Abstract: This paper presents a Delayed Generalized Predictive Controller (DGPC) by internal model for bilateral teleoperation systems in the presence of both communication delays and force feedback. First, the II-freeness algebraic property of mechanical delayed systems, which presents some similarity with the notion of flatness, is used for the slave system to track a master system reference trajectory; the DGPC next ensures stability of the global slave system in spite of delays, disturbances or the slave force feedback (the reference trajectory is not a priori known). Simulation results illustrate the performance of the proposed predictive polynomial controller. Copyright ⓒ 2005 IFAC

Keywords: Bilateral teleoperation systems, Delayed Generalized Predictive Control, Internal model control, Delayed linear systems, II-freeness.

1. INTRODUCTION

Telerobotic systems were developed in order to allow human operators to perform specific tasks in remote, unknown or dangerous environments such as space, undersea or nuclear plants. For bilateral tele-operation systems, human operator controls, set at the master station, are sent through a communication network and then carried out by the remote robot manipulator located at the slave station. The remote manipulator is in direct contact with its external environment, and then, reflects the contact forces to the human operator through a master control device such as a joystick. It was shown that this force feedback considerably improves the teleoperation performances. Unfortunately, the use of a communication network irremediably induces transmission delays that can degrade or even destabilize the closed-loop global system.

To overcome this problem, various solutions have been proposed in the literature: the constant delay case was treated in (Anderson, 1989) with a wave variable approach, the Lyapunov-Krasovski approach was used in (Oboe, 1998) to design a simple control law by pole placement. In (Cho, 2001), the authors have introduced a sliding-mode-based impedance controller. The robust control via $\mu$-synthesis was analyzed in (Leung, 1995) and a $H_\infty$-based impedance controller was given in (Fattouh, 2003). An analysis of the closed-loop stability by using a frequency-domain approach was given in (Niculescu, 2003).

This paper present an alternative predictive polynomial controller for bilateral teleoperation systems. The objective is to develop a control law for the slave system to track the master system trajectory in presence of large and poorly known communication delays and slave force feedback. Our approach may be decomposed in two steps: an open-loop control law design based on the II-
freeness algebraic property (Fliess, 1998) which allows the slave system to track the master system reference trajectory, and a stabilization of the slave system around the master system reference trajectory with a predictive polynomial control using the ideas exhibited in (Boucher, 1996). Contrary to the latter work, our approach is based on the Internal Model Structure (Morari, 1983) to take into account slave model errors. Moreover, the slave force feedback alters the master trajectory dynamics, which is not a priori known, but depends on the II-free output.

The paper is organized as follows: section 2 presents the problem formulation, and section 3 outlines the open-loop control strategy based on the II-freeness algebraic property of the slave system. In section 4, we develop the DGPC stabilization of the slave system around the reference trajectory. Finally, simulations are reported in section 5 to illustrate our approach for the control of bilateral teleoperation systems.

2. PROBLEM FORMULATION

In this paper, the one degree-of-freedom (DOF) master control system and the one DOF remote robot manipulator are modelled by second order linear systems (see figure 1):

![Diagram](image)

Fig. 1. Master, slave and external environment mechanical models

The master and slave systems are given by:

\[ M_m \ddot{x}_m(t) + B_m \dot{x}_m(t) + K_m x_m(t) = F_h(t) - F_c(t - h_{ms}) \]  
(1)

\[ M_s \ddot{x}_s(t) + B_s \dot{x}_s(t) + K_s x_s(t) = u_s(t - h_{ms}) - F_c(t) \]  
(2)

with the external environment forces:

\[ F_c(t) = B_c \dot{x}_s(t) + K_c x_s(t) \]  
(3)

\[ x_m(t) \text{ and } x_s(t) \] are the positions of the master system and of the slave robot manipulator, respectively. \( F_h(t) \) is the applied human operator forces on the master system, \( u_s(t) \) is the remote control of the robot manipulator, \( h_{ms}, h_{sm} \in \mathbb{R}^+ \) are the communication delays from the master to the slave system and from the slave to the master system, respectively. \( M, B \) and \( K \) represent the mass (inertia), the damping coefficient and the stiffness gain, respectively. The position of the robot manipulator is measured in order to deduce the force to apply on the environment.

