Dealing with Unreliabilities in Digital Passive Geometric Telemanipulation

C. Secchi[1], S. Stramigioli[2], C.Fantuzzi[1]

[1]DISMI, Univ. of Modena and Reggio Emilia
Viale Allegri 13, 42100 Reggio Emilia, Italy
secchi.cristian@unimore.it
fantuzzi.cesare@unimore.it

[2]Drebbel Institute, Univ. of Twente
P.O.Box 217, 7500 AE Enschede, NL
S.Stramigioli@ieee.org

***Abstract*— In this paper two problems arising in the digital passive scheme for telemanipulation presented in [5] are addressed. At first, we show how to preserve system passivity in presence of quantization error introduced by position sensor (i.e. encoders) by introducing energy dissipation. Then, we introduce a scheme for a redundant communication channel that will compensate for missed packets improving performances while preserving passivity of the overall scheme.**

I. Introduction

The main problems arising in the control implementation of bilateral telemanipulation are (i) the time delay in communication between master and slave and (ii) the interaction with an unknown environment.

The first problem has been tackled in several ways. In particular, in [1] a very elegant and effective scheme based on scattering theory has been proposed. Successively, in [4], the problem of wave reflection arising in scattering based communication channels has been solved and some adaptive techniques have been proposed to improve system performances. For an overview and a comparison of the various schemes proposed in the literature the reader is addressed, for instance, to [2].

The second problem can be solved effectively by application of passivity theory. As matter of fact, a stable interaction can be obtained by controlling the energy exchange between the interacting systems and this can be done, for instance, by means of *intrinsically passive controllers* (IPC, [6]).

In [9] a general framework for the telemanipulation of port-Hamiltonian systems ([10]) is proposed. In this setting, IPCs are used to obtain a stable interaction and scattering theory is exploited to implement a lossless communication channel, independently of the transmission delay. The problem of wave reflection is considered and solved in a geometric coordinate free way. The whole telemanipulation scheme is intrinsically passive and, therefore, it exhibits a stable behavior.

In [7], [5] the discrete nature of the controller is considered and techniques presented in [8] are used to obtain passive discrete IPCs and to passively interconnect them to the continuous plants. Furthermore a discrete-time scattering based communication channel is defined and its passivity is proven both in case of variable delay and in case of loss of packets. In particular, it is shown that the loss of a packet during the communication corresponds to the dissipation of its energetic content; this leads to a dissipative behavior of the transmission line.

In order to perform a passive interconnection between the discrete controller and the robot, position measurements are needed ([5]). In industrial applications, position information is very often obtained by means of encoders. We will show that the quantization error associated with the encoders leads to the production of some extra energy into the system and, therefore, to loss of passivity. We will propose a strategy to interconnect continuous and discrete port-Hamiltonian systems without the production of any extra energy even in case of unreliable position information.

Loss of packets in the communication doesn't affect the passivity of the overall system but it degrades the performances of the telemanipulation scheme since part of the energy necessary to complete a certain task is not transmitted but dissipated. Thus, we introduce a control scheme able to interpolate the received packets for reconstructing the missed ones, and thus improving control performances.

The paper is organized as follows: in Sec.II we will provide some background on both continuous and discrete port-Hamiltonian systems and on the intrinsically passive scheme of [5]. In Sec.III we will see how to modify the interconnection between continuous and discrete port-Hamiltonian systems in order to preserve passivity even in case of quantization error on the encoders and in Sec.IV we will show a passive communication channel with an embedded interpolation for the missed packets. In Sec.V we will provide some simulations in order to validate our results and in Sec.VI some conclusions and final remarks.

II. BACKGROUND

A. Continuous port-Hamiltonian systems

We will now try to give an intuitive description of port-Hamiltonian systems using coordinates in order to concentrate on the prime contribution of the paper. More formal descriptions can be found in [10]. We can consider a port-Hamiltonian system as composed of a state manifold $\mathcal{X}$, an energy function $H : \mathcal{X} \to \Re$ corresponding to the internal energy, a network structure $D(x) = -D(x)^T$ whose graph has the mathematical structure of a Dirac structure([3]), which is in general a state dependent power continuous interconnection structure, and an interconnection port represented by an effort-flow pair $(e, f) \in V^* \times V$ which is geometrically characterized by dual vector elements. This port is used to interact energetically with the system. The power supplied through a port is equal to $e(f)$ or using coordinates to $e^T f$. We can furthermore split the interaction port in more sub-ports, each of which can be used to model different power flows. We will indicate with the subscript I the power ports by means of which the system interacts with the rest of the world, with the subscript C the power ports associated with the storage of energy and with the subscript R the power ports relative to the dissipative part. Summarizing, we have:

$$\begin{pmatrix} e_I \\ f_C \\ e_R \end{pmatrix} = D(x) \begin{pmatrix} f_I \\ e_C \\ f_R \end{pmatrix}$$

where $D(x)$ is a skew symmetric matrix representing the Dirac structure.

