Cooperative Strategies and Capacity Theorems for Relay Networks

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Course: Information Theory.

June, 2010
Presentation Contents:

- Introduction.
- Models and preliminaries.
- Wireline strategies.
- Wireless models.
- Numerical results of related work.
- Comments.
Introduction:

- The area of relay networks is a very hot area of research in both wireline and wireless communications.

- The Technique:
  source $\rightarrow$ intermediate nodes $\rightarrow$ destination

- Advantages:
  - Achieving broader coverage without the need for a high power transmitter.
- Mitigating fading.
- Providing spatial diversity (cooperative comm.).
- Also, cooperation in wireline networks as in multiaccess relay channels (MARCs).

Applications:
- It is used to communicate via add hoc networks where nodes are communicating without the aid of central control/infrastructure [1].
- Recently, this technique has gained new actuality in collaborative/cooperative wireless communication systems [2].
Two main schemes:
- Decode-and-forward (DAF).
- Compress-and-forward (CAF).

The DAF strategies have as a common feature that the source controls what the relays transmit. For wireless networks, one consequently achieves gains related to multiantenna transmission.

In the CAF strategy, the relay forwards a quantized and compresses version of its channel outputs to the destination.
Models and Preliminaries:

- Relay network model:

Figure 1: Model of relay networks: (a) 1-relay network; (b) 2-relay network.
- **Used parameters:**
  - The previous Figure gives two different relay networks, $T = 3$ and $T = 4$.
  - $T - 2$ relays (nodes $t$ with $t \in \{2, 3, \ldots, T - 1\}$).
  - $W$ is the message.
  - $X_{ti}$, $t = 1, 2, \ldots, T - 1$, $i = 1, 2, \ldots, N$, are the channel inputs.
  - $Y_{ti}$, $t = 2, \ldots, T$, $i = 1, 2, \ldots, N$, are the channel outputs.
  - $\hat{W}$ is the message estimate.
The channel will be assumed memoryless time-invariant channel.

The channel distribution can be given by:

\[ p \left( y_2, \ldots, y_T \mid x_1, \ldots, x_{T-1} \right) \]  \hspace{1cm} (1)

For example: the \( T = 3 \) network (wired)

- Channel inputs: \( X_1 = [X_{11}, X_{12}] \) and \( X_2 \).
- Channel outputs: \( Y_2 \) and \( Y_3 = [Y_{31}, Y_{32}] \).
The channel distribution for this case is:

\[ p(y_2, y_3 | x_1, x_2) = p(y_2 | x_1) \cdot p(y_3 | x_2) \cdot p(y_3 | x_2) \]  (2)

- **Capacity upper bound:**

- Let \( X_S = \{X_t : t \in S\} \) and \( T = \{2, 3, \ldots, T - 1\} \). A capacity upper bound is given in [3] as:

\[
C \leq \max_{p(x_1, x_2, \ldots, x_{T-1})} \min_{S \subseteq T} I \left( X_1 X_S ; Y_S Y_T \mid X_{S^C} \right) \]  (3)

where

\( S^C \) is the complement of \( S \) in \( T \).
Example: for $T = 3$

For this case, we have:

$X_1$ form the source.

$T = \{2\}$.

$S = \{\emptyset, 2\}$.

$C \leq \max_{p(x_1, x_2)} \min \{I(X_1; Y_2 Y_3 | X_2), I(X_1 X_2; Y_3)\}$ \hspace{1cm} (4)

$C \leq \min \{C_{12} + C_{13}, C_{13} + C_{23}\}$ \hspace{1cm} (5)
MARC Model:

- The model is shown in next Figure:

- Node 1 and 2 transmit the ind. messages $W_1$ and $W_2$ with rates $R_1$ and $R_2$

- Three inputs and 2 outputs.

Figure 2: A MARC with two sources.
- **Inputs:**

\[X_1 = [X_{11}, X_{12}]\]
\[X_2 = [X_{21}, X_{22}]\]
\[X_3\]

- **Outputs:**

\[Y_3 = [Y_{31}, Y_{32}]\]
\[Y_4 = [Y_{41}, Y_{42}, Y_{43}]\]
The channel distribution is:

\[
p (y_3, y_4 | x_1, x_2, x_3) = p (y_{31} | x_{11}) \cdot p (y_{32} | x_{21}) \\
\quad \cdot p (y_{41} | x_{12}) \cdot p (y_{42} | x_{22}) \cdot p (y_{43} | x_3)
\]  

(6)
Wireline Strategies:

- Decode–and–forward:

  From [4], the rates for one relay is given by:

  \[
  R_{DF} = \max_{p(x_1, x_2)} \min \{ I(X_1; Y_2|X_2), I(X_1X_2; Y_3) \} \tag{7}
  \]

  \[
  C = \min \{ C_{12}, C_{13} + C_{23} \} \tag{8}
  \]

  \[
  C \leq \max_{p(x_1, x_2)} \min \{ I(X_1; Y_2Y_3|X_2), I(X_1X_2; Y_3) \} \tag{9}
  \]

  \[
  C \leq \min \{ C_{12} + C_{13}, C_{13} + C_{23} \} \tag{10}
  \]
- Regular encoding for one relay:

