Feedback Linearization and Linear Observer for a Quadrotor Unmanned Aerial Vehicle

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Abstract— Performance and characteristics of a Luenberger observer, combined with a classical polynomial controller (based on an accurate model of the plant), are analyzed in this paper. The observer is shown efficient when dealing with bounded uncertainties, disturbances and noise. The analysis is based on the observer and tracking errors during transients and at steady state, and on the performance and robustness with respect to plant uncertainties. Estimation of wind parameters is added to reinforce the robustness. Simulation results are provided and output trajectories analyzed.

Index Terms— Feedback linearization ; Luenberger observer ; Estimation, Observer based control.

I. INTRODUCTION

Unmanned Aerial Vehicles (UAV) are increasingly popular platforms due to their potential use in search and rescue, surveillance, law enforcement, inspection, mapping, and aerial cinematography. For these applications, the ability of helicopters to take off and land vertically, to perform hover flight as well as their agility and controllability, make them ideal vehicles. Unmanned aerial and autonomous vehicles have been under continuous development in last decade, especially in civilian aerial applications such as bridges and buildings supervision, surveillance and/or testing. Their application field becomes very large. An analysis of control design approaches, suitable to this kind of systems, has been developed. The control objective is to be able to follow desired trajectories and allow autonomous motions. The system must obey some desired dynamic behavior. This is important for applications where navigation monitoring is not easy to handle manually.

An unmanned quadrotor aerial vehicle (figure (1)) is required to move in different environments, showing good performance and a great autonomy, under a variety of load conditions and unknown disturbances [1]. Developing a control system that can achieve the aforementioned goal is challenging for a variety of reasons

- The nonlinear behavior of a vehicle subject to aerodynamic forces and moments.
- The multivariable character of a vehicle, leading to interaction between different command channels.
- The consistent amount of uncertainty in both high and low frequencies, due to unknown disturbances introduced by linearization of the nonlinear dynamics.

Fig. 1. The quadrotor Aerial Vehicle (UAV) model.

The measurements of the pertinent system variables are not always available, hence the system state is to be estimated using observers. The main difficulties of the motion control, for high performance positioning, are parametric uncertainties, neglected dynamics, and external disturbances. Since the original work by Luenberger [2], the use of state observers proves to be useful not only in system monitoring and regulation but in failures detection and identification of dynamic systems as well. Almost all observer designs are based on a plant model.

However, the presence of disturbances, dynamic uncertainties and nonlinearities represent great challenges in practical applications. Furthermore, the available state information from measurements, i.e., sensor outputs, usually does not contain full state information and most often, it is corrupted by noise. This complicates further the design of robust controllers for actual systems [3]. A sliding mode observer, yielding insensitivity to unknown parameter variations and noise, has been proposed by Utkin [4]. Dorling and Zinober compared the full and reduced order Luenberger observers with the Utkin observer [5].

In the literature, there exist several observer structures based on different methods such as linearization by coordinate transformation and output injection [6], and variable structure approaches [7] to name but a few. In [8], a nonlinear dynamic model, for a quadrotor helicopter in a form suited for control design, is presented. A comparison between two control approaches, the exact linearization with dynamic extension and backstepping control using a sliding mode observer for the state estimation, as applied to a quadrotor helicopter, is made [8][9]. The input-output decoupling problem is not solvable for this model by means
of a static state feedback control law [1][10][11].

However, these observer structures need to include a plant model in their equations. This situation inevitably generates some practical burdens. Without a model, observers cannot be constructed. Even if a model is available, a reliable state reconstruction could not be expected, unless the model is accurate enough. Even in this case, the observers may become too complicated (due to model complexity) to be of practical use, especially in real time applications. It is expected that the observer will provide robust state reconstruction in the presence of uncertainties and disturbances since the nonlinear system (quadrotor) has been linearized through Lie derivatives.

In this work, a combined Luenberger observer; feedback linearization controller and a disturbance estimator is designed in an overall closed loop system to analyze efficiency when dealing with uncertainties, time delay and wind disturbances. This paper is organized as follows: section II describes the quadrotor dynamics. In section III, we present the linearized Luenberger observer. Simulation results are discussed in section IV and a conclusion is drawn in section V.