Due to the skew-symmetry of $D(x)$, we have, using coordinates:

$$P_I + P_C + P_R := e_I^T f_I + e_C^T f_C + e_R^T f_R = 0 \qquad (1)$$

which is a power balance meaning that the total power coming out of the network structure should be always equal to zero.

A dissipating element of the system can be modeled using as characteristic equations $e_R = R(x) f_R$ with $R(x)$ a symmetric and positive semi-definite tensor.

If we furthermore set $\dot{x} = f_C$ and $e_C = \dfrac{\partial H}{\partial x}$, due to the previous power balance we obtain:

$$\dot{H} + f_R^T R(x) f_R = -e_I^T f_I$$

which clearly says that the supplied power $-e_I^T f_I$ equals the increase of internal energy plus the dissipated one.

B. Discrete port-Hamiltonian systems

For a lot of applications it is meaningful to find a discrete time representation of a physical system which can be used either as a virtual environment in haptics or as an IPC [6] in interacting tasks or in telemanipulation. Hereafter we will briefly review how to discretise a port-Hamiltonian system preserving its passivity. More details can be found in [8].

We can describe a discrete time port-Hamiltonian system as a continuous time port-Hamiltonian system in which the port variables are frozen for a sample interval T. In what follows we indicate with $v(k)$ the value of the discrete variable $v(t)$ corresponding to the interval $t \in [kT, (k+1)T]$.
If we rewrite Eq.(1) for the discrete case, we have:

$$e_I^T(k) f_I(k) + e_C^T(k) f_C(k) + e_R^T(k) f_R(k) = 0 \qquad (2)$$

Furthermore, during the interval k, we have to consider a constant state $x(k)$ corresponding to the continuous time state $x(t)$. This implies that during the interval k, the dissipated energy will be equal to $T f_R^T(k) R(x(k)) f_R(k)$ and the supplied energy will be equal to $-T e_I^T(k) f_I(k)$. In order to be consistent with the energy flows, and as a consequence conserve passivity, we need therefore a jump in internal energy $\Delta H(k)$ from instant kT to instant $(k+1)T$ such that:

$$\Delta H(k) = -T f_R^T(k) R(x(k)) f_R(k) - T e_I^T(k) f_I(k)$$

This implies that the new discrete state $x(k+1)$ should belong to an energetic level such that:

$$H(x(k+1)) = H(x(k)) + \Delta H(k)$$

Solving the previous equations in $x(k+1)$ it is possible to find a set of state the system can jump to preserving passivity. This set can be either empty or have more solutions. In the first case a state is chosen by means of the so-called *energy leap strategy* while in the second case we have to choose the "closest" (the definition of closeness can be made clear by specifying a proper affine connection on the state manifold) state to the current one. For further details the reader is referred to [8].

C. Passive interconnection

Consider the port interconnection of a continuous time Hamiltonian system H_C and a discrete Hamiltonian system H_D (but the result of this subsection is independent of the nature of the energetically interconnected systems) through a sampler and zero-order hold as shown in Fig.1. If the sample&hold is not properly designed, it can happen that the process generates extra energy and that the

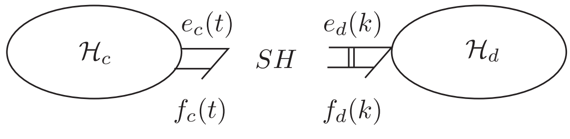


Fig. 1. The interconnection of discrete and continuous port-Hamiltonian systems

passivity of the whole system is not assured even if the two interconnected systems are passive.
Suppose that H_C has an admittance causality (effort in/flow out) and therefore H_D has an impedance causality (flow in/effort out). We will have that:

$$e_c(t) = e_d(k) \qquad t \in [kT, (k+1)T]$$

The following theorem can be proved ([8])

Theorem 1 (Sample Data passivity): If we define for the interconnection port of H_D

$$f_d(k) := \frac{x((k+1)T) - x(kT)}{T}, \tag{3}$$

where $x()$ represents the integral of the continuous flow, we obtain an equivalence between the continuous time and discrete time energy flow in the sense that for each n:

$$E_d(n) = \sum_{i=0}^{n-1} e_d^T(i) f_d(i) = \int_0^{nT} e_c^T(s) f_c(s) ds = E_c(nT) \tag{4}$$

From the previous considerations, it is possible to understand that at each sampling time, we have an EXACT matching between the physical energy going into the continuous time system and the virtual energy coming from the discrete time port independently of the sample time T and of eventual intersample dynamics of the continuous system. It is remarkable that the choice reported in Eq.(3) which is very simple and at the same time attractive due to the fact that it corresponds to position measurements, in practice gives such a powerful and at the same trivial result. This means that we can passively interconnect the two systems in such a way that *independently* of the sampling time and its relation with the characteristics of the interconnected systems, the two systems would be energetically consistent at each sampling time and no energy would be created by the sample and hold procedure. The only energy leakage is due to the fact that the discrete time system has no way what so ever to predict the value of the continuous time system at the interconnection port and this implies that only at the end of the sample period will have an exact measure of the energy it supplied to the continuous time system. But the amount of the eventually produced extra energy is exactly known and it can be compensated with a damping circuit or by a clever book-keeping strategy.