At the station slave, the impedance of the manipulator is \( Z_s = M_s s^2 + B_s s + K_s \), and the impedance of the environment is \( Z_e = B_e s + K_e \). The manipulator is in direct contact with the external environment (see figure 1), then the slave system global impedance is considered as \( Z_{se} = M_s s^2 + B_{se} s + K_{se} \) with \( B_{se} = B_s + B_e \), \( K_{se} = K_s + K_e \). Therefore, the global model of the slave system is described by:

\[ M_s \ddot{x}_s(t) + B_{se} \dot{x}_s(t) + K_{se} x_s(t) = u_s(t - h_{ms}) \]  
(4)

The remote controller directly controls the global slave system. The objective of this controller is to impose the master system behavior on the global slave system by taking into account both the transmission delays and the slave force feedback which alters the dynamics of the master system trajectory. The proposed control is based on two steps: an open loop control which allows to track the reference trajectory (master) and a stabilization of the global slave system around the desired trajectory using a predictive polynomial control.

3. OPEN-LOOP CONTROL STRATEGY

The open-loop control strategy is based on the II-freeness algebraic property (Fliess, 1998) which is an interesting property for the tracking control of linear time-delay systems. This property, which presents a similarity with the flatness concept (Fliess, 1995), allows an explicit parametrization of all trajectories via a finite set of variables, called II-free outputs, and their successive derivatives. An open-loop control law is designed for the slave system based on the II-freeness in order to track the master reference. This open loop control is based on the prediction of the master system trajectory.

Rigid robot manipulators have a number of actuators equal to the number of joint variables. The model is invertible and the input torque of the system is expressed according to the joint variable and its successive derivatives. These mechanical systems are flat with the joint variable flat output. For bilateral teleoperation systems in the presence of communication delays, we use the II-freeness algebraic property that allows to characterize all the dynamic of the system through the dynamic behaviour of its II-free outputs.

From (4), the state-space model can be written on the ring \( \mathbb{R}[\frac{d}{\tau}, \nabla] \) where \( \frac{d}{\tau} \) is the derivation operator and \( \nabla \) is the delay operator (\( \nabla v(t) = v(t - h) \)).
\[ \dot{X}_s(t) = A_{sc1}(\nabla)X_s(t) + B_{sc1}(\nabla)u_s(t) \quad (5) \]
\[ y_s(t) = C_{sc1}(\nabla)X_s(t) \quad (6) \]

where \( X_s(t) \in \mathbb{R}^2 \), \( u_s(t) \in \mathbb{R} \), \( y_s(t) \in \mathbb{R} \). The matrices \( A_{sc1}(\nabla) \in k[\nabla]^{2 \times 2}, B_{sc1}(\nabla) \in k[\nabla]^{2 \times 1} \) and \( C_{sc1}(\nabla) \in k[\nabla]^{1 \times 2} \) can be easily determined.

The controllability of the system with delays on the ring \( \mathbb{R}[\frac{1}{\nabla}, \nabla] \) is studied in an algebraic way. The module \( \Lambda \) is associated to it, so the system is controllable if and only if the module is free (reachable). For the system (5)-(6), the module \( \Lambda \) is not free because the reachability criteria is not verified, i.e. rank \( A_{sc1}(z)/B_{sc1}(z) > 0 \) for \( z = 0 \). But the module is rather weakly controllable because rank \( A_{sc1}(\nabla)/B_{sc1}(\nabla) > 2 \).

