which yields the equilibrium equation for steady-state probability of the system. The early application of hypercube model was to calculate the steady-state probability by solving the equilibrium equation and obtain several features of the system such as loss probability. This model is combined with several other techniques and yields a lot of simulation-based analyses of ambulance system, see, e.g., [13, 14, 15].

In the following sections we briefly report our experience in the study of applying some models to ambulance data in Tokyo metropolis and how those models works in actual situation. They could show a great fertility of the field in operations research, hopefully.

Sec. 2 shows a facility location analysis using some classical models, i.e., MEXCLP and Q-MALP. The solutions of model are investigated in Sec. 3 by simulation-based analysis to verify whether the expected coverage or availability is achieved.

2 Facility location analysis

In order to show how location analysis works for improving an ambulance system, we apply several facility models to the ambulance data in Tokyo metropolis. This analysis partially appeared in [16], after which some extensions of the model, especially constraints on the possible number of relocating ambulances, are enjoyed to make the solution of MEXCLP and Q-MALP fit to the practical situation.

We introduce some notations which are necessary to describe the models. Let I be set of demand points, J

be set of potential ambulance stations, d_{ij} be Euclidean distance between $i \in I$ and $j \in J$, and a_i be demand at $i \in I$, which is the number of calls at demand point i. The busy probability of ambulance, denoted by q, is the probability that each ambulance at any given time is busy. Most of covering location models including MEXCLP and Q-MALP, assume it is constant at any time and in any region. We can obtain the value of q from the arrival rate of calls and the average time of ambulance's turnaround, i.e. the time from dispatch to return to the station. Introducing decision variables x_j, $j \in J$, which is the number of ambulances located at j, and y_{ik}, $i \in I$, which is equal to 1, if the point i is covered by ambulances of more than or equal to k and 0 otherwise, we suppose to locate p ambulances here, then MEXCLEP has the following integer programming formulation.

(MEXCLP)

$$\text{max.} \quad \sum_{i \in I} \sum_{k=1}^{p} a_i (1-q) q^{k-1} y_{ik} \tag{1}$$

$$\text{s.t.} \quad \sum_{j \in N_i} x_j \geq \sum_{k=1}^{p} y_{ik}, \quad i \in I \tag{2}$$

$$\sum_{j \in J} x_j = p \tag{3}$$

$$x_j : \text{ integer}, \quad j \in J \tag{4}$$

$$y_{ik} \in \{0,1\}, i \in I, \, k = 1, \dots, p \tag{5}$$

where N_i is the set of ambulance stations covering the point i and defined as $N_i = \{j \in J : \, d_{ij} \leq D\}$, with a predefined coverage standard distance D. The objective function (1) of model is the expected covered demand, while the constraints (2) says the number of ambulances covering point i (LHS) must be greater than or equal to the number of necessary ambulances for point i (RHS).

Applying this model to the data, then we find that many ambulances are moved to concentrate in heavy demand area and result in too many demand points covered by more than two ambulances. On the other hand, we do not have a remarkable improvement of expected coverage of demand. From this case study we have learned MEXCLP is too ready to concentrate ambulances to heavy demand area. Although this may be apparent from the objective function (1), we have learned that the improvement of expected covered demand is slight at the same time.

We explore another model Q-MALP, where the same decision variables x_j as MEXCLP exist while y_{ik} are replaced by y_i which is equal to 1 if demand point i is covered and 0 otherwise. We do not care about how many ambulances cover the point in Q-MALP, but introduce new parameters b_i which is the number of ambulances necessary to meet the demand at point i and calculated in terms of queueing theory.

(Q-MALP)

$$\text{max.} \quad \sum_{i \in I} a_i y_i \tag{6}$$

$$\text{s.t.} \quad \sum_{j \in N_i} x_j \geq b_i y_i \quad i \in I. \tag{7}$$

$$\sum_{j \in J} x_j = p \tag{8}$$

$$x_j : \text{ integer}, \quad j \in J \tag{9}$$

$$y_i \in \{0,1\}, \quad i \in I \tag{10}$$

Q-MALP is less affected by heavy demand than MEXCLP, because it does not have any increase of objective function by covering heavy demand points multiple times. On the other hand, the solution of Q-MALP greatly depends on the parameters b_i, required ambulances by demand point i, which are hard to know exactly and we often define conventionally. Unless they are specified appropriately, Q-MALP may reach an unreliable solution. In our experiments, we try to make a kind of sensitivity analysis with respect to total number of ambulances to be moved, and successfully find stable solutions.

3 Simulation analysis

Simulation analysis sheds light on facility location models. In both models MEXCLP and Q-MALP, it is a crucial assumption that all the ambulances work independently. This assumption, however, is not realistic yet, and we often observe a correlation for the dispatches of neighboring ambulances in actual situation. It implies those models could make under-estimates of expected covered demand or availability of ambulances. We verify the validity of the assumption with the help of simulation.

Our simulation runs as follows, see [17] for details. Let there exist p ambulances, then define a state vector $\mathbf{S}_k = (S_{k1}, \dots, S_{kp})$ as S_{ki} represents the state of ambulance i at time k: $S_{ki} = 1$ when ambulance i has been dispatched and busy at time k, whereas $S_{ki} = 0$ when ambulance i is on stand-by at the station at time k. Given the current state $\mathbf{S}_k$, there are two possible kinds of state transition:

1. A dispatched ambulance i finishes taking a patient to the hospital and returns to the home station. The consequent state transition is given by $S_{k,i} = 1 \to S_{k+1,i} = 0$, and $S_{k+1,i'} = S_{k,i'}$ for $i' \neq i$.
2. A new call arrives and the ambulance i nearest to the patient is dispatched. The state transition is $S_{k,i} = 0 \to S_{k+1,i} = 1$, and $S_{k+1,i'} = S_{k,i'}$ for $i' \neq i$.

Each probability of transition is given in terms of parameters of arrival rate at each point and service rate of ambulances.

We try to apply this simulation to the solution of MEXCLP to estimate the expected covered demand directly. Our simulation estimate is approximately 20%

lower than MEXCLP gives. This large difference in estimates suggests mutual independence of ambulances is open to question.

4 Summary and concluding remarks

We reported our experimental studies on ambulance system, focusing on facility location models and simulation models. They are relatively simple and straightforward application of classical models and can develop into more advanced analysis. One possible future work could be to combine two methods to give a more realistic optimization model, as is tried by several researchers, although it would require to device an efficient algorithm to deal with such a large-scale problem as we presented.

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