D. The Intrinsically Passive Telemanipulation Scheme

The sampled data intrinsically passive telemanipulation scheme is represented in Fig.2. We can see that the continuous plant (i.e. a port-Hamiltonian system representing either master or salve) is interconnected in a passive way (as described in Sec.II-C) to the discrete passive controller (represented by a discrete port-Hamiltonian system, obtained by means of the discretization method described in Sec.II-B). Master and slave will communicate by means of a discrete communication channel. It is possible to define a discrete time scattering. Each discrete time ports (either the master or the slave one) of the discrete communication channel is characterized by and effort $e(k)$ and by a flow $f(k)$. The energy flowing into the system in one sample period is equal to:

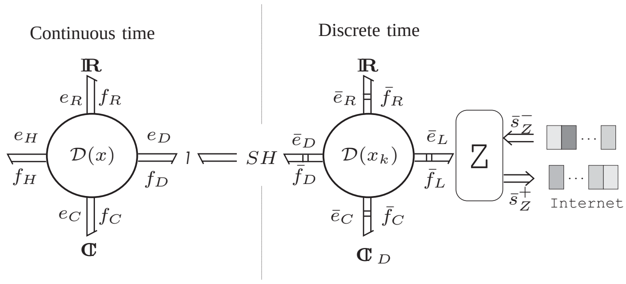


Fig. 2. The Passive Sample Data Telemanipulation Scheme.

$$E_L = T e^T(k) f(k)$$

We can always make the following decomposition of the power flow into an incoming power wave and an outgoing power wave in such a way that:

$$e^T f = \frac{1}{2}\|s_Z^+\|^2 - \frac{1}{2}\|s_Z^-\|^2$$

Integrating in a discrete sense the quantities we get that the energy flow during one sample period is:

$$E_L(k) = T e^T f = \frac{T}{2}\|s_Z^+\|^2 - \frac{T}{2}\|s_Z^-\|^2$$

where T is the sample period and we can interpret $\frac{T}{2}\|s_Z^+\|^2$ and $\frac{T}{2}\|s_Z^-\|^2$ respectively as incoming and an outgoing energy packages.
At each sample time the system will read the incoming energy quantum $\frac{T}{2}s_Z^+(k)$ and the discrete effort $e(k)$ and will calculate the discrete flow $f(k)$ and the discrete energy quantum $\frac{T}{2}s_Z^-(k)$ to transmit through the communication channel.
In [9] the mappings which allow to compute $s_Z^-(k)$ and $f(k)$ from $s_Z^+(k)$ and $e(k)$ are reported. For further details the reader is addressed to [5].

III. QUANTIZATION ERROR ON THE ENCODERS

Encoders are affected by quantization errors that causes a position measurement error which can be modeled by an additive bounded disturbance $w(t)$. Let

$$\|w(t)\| \leq W \quad \forall t$$

where W is a finite positive constant.
We have that:

$$x(t) = \bar{x}(t) + w(t) \tag{5}$$

where $\bar{x}(t)$ and $x(t)$ represent the real value of the position and the measure respectively.

If we use the output of the encoders to calculate $f_d(kT)$ as in Eq.(4), we obtain:

$$\begin{aligned} f_d(k) &= \frac{x(k+1)-x(k)}{T} = \\ &= \bar{f}_d(k) + \frac{w(k+1)-w(k)}{T} = \bar{f}_d(k) + \delta(k) \end{aligned} \tag{6}$$

Because of the quantization error the discrete flow in Eq.(6) is the sum of two terms: the ideal flow ($\bar{f}_d(k)$) and a spurious term ($\delta(k)$). Since $w(\cdot)$ is bounded, the spurious term is bounded as well and we have:

$$\|\delta(k)\| \leq \frac{2W}{T}$$

Let us investigate the energetic behavior of the interconnection during a sample period $[kT,(k+1)T]$. We have that, referring to Fig.1, the continuous energy increment $\Delta E_c(k)$ is:

$$\begin{aligned} \Delta E_c(k) &= E_c((k+1)) - E_c(k) = \\ &= \int_{kT}^{(k+1)T} e_c^T(\tau) f_c(\tau) d\tau = e_d^T(k)\bar{f}_d(k)T = \\ &= \bar{E}_d(k+1) - \bar{E}_d(k) = \Delta\bar{E}_d(k) \end{aligned} \tag{7}$$

On the other hand, the discrete energy increment $\Delta E_d(k)$ is equal to:

$$\begin{aligned} \Delta E_d(k) &= E_d(k+1) - E_d(k) = \\ &= e_d^T(k) f_d(k)T = e_d^T(k)\bar{f}_d(k)T + \\ &+ e_d^T(k)\delta(k)T = \Delta\bar{E}_d(k) + \Delta E_s(k) \end{aligned} \tag{8}$$

Comparing Eq.(7) and Eq.(8), we can see that:

$$\Delta E_d(k) = \Delta E_c(k) + \Delta E_s(k)$$

We have an additional term ($\Delta E_s(k)$) due to the spurious term in $f_d(k)$ and, therefore, there is no more energetic consistency between continuous and discrete domains. This term can lead to production of extra energy in the interconnection and, therefore, to the loss of passivity.