Then, the system (5)-(6) is not free but is weakly controllable. This system is called \( \Pi \)-free with a basis \( x_s \) (Fiess, 1998). Then, the open-loop control law is obtained allowing to track a reference trajectory \( x_{s,ref} \) of the global slave system with \( x_{s,ref} = x_m \):

\[ u_{s,ref}(t) = M_l\dot{x}_m(t + h_m) + B_{sc1}\dot{x}_m(t + h_m) + K_{sc1}x_m(t + h_m) \quad (7) \]

For the open loop control (7) of the global slave system, the prediction of the master system trajectory is needed at time \( t + h_m \). In the following, we suppose that delays are constant and equal: \( h_m = h_m = h \). Note that constant time-delay may be obtained from time-varying or random delay by using queuing mechanisms (First-in-First-out). Furthermore, the operator force \( F_h \) is assumed to be constant on prediction horizon and is updated at each sampling time. We will show that this condition is not restrictive through the numerical simulations presented in sections 5.

System (1) is also weakly controllable but it is not reachable, this system is \( \Pi \)-free too with a base \( x_m \). Therefore:

\[ M_m\dot{x}_m(t + h) + B_m\dot{x}_m(t + h) + K_mx_m(t + h) = F_h(t + h) - F_c(t) \quad (8) \]

Note that the slave force feedback \( F_c(t - h) \) must be calculated at time instant \( t \) (\( F_c(t) \) where the bar stands for calculated variables) in order to predict the master system trajectory. From the state delayed measurements of the global slave system (i.e with initial conditions \( X_s(t - h) = [x_s(t - h), \dot{x}_s(t - h)]^T \)), we calculate the general solution of the equation (5) at time instant \( t \).

\[ \dot{X}_s(t) = e^{A_{sc1}h}X_s(t - h) + \int_{t - h}^{t} e^{A_{sc1}(t - \nu)}B_{sc1}u_s(\nu - h) \, d\nu \quad (9) \]

So, the exterior environment force \( F_c(t) \) is deducted with (3):

\[ \dot{F}_c(t) = B_s\dot{x}_s(t) + K_c\ddot{x}_s(t) \quad (10) \]

Then, the open loop control (7) allows to impose the master system behavior on the global slave system and to transform it into the equivalent double integrator system:

\[ y_s(t + h) = w_s(t + h) \quad (11) \]

where \( w_s \) is the new control and \( y_s = x_s \) is the \( \Pi \)-free output.

4. STABILIZATION WITH A DELAYED GENERALIZED PREDICTIVE CONTROL

The open-loop control (7) is the reference control that allows the slave system to track the master system trajectory without taking into account the model errors, prediction errors or the disturbances. To ensure stability of the global slave system around the desired trajectory, we use the delayed generalized predictive control approach (DGPC) introduced in (Gomma, 1998).

Generalized predictive control (GPC), suggested by (Clarke, 1987), is based on the minimization of a quadratic cost function including a sequence of future inputs. The setting of the generalized predictive control requires the definition of a numerical model of the system like the CARIMA structure (Combined Auto-Regressorive and Integrated Moving Average):

\[ A(z^{-1})y(k) = B(z^{-1})u(k - 1) + v(k) \quad (12) \]

with,

\[ A(z^{-1}) = 1 + a_1z^{-1} + a_2z^{-2} + ... + a_\nu z^{-\nu} \]
\[ B(z^{-1}) = z^{-d}(b_0 + b_1z^{-1} + ... + b_\mu z^{-\mu}) \]
\[ C(z^{-1}) = 1 + c_1z^{-1} + c_2z^{-2} + ... + c_\nu z^{-\nu} \]

where \( y(k) \) and \( u(k) \) are the output and the control of the system, respectively. The disturbance term \( v(k) = G(z^{-1})\xi(k) \) is in the moving average form. \( \Delta(z^{-1}) = 1 - z^{-1} \) is the differencing operator and \( \xi(k) \) is an uncorrelated random sequence. \( A, B \) and \( C \) are polynomials in the backward operator \( z^{-1} \). The parameter \( d \in \mathbb{N} \) is the pure system delay.