1. The message $w$ is divided into $B$ blocks of length $nR$ each.

2. The transmission is performed in $B+1$ blocks by transmitting codewords of length $n$.

3. Node 1 transmits $x_1(i, j)$ and node 2 transmits $x_2(j)$.

4. Quantities:

$$B_w = BnR, \quad N = (B + 1)n, \quad \text{and} \quad R_w = R \cdot B / (B + 1)$$
The DAF model and the information transfer are shown in Figures below:

<table>
<thead>
<tr>
<th>Block 1</th>
<th>Block 2</th>
<th>Block 3</th>
<th>Block 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1(1, w_1)$</td>
<td>$x_1(w_1, w_2)$</td>
<td>$x_1(w_2, w_3)$</td>
<td>$x_1(w_3, 1)$</td>
</tr>
<tr>
<td>$x_2(1)$</td>
<td>$x_2(w_1)$</td>
<td>$x_2(w_2)$</td>
<td>$x_2(w_3)$</td>
</tr>
</tbody>
</table>

Figure 3: DAF for one relay.

- $I(X_1; Y_2|X_2)$
- $I(X_2; Y_3)$
- $I(X_1; Y_3|X_2)$

Figure 4: The information transfer for regular encoding/sliding window decoding.
Strategy steps:

1. In the 1\textsuperscript{st} block: node 1 and 2 transmit their codewords.

2. Node will reliably decode \( w_I \) if \( n \) is large and:

\[
R < I \left( X_1; Y_2 \mid X_2 \right) \tag{11}
\]

3. In the 2\textsuperscript{nd} block.

4. Finally, depending on the used decoding strategy:
- If the backward decoding is used:
  
a. Let $y_{3b}$ be the $b$th block of channel outputs of node 3.

b. At block $b+1$, we have $y_{3(b+1)}$ and all the previous outputs (stored).

c. Now, node 3 can decode $w_b$ reliably if $n$ is large and:

$$R < I(X_1X_2; Y_2)$$

(12)

d. One continues in this fashion until all message blocks have been decoded.
e. To have a rate close to $R$, one may use large number of blocks:

$$R_w = R \cdot B / (B + 1)$$

- Using the sliding window decoding:

At node 3: after receiving $\underline{y}_{3b}$, $b \geq 3$, it similarly decodes $w_{b-1}$ by using $\underline{y}_{3(b-1)}$ and $\underline{y}_{3b}$, all the while assuming its past message estimate $\hat{w}_{b-2}^{(3)}$ is $w_{b-2}$.
Wireless Models:

- No fading and one relay:

\[
R_{DF} = \max_{0 \leq \rho \leq 1} \min \left\{ \log \left( 1 + \frac{P_1}{d_{12}^{\alpha}} (1 - \rho^2) \right) \right\},
\]

\[
\log \left( 1 + \frac{P_1}{d_{13}^{\alpha}} + \frac{P_2}{d_{23}^{\alpha}} + \frac{2\rho \sqrt{P_1 P_2}}{d_{13}^{\alpha/2} d_{23}^{\alpha/2}} \right) \right\}.
\]

Figure 5: A single relay on a line.
\[ R_{CF} = \log \left( 1 + \frac{P_1}{d_{12}^\alpha (1 + \hat{N}_2)} + \frac{P_1}{d_{13}^\alpha} \right) \]

- Special case 1: \( d \to 0 \)

\[ R_{DF} = \log (1 + P_1 + P_2 + 2\sqrt{P_1 P_2}) \]
\[ R_{CF} = \log (1 + P_1 + P_2) \]

- Special case 2: \( d \to 1 \)

\[ R_{DF} = \log (1 + P_1) \]
\[ R_{CF} = \log (1 + 2P_1) \]
Curve for one relay: $P_1 = P_2 = 10$, alpha = 2:

![Figure 6: Rates for one relay.](image)
Numerical Results of Related Work:

1) $P_{\text{out}}$ vs. normalized SNR:

Figure 7: Comparison of $P_{\text{out}}$ of reg. and non-reg. systems.
2) **BER vs. average SNR per hop:**

![Comparison of Bit Error Rates for Regenerative and Non-Regenerative Systems](image)

Figure 8: Comparison of the BER of reg. and non-reg. systems (balanced).
This paper gives many basic theorems and relations of channel capacity and system rates for both wireline and wireless system.

Various schemes were presented and compared in this work: DAF, CAF, MARC and BRC channels, single and multi relays.

Other parts of the papers dealt with the effect of antenna on the wireless systems and the fading.
References:


Thanks for listening

Any questions??

Anas...