In order to recover passivity in the interconnection, we need to somehow dissipate the extra energy produced during the sampling. We will, therefore, consider the scheme represented in Fig.3 (in a bond graph notation) where we endowed the interconnection with a dissipative element which must be properly designed. Since the spurious term

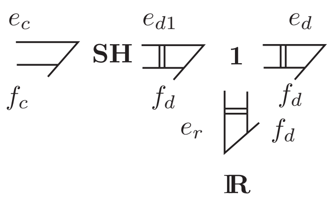


Fig. 3. The sample & hold plus dissipation

$\delta(\cdot)$ is bounded, we can dissipate the maximum amount of energy introduced because of quantization error. Since:

$$\Delta E_s(k) = e_d^T(k)\delta(k)T$$

we have that in the worst case $\Delta E_{sw}(k) = \|e_d^T(k)\frac{2W}{T}T\| = \|e_d^T(k)2W\|$. We can design the dissipative element such that it dissipates E_{sw} at each sample period. We have that some energy can be produced, because of quantization error, in the energetic flow between the port (e_c, f_c) and the port (e_{d1}, f_d). On the other hand, the maximum amount of energy that can be produced is dissipated through the port (e_r, f_d) and, therefore, the energy flowing through the port (e_d, f_d) will be lower or equal than that flowing through (e_c, f_c). This means that the behavior of the interconnection between the continuous port (e_c, f_c) and the discrete port (e_d, f_d) is passive.

This approach is working fine but it is somehow over conservative and it could lead to poor performances. In fact, the quantization error on the encoders does not always cause production of energy but it can also cause dissipation. It is clear that the latter behavior does not affect the passivity of the interconnection. With the proposed approach, instead, the worst case produced energy is always dissipated, disregarding the contribution introduced by the disturbances.

A less conservative scheme for the interconnection can be obtained if we have a measure of the continuous flow f_c; we can get this measure by means of analogic flow sensors (e.g. tachometric dynamos) or of some estimation algorithms such as a state variable filter.

By means of flow measure/estimation we can compute:

$$\Delta E_c(k) = \int_{kT}^{(k+1)T} e_c^T(\tau) f_c(\tau) d\tau$$

and compare it with

$$\Delta E_d(k) = e_d(k)^T f_d(k)T$$

If $\Delta E_d(k) > \Delta E_c(k)$, the quantization error produced some extra energy and the amount $\Delta E_d(k) - \Delta E_c(k)$ must be dissipated. If $\Delta E_d(k) \leq \Delta E_c(k)$ the quantization error led to to some energy dissipation and, therefore, there is no need to activate the dissipative element of the interconnection. The main advantage of this approach with respect to the previous one, is that now we know exactly when there is need of dissipation and the exact amount of energy we need to dissipate and, therefore, we minimally act on the system degrading the performances as less as possible.

On the other hand, even the measure/estimation of the flow can be imprecise and therefore we will have:

$$f_c(t) = \bar{f}_c(t) + n(t)$$

where $n(t)$ represents the error on the measure/estimation and $\bar{f}_c(t)$ the real flow. We can assume that the error is bounded:

$$\|n(t)\| \leq N$$

where N is a finite positive constant. In this case we have that:

$$\begin{aligned} \int_{kT}^{(k+1)T} e_c^T(\tau) f_c(\tau) d\tau = \Delta \bar{E}_c(k) + \\ + e_d^T(k) \int_{kT}^{(k+1)T} n(t) = \Delta \bar{E}_c(k) + \Delta E_n(k) \end{aligned} \tag{9}$$

We have a spurious term $\Delta E_n(k)$ due to the error on the measurement of the flow. This term is bounded and, similarly to $\Delta E_s(k)$ depends on the effort:

$$\|\Delta E_n(k)\| \leq \|e_d^T(k) NT\| := \Delta E_{nw}(k)$$

Because of the uncertainty introduced on the measure of $\Delta E_c(k)$ we can not any longer exactly predict when there has been production of energy. We can write the following algorithm for the design of the dissipative element in the interconnection:

If $\Delta E_d(k) - \Delta E_c(k) > 0$ *then*
dissipate $\Delta E_d(k) - \Delta E_c(k) + \Delta E_{nw}(k)$
else
dissipate $\Delta E_{nw}(k)$

We always need to dissipate ΔE_{nw} to be assured that there is no production of energy.

Remark 1: The algorithm gets less and less conservative the more the flow measure is reliable. Notice that if ΔE_{nw} is bigger than ΔE_{sw}, the proposed algorithm is over conservative with respect to the constant dissipation of ΔE_{sw}. We have to choose which algorithm to use depending on the reliability of flow measure/estimation.