II. QUADROTOR DYNAMICS

The quadrotor helicopter is assumed to be a rigid body, having 6 degrees of freedom and subject to external efforts. The model includes kinematics and dynamic equations [8]. The quadrotor helicopter is shown in figure (1). The two diagonal motors 1 and 3, are running in the same counter-clockwise direction, whereas motors 2 and 4, run in the clockwise direction to eliminate the anti-torque. By varying the rotor speeds altogether with the same quantity, the lift forces will change, affecting the altitude $z$ of the system and enabling vertical take-off / landing. Yaw angle is obtained by speeding up or slowing down the clockwise motors depending on the desired angle direction. Tilting around $x$ axis (roll angle), allows the quadrotor to move in the $y$ axis direction. The motion direction depends on the angle value whether it is positive or negative. Tilting around $y$ axis (pitch angle), allows the quadrotor to move toward $x$ direction. The rotor is the primary UAV source of control and propulsion. The Euler angle orientation of the flow allows the forces and moments to control the altitude and position of the system. The absolute position is described by three coordinates $(x_0, y_0, z_0)$, and its altitude by Euler angles $(\psi, \theta, \phi)$, under the conditions $(-\pi \leq \psi \leq \pi)$ for yaw, $(-\pi < \theta < \frac{\pi}{2})$ for pitch, and $(-\pi < \phi < \frac{\pi}{2})$ for roll.

Using Newton’s law, if $\sum F_{ext}$ and $\sum T_{ext}$ represent the external forces and moments respectively, the dynamic equations of the system may be represented as:

$$m\ddot{V}_0 = \sum F_{ext}$$

$$J\ddot{\omega} = -\omega \times J\omega + \sum T_{ext}$$

$\ddot{u}_i$ represent the inputs of the system. $u_1$ is the simultaneous 4 rotors thrust; $u_2$ is the thrust of the left and right rotors; $u_3$ is the thrust of the front and the rear rotors; $u_4$ is the momentum difference between the clockwise turning rotors and the counter clockwise ones.

The MIMO nonlinear system has the form [12]

$$\dot{x} = f(x) + \sum_{i=1}^{4} \bar{g}_i(x)\bar{u}_i$$

$$y = h(x) = col(x_0, y_0, z_0, \psi)$$

$$\dot{x}^T = (x_0, y_0, z_0, \psi, \theta, \phi, w_0, v_0, w_0, \zeta, \pi, q, r)$$

$$\bar{g}_1(x) = (0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)^T$$

$$\bar{g}_2(x) = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T$$

$$\bar{g}_3(x) = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T$$

$$\bar{g}_4(x) = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T$$

$$\bar{g}_1^r = -\frac{1}{m} (cos \phi cos \psi sin \theta + sin \phi sin \psi)$$

$$\bar{g}_1^q = -\frac{1}{m} (cos \phi sin \theta sin \psi - cos \psi sin \phi)$$

$$\bar{g}_1^q = -\frac{1}{m} (cos \theta sin \phi)$$

The real control signals $(\bar{u}_1, \bar{u}_2, \bar{u}_3, \bar{u}_4)$ are replaced by $(u_1, u_2, u_3, u_4)$ to avoid singularity in Lie transformation matrices when using exact linearization. In this case $u_1$ is delayed by a double integrator. The other control signals remain unchanged.

$$u_1 = \zeta$$

$$\zeta = \xi$$

$$\xi = \bar{u}_3$$

$$u_2 = \bar{u}_2$$

$$u_3 = \bar{u}_3$$

$$u_4 = \bar{u}_4$$

The Input-Output linearization uses full state feedback to globally linearize the nonlinear dynamics of selected controlled outputs. Each of the output channels is differentiated, a sufficient number of times, until an input control component appears in the resulting equation. Using the Lie derivative, feedback linearization will transform the
nonlinear system into a linear and non-interacting system known as the Brunovsky form [13]

\[
\begin{align*}
\frac{d^4x_0}{dt^4} &= v_1 \\
\frac{d^4y_0}{dt^4} &= v_2 \\
\frac{d^4z_0}{dt^4} &= v_3 \\
\frac{d^4\psi}{dt^4} &= v_4
\end{align*}
\]