Using the operator \( \Delta(z^{-1}) \) in (12), we have:

\[ A(z^{-1})\Delta y(k) = B(z^{-1})\Delta u(k - 1) + C(z^{-1})\xi(k) \quad (13) \]

This equation describes the evolution of the system output variation with respect of the input variation between two successive sampling time. Hereafter, \( z^{-1} \) is omitted for clarity.
For bilateral teleoperation systems, the slave force feedback modifies the master system dynamics, and the slave reference trajectory is not a priori known but depends on the II-free slave output. Then, the master model can be written according to the slave system control in order to solve the minimization problem. On the other hand, using the traditional GPC in the bilateral teleoperation systems, the control signals do not depend on the past output values. The DGPC allows to overcome this point (Gomma, 1998) by taking account delayed output values. Let us introduce the following notations: \( \tilde{y}_s(k) = y_s(k - h) \), \( \xi_s(k) = \xi_s(k - h) \), \( \tilde{y}_{m,ref}(k) = y_{m,ref}(k - 1) \) and \( \xi_m(k) = \xi_m(k - 1) \). The slave and master models (11)-(1) can be transformed to the CARIMA form:

\[
A_{se2} \Delta \tilde{y}_s(k) = B_{se2} \Delta w_s(k - 1) + C_{se} \xi_s(k) \quad (14)
\]

\[
A_m \Delta \tilde{y}_{m,ref}(k) = B_m \Delta u_m(k - 1) + C_m \xi_m(k) \quad (15)
\]

with \( u_m(k - 1) = F_h(k - 1) - z^{-h}F_c(k - 1) \), where the subscript \( s \), \( m,ref \) stand for the slave system, and the master reference system. The polynomials \( A_{se2} \) (resp. \( A_m \)) and \( B_{se2} \) (resp. \( B_m \)) are obtained from the discretization of (11) (resp. (1)).

The external force (3) is going with respect to the II-free slave output and the operator force \( F_h \) is always assumed constant. The master model according to the slave system control can be then determined. Equation (15) can be written as:

\[
A_m \Delta \tilde{y}_{m,ref}(k) = -B_m (K_c \Delta y_s(k - h - 1) + B_s \Delta v_s(k - h - 1) + C_m \xi_m(k) \quad (16)
\]

where \( v_s \) is the velocity. From (14), we determine \( \Delta y_s(k - h - 1) \) which depends on the noise at time \( k - h - 1 \). As the noise \( \xi_s(k - h - 1) \) is, by definition, independent of the measurement signal at time \( k - h \), equation (17) is obtained. The transfer function of the global system slave is expressed between the velocity \( v_s(k - h - 1) \) and the input \( w_s(k - 1) \), then \( \Delta y_s(k - h - 1) \) is expressed in (18).

\[
\Delta y_s(k - h - 1) = A_{se2}^{-1} B_{se2} \Delta w_s(k - 2) \quad (17)
\]

\[
\Delta v_s(k - h - 1) = A_{se2}^{-1} B_{se2} \Delta w_s(k - 2) \quad (18)
\]

Thus, the master model depends of the slave control:

\[
A_{sem} \Delta \tilde{y}_{m,ref}(k) = -B_{mex} \Delta w_s(k - 1) + C_{sem} \xi_m(k) \quad (19)
\]

with \( A_{sem} = A_{se2} A_m \), \( C_{sem} = A_{se2} C_m \) and \( B_{mex} = B_m \times (K_c B_{se2} + B_s B_{se2}) z^{-1} \).

The structure of the predictive control by internal model is used to take into account the model errors of the global slave system (4) and to reject the additive disturbances on system outputs. The slave plant output \( y_p(k) \) is determined from the global slave model output and the error signal, \( y_p(k) = y_s(k) + e(k) \) (see figure 2). This structure of the CMI constitutes the characteristic of an integrator type controller.

For the slave model (14) and for the master model (19), the following minimization problem is solved:

\[
J(\tilde{\tilde{w}}, k) = \frac{1}{2} \left( \sum_{j=H_u}^{H_p} \| \Delta w_s(k + j - H_w) \|^2_R(j) + \sum_{j=H_u}^{H_p} \| y_p(k + j, \tilde{\tilde{w}}) - y_{m,ref}(k + j, \tilde{\tilde{w}}) \|^2_Q(j) \right) \quad (20)
\]

where \( H_p, H_w \) are the prediction horizon and the initial horizon, respectively, \( H_u \) is the control horizon with \( H_u < H_p \) and \( \Delta w_s(k + j) = 0 \), \( \forall j \geq H_u \). \( Q(j) \geq 0 \) and \( R(j) > 0 \) are the diagonal elements of the weighting matrices \( Q \) and \( R \). \( \tilde{\tilde{w}} \) is a sequence of future controls with \( \tilde{\tilde{w}} = [\Delta w_s(k)...\Delta w_s(k + H_u - 1)]^T \).