IV. Passive interpolation of lost packets

The aim of this section is to build a redundant communication channel in order to be able to replace the packets missed in the communication by interpolated packets. We will start from the discrete-time scattering based communication channel reported in Sec.II-D. The transmitted information represents a power wave, therefore, the packets are not totally uncorrelated and it does make sense to obtain the missed packets by interpolating the received ones. Owing to keep the method simple, we focus on linear interpolation algorithm in our control strategy.

When we produce an interpolated packet we replace the energy content of the missed packet with the one of the interpolated one. If the energy associated to the interpolated packet is greater than the one associated to the lost one, the interpolation process introduces some extra energy in the communication line. The interpolation process, therefore, must be carefully addressed in order to preserve the passivity.

By means of loss rate and other statistical indicators deriving from an analysis of the communication channel, we know which is, in average, the maximum number of consecutive packets that can be lost in the communication and, consequently, the maximum number of consecutive packets that have to be interpolated. Let assume that this number is n. We will endow the communication channel with transmission and receiving buffers.

In order to avoid energy production in the interpolation process, we need to know the maximum amount of energy that can be used to perform the eventual interpolation of missed packets. The aim of the transmission buffer is to endow each packet with this extra information: the total energy E^a of the next n packets that will be subsequently transmitted. The transmission buffer, therefore, will introduce an extra delay on $n+1$ sample periods in order to endow each packet with the extra information required.

The interpolation process will be performed in the receiving buffer and, since the maximum number of packets to be interpolated is n, its dimension will be, therefore, $n+2$ and it will introduce a delay of $n+2$ sample periods. Assume that we lose n packets, namely we receive nothing between $t = (k-n-1)T$ and $t = (k-1)T$. At $t = kT$ we can perform the interpolation between the received packets $s^+(k-n-2)$ and $s^+(k)$ (both present in the receiving buffer) to obtain the lost packets. In order to avoid any extra energy production, we use the information $E^a(k-n-2)$ embedded in the packet received in $t = (k-n-2)T$. We calculate the maximum energetic content ϵ that each packet can have by:

$$\epsilon = \frac{E^a(k-n-2)}{n}$$

Then we tune each interpolated packet in order to meet the energetic constraints.

Remark 2: Since there is not any information on the distribution of the energy among the lost packets, we fix the same energy bound for each packet to interpolate.

We can write the following algorithm:

1) Read $E^a(k-n-2)$, the energy available for the interpolation
2) Calculate the energetic content of each interpolated energy quantum: $\epsilon = \frac{E^a(k-n-2)}{n}$
3) By linear interpolation between $s^+(k)$ and $s^+(k-n-2)$ obtain $\bar{s}_I^+(i)$, with $i \in [k-n-1, k-1]$.
4) Obtain the packets to replace the missed ones, keeping into account energetic constraints:
   a) if $\frac{1}{2}\|\bar{s}_I^+(i)\|^2 T > \epsilon$ then choose $\alpha = \sqrt{\frac{2\epsilon}{\|\bar{s}_I^+(i)\|^2 T}}$

else
$\alpha = 1$
end

b) $s_I^+(i) = \alpha \bar{s}_I^+(i)$

The total amount of energy E_I of the interpolated packets is:

$$E_I = \sum_{j=k-n-1}^{k-1} \frac{1}{2}\|s_I^+(j)\|^2 \leq E^a = \sum_{j=k-n-1}^{k-1} \frac{1}{2}\|s^+(j)\|^2$$

The energy introduced in the communication channel by the interpolation process is bounded by the energy content of the lost packets; no extra energy is introduced and, therefore, passivity is, intutively, preserved.

Remark 3: If $m < n$ packets get lost it is possible to recover from the energy content of the $m - n$ received packets the energy available for the interpolation of the missed packets. If $p > n$ packets get lost, the interpolation algorithm fails and nothing is done to replace the lost packets; passivity is preserved and, in this case, the communication channel dissipates the energy associated to the missed packets.

The communication channel we are using for reliable telemanipulation is represented in Fig.4.

TX_m and TX_s represents the transmission buffer at

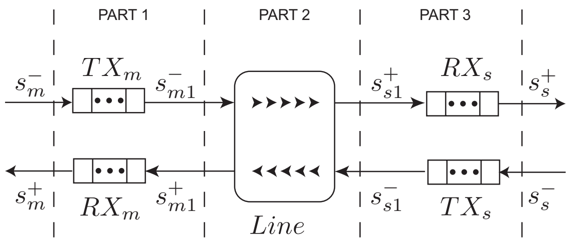


Fig. 4. The Communication Channel

master and slave side respectively and RX_m and RX_s the receiving buffers at master and slave side respectively. Let δ_{ms} and δ_{sm} the transmission delays between master and slave and slave and master respectively. Moreover, let δ_{Tm} and δ_{Ts} be the delays introduced by the transmission buffer at master and slave side respectively and δ_{Rm} and δ_{Rs} the delays introduced by the receiving buffer at master and slave side respectively.

We will formally prove that the reliable channel proposed is passive. Suppose that 1 packet gets lost in the transmission between master and slave. We will use the following notation for the discrete derivative and the discrete integral:

$$dg(k) = \frac{g(k+1) - g(k)}{T} \quad I_h^k g = \sum_{i=h}^{k-1} g(i)T$$

where g is a generic sequence.