\(v_1, v_2, v_3, \text{ and } v_4\), represent the new control inputs. Obviously, when developing the control law, the system requires the primary output vector \((x_0, y_0, z_0, \psi)\) and its successive derivatives to be compared with the desired state trajectories. Adopting a classical polynomial control law, the closed loop system is represented by

\[
\begin{align*}
v_1 &= x_d^{(4)} - \lambda_1(x_0^{(3)} - x_d^{(3)}) - \lambda_2(x_0^{(2)} - x_d^{(2)}) \\
v_2 &= y_d^{(4)} - \lambda_1(y_0^{(3)} - y_d^{(3)}) - \lambda_2(y_0^{(2)} - y_d^{(2)}) \\
v_3 &= z_d^{(4)} - \lambda_1(z_0^{(3)} - z_d^{(3)}) - \lambda_2(z_0^{(2)} - z_d^{(2)}) \\
v_4 &= \psi_d - \lambda_1(\psi - \psi_d) - \lambda_4(\psi - \psi_d)
\end{align*}
\]

(10)

The poles placement is based on the choice of \(\lambda_i\), making its choice decisive in defining the system dynamics.

### III. Luenberger State Observer

When dealing with real time dynamic systems, it is necessary to manipulate the state vector \(x\) and the complete measure is either expensive or difficult to implement. In this case, an observer may be used to obtain an estimate to replace the non-measured state components. A reliable state estimation is mainly required not only for control purpose, but also for other applications such as spacecraft navigation, monitoring, and fault diagnosis in mechanical systems as well. However, the mathematical model (3) is only an approximation to the physical process and the actual plant is usually affected by external disturbances.

For control implementation, the measured quantities are state variables \(x_0, y_0, z_0\) and \(\psi\) representing the translational motion and rotation around \(z\) axis respectively. Non measurable signals can be obtained by successive differentiation. Unfortunately they are contaminated by the measurement noise to such a degree that they can no longer be used. In fact, the accuracy of the state estimation depends largely on the goodness of the actual plant physics model and on the estimator structure [8][13].

#### A. Observer Model

After linearization, the quadrotor model may be represented in a state space form where \([x_1, x_5]\) is the measured vector

\[
\begin{align*}
x_1 &= \text{col}(x_0, y_0, z_0) \\
x_2 &= x_2 = \text{col}(\ddot{x}_0, \ddot{y}_0, \ddot{z}_0) \\
x_3 &= x_3 = \text{col}(\dddot{x}_0, \dddot{y}_0, \dddot{z}_0) \\
x_4 &= x_4 = \text{col}(\ddot{x}_0, \ddot{y}_0, \ddot{z}_0) \\
x_5 &= \psi \\
x_6 &= \dot{\psi} \\
x_7 &= \dot{\psi}
\end{align*}
\]

For a vector \(z = [x_1, x_2, x_3, x_4, x_5, x_6]\), a well known result from linear system theory is that, for a linear time-invariant (LTI) system (11) with the dynamics

\[
\begin{align*}
\dot{z}(t) &= A z(t) + B v(t) \\
y(t) &= C z(t) + D v(t)
\end{align*}
\]

\(A = \begin{bmatrix} A_1 & 0_{4 \times 4} & 0_{4 \times 2} \\ 0_{4 \times 4} & A_1 & 0_{4 \times 2} \\ 0_{4 \times 4} & 0_{4 \times 4} & A_2 \end{bmatrix} \), \(B = \begin{bmatrix} 0_{4 \times 4} \\ 0_{4 \times 4} \\ 0_{4 \times 4} \\ 0_{4 \times 4} \end{bmatrix} \), \(C = \begin{bmatrix} C_1 & 0_{4 \times 3} & C_2 & 0_{4 \times 3} & C_3 & 0_{4 \times 3} & C_4 & 0_{4 \times 1} \end{bmatrix} \)