The objective is to determine the control sequence \( \tilde{\tilde{w}} \) minimizing the quadratic error between the future predictions of the master system output and of the slave system output. Future values of the slave plant output \( y_p(j), j \in H_p \), are estimated at time instant \( k - h \). The error \( e(k - h) \) is considered constant on the horizon of prediction and updated at each sampling period (Morari, 1983). We thus obtain \( y_p(j) = y_s(j) + e(k - h) \) with \( j \in H_p \).

The polynomials \( C_m \) and \( C_{se2} \) are chosen to be equal to 1 in order to simplify the algorithm, and thus \( C_{m1}^{-1} \) and \( C_{se2}^{-1} \) are absorbed into the polynomials \( A \) and \( B \) (Clarke, 1987). It means that noises will be considered like random step. To calculate the predictions of the outputs \( \tilde{y}_{m,ref} \) and \( \tilde{y}_s \), two Diophantine equations for each model were solved. For the master model (19), we consider the Diophantine equation (21) with the unknown couple of polynomials \( (E_{mj}, F_{mj}) \) and the Diophantine equations (22) in \( (G_{mj}, H_{mj}) \):

\[
C_{sem} = E_{mj} A_{sem} \Delta + z^{-j} F_{mj} \quad (21)
\]

\[
E_{mj} B_{mex} = C_{sem} G_{mj} + z^{-j} H_{mj} \quad (22)
\]

Likewise, for the slave model (14):

\[
1 = E_{sj} A_{se2} \Delta + z^{-j} F_{sj} \quad (23)
\]

\[
E_{sj} B_{se2} = G_{sj} + z^{-j} H_{sj} \quad (24)
\]

where \( E_j(z^{-1}), F_j(z^{-1}), G_j(z^{-1}) \) and \( H_j(z^{-1}) \) are real polynomials with degrees \( (j - 1) \), \( n_{j}^{(f)} = \max(n_a, n_c - j), (j - 1) \) and \( n_h^{(j)} = \max(n_c, n_b + h - 1, \max(n_a, n_c - j), (j - 1) \).
By combining (23), (24) and (14), we obtain the prediction of the slave model output:

\[
\hat{y}_s(k + j + h) = G_{m_{s,i}}\Delta w_s(k + j + h - 1) + H_{s_{j,i}}\Delta w_s(k - 1) + F_{s_{j,i+h}}\hat{y}_s(k)
\]  

(25)

Similarly, for the prediction of the master model output with (21), (22) and (19):

\[
\hat{y}_{m_{ref}}(k + j + h) = -G_{m_{j,i}}\Delta w_s(k + j) + C_{sem}^{-1}[F_{m_{j,i}}\hat{y}_{m_{ref}}(k) - H_{m_{j,i}}\Delta w_s(k - 1)]
\]  

(26)

Next, by substituting \(\hat{y}_s(k) = y_s(k - h)\) in (25) and \(\hat{y}_{m_{ref}}(k) = y_{m_{ref}}(k - 1)\) in (26) we have the matrix-vector form:

\[
Y_{m_{ref}} = -G_m\hat{w}_s + I_1
\]

(27)

\[
Y_s = G_s\hat{w}_s + I_2
\]

(28)

where \(Y_{m_{ref}}, Y_s \in \mathbb{R}^{(H_p - H_w + 1) \times 1}\) are the predicted output vectors, \(I_1, I_2 \in \mathbb{R}^{(H_p - H_w + 1) \times 1}\) include all past values, and the matrices \(G_m, G_s \in \mathbb{R}^{(H_p - H_w + 1) \times H_s}\).