The communication channel can be divided in three parts as shown in Fig.4. Consider the first part of the communication channel; we have that the power flow is:

$$P_1(k) = \tfrac{1}{2}\|s_m^-(k)\|^2 - \tfrac{1}{2}\|s_{m1}^-(k)\|^2 + \tfrac{1}{2}\|s_{m1}^+(k)\|^2 - \\ -\tfrac{1}{2}\|s_m^+(k)\|^2 \tag{10}$$

Since there is no loss of packets between slave an master, we can write:

$$\begin{cases} s_{m1}^-(k) = s_m^-(k - \delta_{Tm}) \\ s_m^+(k) = s_{m1}^+(k - \delta_{Rm}) \end{cases}$$

and, therefore, if we pose:

$$E_1(k) := [I_{k-\delta_{Rm}}^k(\frac{1}{2}\|(s_{m1}^+)\|^2) + I_{k-\delta_{Tm}}^k(\frac{1}{2}\|(s_m^-)\|^2)]$$

we have that:

$$P_1(k) = dE_1(k)$$

and therefore the first part of the communication channel is lossless.

Let us now consider the second part of the communication channel; we have that:

$$P_2(k) = \tfrac{1}{2}\|s_{m1}^-(k)\|^2 - \tfrac{1}{2}\|s_{s1}^+(k)\|^2 + \tfrac{1}{2}\|s_{s1}^-(k)\|^2 - \\ -\tfrac{1}{2}\|s_{m1}^+(k)\|^2 \tag{11}$$

It can be proven ([5]) that when a packet gets lost the channel becomes dissipative instead of lossless. Suppose that we do not receive anything at the slave side at time $t = kT$. Letting:

$$E_2(k) := [I_{k-\delta_{ms}}^k(\frac{1}{2}\|(s_{m1}^-)\|^2) + I_{k-\delta_{sm}}^k(\frac{1}{2}\|(s_{s1}^-)\|^2)]$$

we have that

$$P_2(k) = dE_2(k) + P_{d2}(k)$$

We have a dissipative behavior and

$$P_{d2}(k) = \frac{1}{2}\|s_{m1}^-(k - \delta_{ms})\|^2 > 0$$

and we dissipate an energy amount equal to:

$$E_{d2}(k) = \frac{1}{2}\|s_{m1}^-(k - \delta_{ms})\|^2 T$$

Let us now consider the third part of the communication channel. Since one packet is lost in the transmission between master and slave, suppose nothing is received at $t = kT$ at the slave side. We have that the power $P_3(k)$ flowing in the third part of the communication channel is:

$$P_3(k) = -\frac{1}{2}\|s_s^+(k)\|^2 + \frac{1}{2}\|s_s^-(k)\|^2 - \frac{1}{2}\|s_{s1}^-(k)\|^2$$

We can write:

$$P_3(k) = \frac{1}{2}\|s_{s1}^+(k)\|^2 - \frac{1}{2}\|s_s^+(k)\|^2 + \frac{1}{2}\|s_s^-(k)\|^2 -$$

$$-\frac{1}{2}\|s_{s1}^-(k)\|^2 - \frac{1}{2}\|s_{s1}^+(k)\|^2 = dE_3(k) - \frac{1}{2}\|s_{s1}^+(k)\|^2$$

where

$$E_3(k) := d[I_{k-\delta_{Rs}}^k(\frac{1}{2}\|(s_{s1}^+)\|^2) + I_{k-\delta_{Ts}}^k\frac{1}{2}\|(s_s^-)\|^2)]$$

We will have, therefore, an energy production at $T = kT$:

$$E_{p3} = P_{p3}(k)T = \frac{1}{2}\|s_{s1}^+(k)\|^2 T$$

At time $t = (k + \delta_{Rs})T = hT$ the lost packet is replaced with the interpolated one and, therefore, instead of giving out $s_s^+ = 0$ for the missed packet I will give out $s_s^+ = s_{sI}^+$. When s_s^+ is replaced with an interpolated packet I have that:

$$P_3(h) = \frac{1}{2}\|s_{s1}^+(h)\|^2 - \frac{1}{2}\|s_{sI}^+(h)\|^2 + \frac{1}{2}\|s_s^-(h)\|^2 -$$

$$-\frac{1}{2}\|s_{s1}^2(h)\|$$

We can always write:

$$P_3(h) = \frac{1}{2}\|s_{s1}^+(h)\|^2 - \frac{1}{2}\|s_s^+(h)\|^2 + \frac{1}{2}\|s_s^-(h)\|^2$$

$$-\frac{1}{2}\|s_{s1}^2(h)\| + \frac{1}{2}\|s_s^+(h)\|^2 - \frac{1}{2}\|s_{sI}^+(h)\|^2$$

and, therefore,

$$P_3(h) = dE_3(h) + \frac{1}{2}\|s_s^+(h)\|^2 - \frac{1}{2}\|s_{sI}^+(h)\|^2 =$$

$$= dE_3(h) + P_{d3}(h)$$

By construction $P_{d3} > 0$ and, therefore, the interpolation algorithm introduces dissipation into the system.