\(A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \),

\(B_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \), \(B_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \), \(B_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \), \(B_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \)

\(C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} ; C_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} ; C_3 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} ; C_4 = \begin{bmatrix} 0 & 0 \end{bmatrix} \)

and an observable \((A, C)\) pair, a stable linear Luenberger observer given by

\[
\begin{align*}
\dot{\hat{z}}(t) &= A \hat{z}(t) + B v(t) + L (y(t) - \hat{y}(t)) \\
\dot{\hat{y}}(t) &= C \hat{z}(t) + D v(t) ; \quad \hat{z}(0) = \hat{z}_0
\end{align*}
\]

can be designed by placing the poles of the observer at any desired location such that the error signals exhibit the desired dynamics [14].

\[
\begin{align*}
\dot{\hat{z}}(t) &= (A - LC) \hat{z}(t) + (B - LD)v(t) + Ly(t)
\end{align*}
\]
The estimation error is \( e = z - \hat{z} \), its dynamic equation is given by
\[
\dot{\hat{e}} = (A - LC)e = \hat{A}e
\] (19)
The estimation error \( e \) will converge to zero if all eigenvalues of \( \hat{A} = (A - LC) \) are in the left half plane. The observer design refers to selection of the gain matrix \( L \), using the pole placement method. The main challenge in these applications is the extensive observer dependence on the accuracy of the plant mathematical model (\( A, B, \) and \( C \) matrices). Therefore, the convergence rate of \( \hat{z}(t) \to z - \hat{z} \) to zero, can arbitrarily be chosen by appropriate design of \( L \). It follows that \( \hat{z}(t) \) converges exponentially to \( z(t) \) as \( t \to \infty \) with a rate depending on matrix \( \hat{A} \). This result is valid for any matrix \( A \) and any initial condition \( z_0 \) as long as \( (C,A) \) is an observable pair and \( A, C \) are known.

\section*{B. Output States Reconstruction}

The observer, described in the previous section, is a state estimator with partial state \((x_0, y_0, z_0, \phi)\) taken as a measured output. The observer makes an estimation of the state needed by the control law to calculate the tracking error between the desired trajectories \((x_{1d}, x_{2d}, x_{3d}, x_{4d}, x_{5d}, x_{6d})\) and the estimated trajectories \((x_1, x_2, x_3, x_4, x_5, x_6)\). Unfortunately, the estimated states do not involve all the output states. To obtain the entire state output, the missing variables \((\theta, \phi, p, q, r)\) from \( \ddot{x} \) vector(3) are calculated through the estimated values from the nonlinear system (3), without taking the perturbation into account. From (12), \( \theta \) and \( \phi \) can be deduced
\[
\phi_e = \arcsin\left(-\frac{m(\sin(\hat{\phi})\dot{x} - \cos(\hat{\phi})\dot{y})}{u_1}\right)
\] (20)
\[
\theta_e = \frac{1}{\cos \phi_e} \arcsin\left(-\frac{m(\cos(\hat{\phi})\dot{x} + \sin(\hat{\phi})\dot{y})}{u_1}\right)
\] (21)
\[
(p, q, r) \text{ can be determined from the transformation matrix [7] which needs } (\hat{\psi}, \hat{\theta}, \hat{\phi}). \text{ These parameters can be evaluated from (12) and the third derivatives } (\dddot{x}_0, \dddot{y}_0) \text{ i.e.,}
\]
\[
\dot{\phi}_e = \frac{-m\dddot{x}_0 \sin(\hat{\phi}) + \dddot{y}_1 \cos(\hat{\phi})}{u_1 \cos(\hat{\phi})}
\] (22)