Then, the sequence of optimal control \(\hat{w}_s\) is determined by minimizing the quadratic cost function (20):

\[
\hat{w}_{s,\text{opt}} = K_{opt}[C_{sem}^{-1}F_mY_{m_{ref}}(k - 1) - (C_{sem}^{-1}H_m + H_s)\Delta w_s(k - 1) - F_s\hat{y}_s(k - h) - E(k - h)]
\]

(29)

where \(F_i = [F_{i,H_u} ... F_{i,H_w}]^T\), \(H_i = [H_{i,H_u} ... H_{i,H_w}]^T\) with \(i = (m,s)\), \(E(k - h) = [e(k - h), e(k - h)]^T \in \mathbb{R}^{(H_p - H_w + 1)}\), and \(K_{opt} = (\Theta^TQ\Theta + R)^{-1}\Theta^TQ \in \mathbb{R}^{(H_s \times (H_p - H_w + 1))}\) with \(\Theta = G_m + G_s\).

The strategy of the receding horizon control uses the first row of \(\hat{w}_{s,\text{opt}}\):

\[
\Delta w_{s,\text{opt}}(k) = [1 0 ... 0] \hat{w}_{s,\text{opt}}
\]

(30)

So, the feedback control (DGPC) of the slave system is:

\[
w_{s,\text{opt}}(k) = w_{s,\text{opt}}(k - 1) + \Delta w_{s,\text{opt}}(k)
\]

(31)

From the control scheme (see figure 2) and optimal control (30), the structure of the predictive control by internal model with the polynomials is deduced:

\[
R(z^{-1}) = C_{sem}k_dgpcF_s
\]

(32)

\[
T(z^{-1}) = k_dgpcF_m
\]

(33)

\[
W(z^{-1}) = C_{sem}k_dgpc
\]

(34)

\[
D(z^{-1}) = C_{sem} + k_dgpcH_{ms}
\]

(35)

with \(H_{ms} = (H_m + C_{sem}H_s)z^{-1}\) and \(k_{dGPC}\) is the first row of \(K_{opt}\).

By appropriate choices of the horizon lengths \(H_p, H_w, H_u\) and of the weighting matrices \(Q, R\) in the DGPC, an excellent master reference trajectory tracking may be obtained for the slave system.

5. SIMULATION RESULTS

In the simulation, the plant parameters are set to:

<table>
<thead>
<tr>
<th></th>
<th>master</th>
<th>slave</th>
<th>external environment</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M(Kg))</td>
<td>1.5</td>
<td>2.5</td>
<td>0</td>
</tr>
<tr>
<td>(B(Ns/m))</td>
<td>1.5</td>
<td>2.5</td>
<td>0.1</td>
</tr>
<tr>
<td>(K(N/m))</td>
<td>1</td>
<td>0.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The plants are discretized with a sampling period of \(T_s = 50ms\), the time-delays are \(h_{ms} = h_{sm} = 1s\), and zero initial conditions are considered.

Several simulation configurations are considered in order to illustrate robustness of the proposed DGPC. In simulation 1, without uncertainties, the DGPC parameters \((H_u, H_p, H_w, R, Q)\) are set to \((5, 10, 0.25, 1)\). The resulting bilateral teleoperation system response is presented in figure 3. The simulation shows the effectiveness of the proposed DGPC for the master reference trajectory tracking. The assumption on the human operator force \((F_h\) is constant) is not restrictive since we have good tracking abilities in spite of non constant operator forces.

Fig. 3. Simulation 1 of the DGPC

To illustrate robustness of our approach, in simulation 2, time-delays errors of 50\% has been introduced, i.e. \(h_{ms} = h_{sm} = 2s\) in the system. The DGPC parameters are set to (5, 10, 6, 1, 1). Moreover, we have performed with success several simulations with large and poorly known time-delays. Note that in figure 4 the slave system tracks the master reference trajectory.

6. CONCLUSION

A delayed generalized predictive controller using the II-freeness algebraic property of delay mechanical systems has been proposed for the control of bilateral teleoperation systems. The given
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