Now, we will put all the parts of the communication channel together in order to prove the passivity of the overall transmission line.

We have that the power flowing through the communication channel is:

$$P(k) = \frac{1}{2}\|s_m^-(k)\|^2 - \frac{1}{2}\|s_s^+(k)\|^2 + \frac{1}{2}\|s_s^-(k)^2\| -$$

$$-\frac{1}{2}\|s_m^+(k)\|^2 = P_1(k) + P_2(k) + P_3(k) \tag{12}$$

When a packet is lost in the communication between master and slave, we have that:

$$P(k) = d(E_1(k) + E_2(k) + E_3(k)) + P_{d2}(k) - P_{p3}(k) =$$

$$= dE(k) + P_{d2}(k) - P_{p3}(k)$$

Since $P_{d2}(k) = P_{p3}(k)$ we have that

$$P(k) = dE(k)$$

and therefore the channel has a lossless behavior. At time $t = hT = (k + \delta_{Rs})T$ we have that:

$$P(h) = dE(h) + P_{d3}(h)$$

We can therefore conclude that the loss of a packet in the transmission line introduces a dissipative behavior in the communication channel. This result can be easily generalized both in case more than one packet is lost and in case the loss is in the communication between slave and master.

We can therefore state the following:

Proposition 1: The interpolated communication channel is passive.

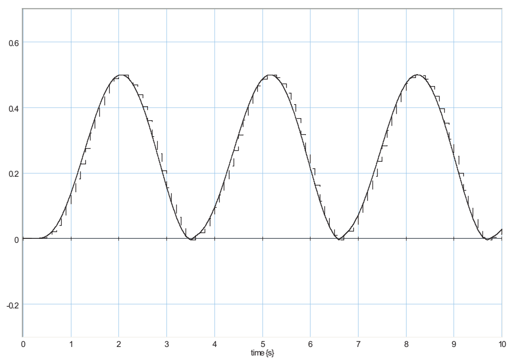
Remark 4: Both in case of interpolated and non interpolated communication channel, we obtain a dissipative communication channel. The improvement of performances obtained by means of the interpolation relies in the fact that the dissipated energy is much lower. This brings to a communication channel closer to the ideal case (i.e. losslessness) and therefore to better performances.

Remark 5: Both transmission and reception buffers introduce some extra delays necessary to endow packets with energetic information and to implement the interpolation. If the maximum number of possibly consecutive missed packet is quite small, the delay introduced is not so big and performances improvement is worth of it. In case bigger delays have to be introduced, it could be better, depending on the application, to maintain the dissipative behavior due to loss of packets rather than introducing a big extra delay.

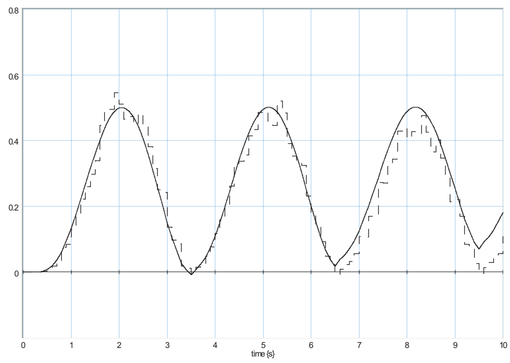
V. SIMULATIONS

The aim of this section is to provide some simulations in order to validate our results.

The first simulations are intended to validate the passive interconnection proposed in Sec.III. We connected a simple mass with an initial state $p = 1Kgm/sec$ connected to a discrete spring, obtained by discretizing a continuous spring by means of the algorithm proposed in Sec.II-B; the sample period is $T = 0.1sec$. In Fig.5 we can see the energetic behavior at the interconnection both in case of ideal encoder and when there is a quantization error with maximum amplitude of 0.05 In Fig.6 we can see the energetic behaviors in case compensated interconnection is activated. Picture (a) shows the energetic behavior in case the worst case produced energy is dissipated. We can see that discrete energy is always lower than continuous one and, therefore, passivity has been recovered despite of quantization noise on the encoders. Nonetheless, the energetic behavior is quite different from the ideal one. In picture (b) we can see the energetic behavior in the compensated interconnection in case the information of flow sensors (or estimation algorithms) is unreliable (we simulated an unreliability with maximum amplitude of 0.01). We can see that the behavior is passive closer to the ideal case than the one we have in case we dissipate the worst case produced