\section*{C. Wind Parameters Estimation}

The final model, obtained using feedback linearization, differs from (4) in presence of perturbation since linearization is not exact. Considering perturbation, the system is represented by
\[
\dot{x} = Ax + B_1 \eta_1 + B_2 \eta_2
\] (24)
with \( x \) being the measured state vector and its successive derivatives, \( \eta_1 \) and \( \eta_2 \) are the wind disturbances vector \((\dot{\bar{A}}_x, \dot{\bar{A}}_y, \dot{\bar{A}}_\theta) \) and \((\dot{A}_p, \dot{A}_q, \dot{A}_r) \) respectively.
\[
x = \text{col}(x_1, x_2, x_3, x_4, x_5, x_6)
\] (25)
\[
A = \begin{bmatrix}
 0 & I & 0 & 0 & 0 \\
 0 & 0 & I & 0 & 0 \\
 0 & 0 & 0 & I & 0 \\
 -\lambda_0 & -\lambda_1 & -\lambda_2 & -\lambda_3 & 0 \\
 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & -\lambda_4 & -\lambda_5
\end{bmatrix}
\] (26)
\[
M_1 = \begin{bmatrix}
 \frac{1}{m} & 0 & 0 \\
 \frac{1}{m} & 0 & 0 \\
 0 & 0 & 0
\end{bmatrix};
M_2 = \begin{bmatrix}
 a_{14} & a_{15} & 0 \\
 a_{24} & a_{25} & 0 \\
 a_{34} & a_{35} & 0
\end{bmatrix}
\] (27)
\[
a_{14} = (\zeta \dot{S} \phi \dot{C} \theta - \zeta \dot{C} \phi \dot{S} \theta)/(mI_x); a_{15} = (\zeta \dot{S} \phi \dot{C} \theta)/(mI_x)
\]
\[
a_{24} = -(\zeta \dot{C} \psi \dot{C} \theta)/mI_y; a_{25} = (\zeta \theta)/(mI_y)
\]
\[
a_{34} = -(\zeta \psi \dot{C} \theta)/(mI_y); a_{35} = \dot{C} \phi/(I_y \dot{C} \theta)
\] (28)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{closed_loop_system.png}
\caption{The overall closed loop system.}
\end{figure}

A problem of primary importance is the selection of the quadratic Lyapunov function. From (12), the Lyapunov function which ensures convergence is chosen as
\[
V = \dot{x}^T P \dot{x} + \frac{1}{2} \eta_1^T \Gamma_1 \eta_1 + \frac{1}{2} \eta_2^T \Gamma_2 \eta_2
\] (29)
\( \tilde{x} \) represents the tracking errors between the estimated and desired values. By computing \( \dot{V} \) and making \( \dot{V} < 0 \) we deduce the adaptation law for parameters tuning

\[
\begin{align*}
\dot{\tilde{\eta}_1} &= -2\Gamma_1^{-1}B_1^TP\tilde{x} \\
\dot{\tilde{\eta}_2} &= -2\Gamma_2^{-1}B_2^TP\tilde{x}
\end{align*}
\]

(30)

Finally, the closed loop system with the combined controller-observer-estimator is represented in figure (2).

IV. Simulation and Results

Simulation is carried out using the following quadrotor parameters:

\[
\begin{align*}
I_x &= I_y = I_z = 1.241 N/\text{rad/s}^2, \\
m &= 2.06 kg, \\
d &= 0.1 \text{m}, \text{and} \\
g &= 9.81 \text{m/s}^2.
\end{align*}
\]

Fig. 3. Trajectories behavior without disturbances

To evaluate the performance of the proposed observer and control, the reference (desired) trajectory used to carry out simulation, is a vertical helix, see figure (3), with equation

\[
\begin{align*}
x_{od} &= \frac{1}{2} \cos\left(\frac{t}{2}\right) \\
y_{od} &= \frac{1}{2} \sin\left(\frac{t}{2}\right) \\
z_{od} &= -1 - \frac{t}{10} \\
\psi_{od} &= \frac{\pi}{3}
\end{align*}
\]

(31)

Fig. 4. Yaw trajectory

\((\lambda_0, \lambda_1, \lambda_2, \lambda_3)\) are the coefficients of the polynomial \((s + 10)^2\) and \((\lambda_4, \lambda_5)\) the ones of \((s + 5)^2\).