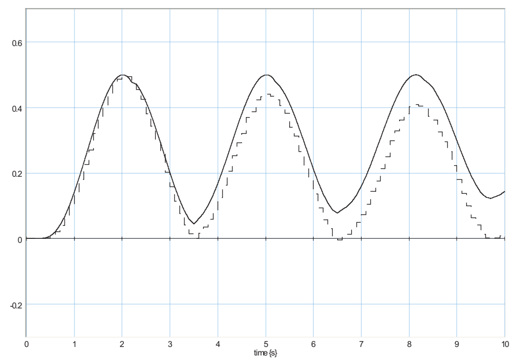


(a) Ideal energetic behavior

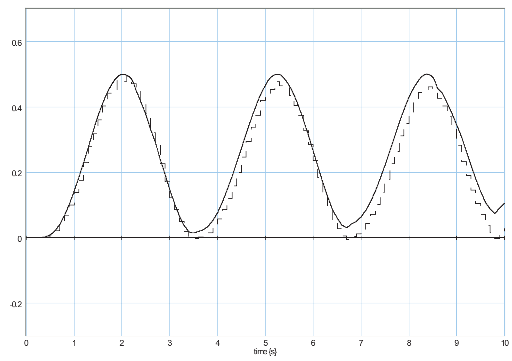


(b) Energetic behavior with quantization noise

Fig. 5. Energetic behavior in the ideal and in the noisy case. Continuous Energy (solid) and Discrete Energy (dashed)

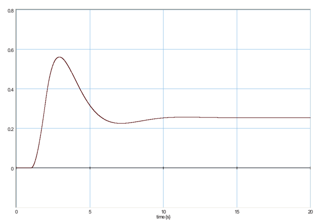


(a) Worst Case produced energy dissipation

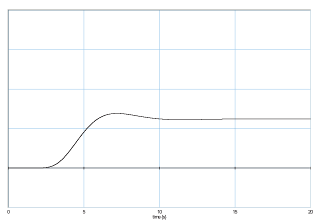


(b) Using Unreliable flow sensors

Fig. 6. Compensations. Continuous (solid) and discrete (dashed) energy



(a) Master Position



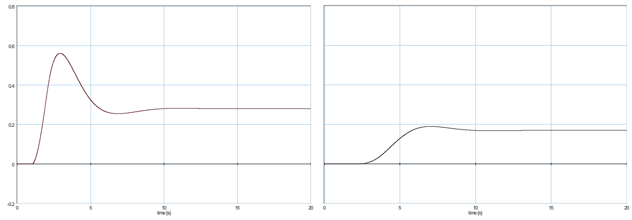
(b) Slave Position

Fig. 7. Positions of Master and Slave

energy.
In order to show the effectiveness of the interpolation algorithm, we simulated a simple 1-dof telemanipulation system. The master and the slave are simple masses and they are controlled by a discrete IPC and they communicate by means of a packet switching communication channel. The transmission delay is 0.5 seconds and the sample time is $T = 5msec$.

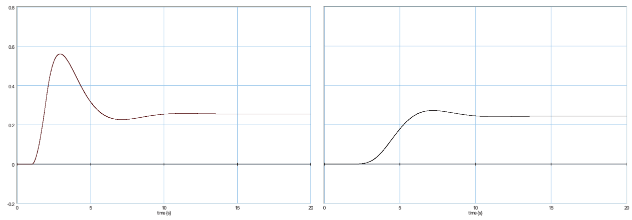
In Fig.7 is represented the behavior of the system in case there is no loss of packets during the transmission. We can see that the slave follows the master after a certain delay.
Next we simulated a loss of packets in the communication between master and slave. The loss rate is 30%, namely one packet is lost per each three packets transmitted.
In Fig.8 the behavior of the system is shown. We notice that the behavior of the system is stable since the loss of packets implies a dissipation of energy within the communication channel and, therefore, the passivity of the overall system is not compromised. Nevertheless the performances are affected by unreliability of the transmission line. In fact, we can see that the slave position is quite different from the master one. This happens because a lot packets are not delivered to the slave and, therefore, the remote robot does not receive enough energy to perform the task.

We can improve the performances of the telemanipulation system by introducing the interpolation scheme illustrated in Sec.IV. In Fig.9 we can see the behavior of the system in case there is loss of packets in the communication between master and slave and the interpolation algorithm is enabled. We can see that the performances of the systems increase and that the position of the slave is much closer than in the case no interpolation was performed. Furthermore, the passivity of the overall system is preserved by construction of the interpolation algorithm.



(a) Master Position (b) Slave Position

Fig. 8. Positions of Master and Slave in case of loss of packets



(a) Master Position (b) Slave Position

Fig. 9. Positions of Master and Slave in case of loss of packets and interpolation algorithm enabled

VI. Conclusions and Future Work

In this paper we have shown how it is possible to modify the intrinsically passive telemanipulation scheme proposed in [7], [5] in order to take into account quantization error on the encoders needed to perform a passive interconnection between continuous and discrete domain and to consider an interpolation algorithm to obtain the packets missed in the transmission. We recovered the passivity of the interconnection by dissipating the extra energy produced because of the quantization noise on the encoders. By means of flow measure/estimation, it has been possible to obtain a less conservative behavior of the interconnection. We proposed an interpolation scheme to interpolate missed packets preserving the passivity of the communication scheme. Performances are increased but some extra delay has been added to the communication.

Future work will be devoted to the study of other interpolation algorithms in order to be able to improve performances introducing no delay. From a more practical point of view, we would like to implement on a real setup the proposed scheme.

Acknowledgments

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