The desired poles for the closed loop system used to determine \( L \) are

\[
P_{14 \times 1} = \begin{cases}
-5.5; -5.5; 5.5; -6.6; -6.6; -7.7; \\
-7.7; -7.7; -8.8; -8.8; -8.8; -4.95; -4.95
\end{cases}
\]

To evaluate the performance and robustness of the sliding observer, simulation is carried out considering the following cases: without disturbance, with disturbance, and ultimately with uncertainties.

1) Without disturbance: In this case, we use \((A_x = A_y = A_z = 0); (A_p = A_q = A_r = 0)\). Figure 4 shows a good convergence of the yaw trajectory. Figures 5 and 6 show a smooth behavior of tracking errors and the control signals respectively.

Fig. 5. Tracking error for x, y, z, \( \psi \)

2) With disturbances: The estimation gains for wind disturbance \([A_p; A_q; A_r]\) are \(\Gamma_2 = [2.10^{-4}; 2.10^{-4}; 4]\) around \(x\) axis \((A_p)\), \(y\) axis \((A_q)\) and \(z\) axis \((A_r)\). Using the aerodynamic disturbances (Moments) \(A_p = 0.8; A_q = 1; A_r = 0.8\); the following results are obtained: In fig-7 we present tracking errors showing that disturbance is well rejected and fig-8 shows a good convergence of parameters estimation.

Fig. 6. Control signals \(v_1, v_2, v_3, v_4\)

3) With uncertainties: Uncertainties of 20% are introduced on the mass \(m\) and inertial coefficients \(I_x, I_y, I_z\) to show the behavior of the observer-control law combination toward error modelling. The obtained results are shown in fig-9.

4) Concluding remarks: It is noted from the obtained results without perturbation, (fig-3, 4, 5 and 6), that the Luenberger observer gives quite satisfactory results, especially when the poles placement is chosen judiciously. This can be seen from trajectories tracking error vanishing after a finite time with a perfect convergence shape.

-When wind disturbances are introduced, the results presented in fig-7 reflect the good robustness of the mixed
observer-controller. This is confirmed by the convergence of tracking error (fig-7) and disturbance rejection. The estimation of wind parameters, around x axis ($A_x$), y axis ($A_y$) and z axis ($A_z$), exhibits a very good convergence as presented in fig-8. This shows that the system dynamic behavior is sensitive to aerodynamic moments disturbances.

-However, the estimation of translational wind forces (given by $A_x$, $A_y$ and $A_z$) is not represented here since the observer-controller exhibit an efficiency to overcome this type of perturbation without need to an estimation-compensation procedure.
-The observer robustness is also tested by introducing 20% uncertainties on the system parameters $m, I_x, I_y, I_z$ (fig-9).

- The output state vector convergence is obtained despite the (well known) non-robustness of the exact linearization approach when system parameters suffer severe uncertainties. However attention must be paid on the choice of observer gain $L$ to avoid noise amplification around the desired trajectories.

![Wind parameters estimation for $A_x = 0.8; A_y = 0.5; A_z = 1$](image)

V. CONCLUSION

In this paper, a kino-dynamic model of a quadrotor helicopter is presented. We developed a nonlinear controller with observers. A feedback linearization using Luenberger observer is applied to the quadrotor UAV. The linear observer is used to rebuilt the non measured variables required to enhance robustness of the control law. An adaptive estimator is added to the overall system to estimate the effect of the external disturbances such as wind. The whole control system (observer-estimator-controller) constitutes an interesting contribution to control systems equipped with a minimum number of sensors. This approach shows a good robustness of the controller and permits to reduce the number of sensors to be used. Intensive simulations were carried out to validate the performance and stability of the controller. The robustness study was realized in simulations taking into account uncertainties and disturbances with a noise corrupted measured state. The obtained results show good convergence of estimated values and satisfying tracking errors of desired trajectories. Also, they show that the estimator added to the control reinforces the robustness and stability of the overall system. The adaptive part allows estimation of parameters which may not be well known and may change during operation.

![Tracking errors for 10 % uncertainties on $m; I_x; I_y; I_z$](image)

REFERENCES


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