ABSTRACT

Multibody simulation models often require residual flexibility in order to obtain proper model response. Accurate finite element models, validated by experimental modal tests and updated based on that test data, may be necessary to achieve the performance accuracy desired. Development of these models and performance of the test and model updating can be difficult and time consuming.

An alternate approach is proposed which is based on experimental measurements used to update a reduced order model. This reduced order model is then used to formulate the residual flexibility of the system without the need for a detailed test verified finite element model. The system models are developed from both modal and response based modeling approaches. In addition, effects of component interaction are introduced using modal based and impedance based modeling approaches. The technique is compared to the traditional full model approach to show the suitability of the proposed approach. Flexibility of the overall system as well as time response comparisons of the system are presented. An example structure is used to show the use of the proposed techniques.

INTRODUCTION

Multibody dynamic simulations are a critical part of the design process for many applications. For many cases, the flexibility of the attachment points is necessary in order to provide the proper response of the system. Finite element models are often used for this purpose. These models must be accurate in order to provide the proper boundary conditions for the multibody simulation. Often experimental tests are needed to adjust the models to reflect the actual system. For the study identified herein, a helicopter-wing-missile configuration shown in Figure 1 was the impetus for this work.

In order to provide a better approximation of the helicopter and wing configuration, typically modal testing is performed followed by updating of the finite element model. This can be very time consuming, since the models of the wing and
helicopter are complicated and model updating is not a trivial task. A typical wing configuration is shown in Figure 2 to illustrate the complexity of the model; the helicopter model is significantly more complicated. These models are very detailed and significant effort is required to perform model correlation and updating. Using commercially available sensitivity based updating methodologies, the exact solution is never obtained and the updating process only converges to the experimentally obtained data. Clearly, a simpler and more direct method of obtaining flexibility would be highly desirable.

In this paper, several alternate approaches are proposed using a combination of model reduction, modal modeling, impedance modeling and direct model updating to provide a suitable representation of the structure. This approach eliminates the need for detailed updating of individual model features which offers very significant computational and procedural advantages. The approaches proposed in this paper allow for the structure to be evaluated (in either the assembled or component configurations) are described to allow for alternate approaches to be utilized for different design scenarios. In order to understand the modeling scenario proposed, several aspects of model reduction, impedance modeling and model updating need to be identified. This is followed by a description of the modeling scenario proposed. Test cases are then presented to show the proposed technique, and the results are compared to those from a more traditional approach using a full structure model to describe the system. For purposes of the work performed herein, a simpler representation of the helicopter configuration (shown in Figure 1) will be employed as seen in Figure 3. The work presented here extends the work presented earlier [1,2] where only system flexibility was the focus of that work.
THEORY

In order to develop appropriate models for the proposed methods to determine changes in the system characteristics, reduced order models along with model updating techniques and impedance modeling techniques are necessary. Each of these methods is only summarized here; details of the techniques are contained in their respective references.

Model Reduction:

Model reduction is necessary in order to develop very brief but accurate models for system description. The methods used have been previously presented in many applications. For this work, the reduction is performed to produce a smaller model which is still able to accurately represent the features of interest.

Several methods for reducing analytical models are available. Three common methods are Guyan [3], SEREP [4], and a Hybrid method [5]. In these methods, the relationship between the full set of degrees of freedom and a reduced set of degrees of freedom can be written as:

\[ \{X_n\} = [T]\{X_a\} \]  

(1)

All three methods require the formation of a transformation matrix, \( T \), that can project the full mass and stiffness matrices to a smaller size. The reduced matrices can be formulated as:

\[ [M_a] = [T]^T [M_n] [T] \]  

(2)

\[ [K_a] = [T]^T [K_n] [T] \]  

(3)

The Guyan reduction process [3] forms the transformation matrix as:

\[ [T_G] = \begin{bmatrix} I \\ -[K_{dd}]^{-1}[K_{da}] \end{bmatrix} \]  

(4)

The SEREP reduction process [4] produces reduced matrices for mass and stiffness that yield the exact frequencies and mode shapes as obtained from the eigensolution of the full size matrix. The SEREP transformation is formed as:

\[ [T_U] = [U_n] [U_a]^T \]  

(5)

The Hybrid method [5] utilizes the accuracy of the SEREP method and seeds the reduced matrices with reduced Guyan matrices to insure that the resultant reduced matrices are fully ranked for all cases. The Hybrid method transformation matrix is:

\[ [T_H] = [T_G] + [[T_U] - [T_G] [U_a] [U_a]^T [T_U]^T [M_n] [T_U]] \]  

(6)

Any of these model reduction schemes can be employed and have various advantages and disadvantages. For this study, the more commonly used Guyan reduction will be employed for most studies (but comparison to other techniques are presented). For the analyses of this work, the accuracy of the reduced model is not as critical in studies such as correlation because these matrices are only used as seed matrices for the model updating analyses. It is also important to note that the reduced matrices will be fully populated and will not have any element topology that can be identified.

Impedance Modeling Approach:

Impedance models [6,7] are commonly used to develop system models where components are each tested independent of the assembled system. In the case of this model, a free-free component can be attached to another component using only the attachment DOF measured FRFs. This technique has been widely used and the basic equation describing the response of the system can be defined as
The form of this equation specifically describes the transfer functions of the outputs on Component A due to inputs on Component A as influenced by the dynamic effects of Component B. Variations on this one equation cover the range of possibilities that can exist in the development of a system model.

Analytical Model Improvement:

These analytical models are adjusted using experimental results through a direct model updating approach. The development of these techniques are contained in [8, 9, 10, 11]. The reduced models are used for this process as a description of the original FEM. From the basic modal representation of the system, modal mass and stiffness are evaluated from experimental data [10] as:

\[
\begin{align*}
\begin{bmatrix} E \end{bmatrix}^T \begin{bmatrix} M \end{bmatrix} E &= \begin{bmatrix} M \end{bmatrix} = [I] \\
\begin{bmatrix} E \end{bmatrix}^T \begin{bmatrix} K \end{bmatrix} E &= \begin{bmatrix} K \end{bmatrix} = \Omega^2
\end{align*}
\]

Using a generalized inverse of these equations, the mass and stiffness can be estimated as:

\[
\begin{align*}
\begin{bmatrix} E^g \end{bmatrix}^T \begin{bmatrix} I \end{bmatrix} \begin{bmatrix} E^g \end{bmatrix} &= \begin{bmatrix} M \end{bmatrix} \\
\begin{bmatrix} E^g \end{bmatrix}^T \begin{bmatrix} \Omega^2 \end{bmatrix} \begin{bmatrix} E^g \end{bmatrix} &= \begin{bmatrix} K \end{bmatrix}
\end{align*}
\]

This works fairly well for cases where the number of relevant modes is equal to the number of degrees of freedom. However, this is often not the case and there are generally more measured DOF than measured modes. The experimentally derived mass and stiffness matrices are developed using the Analytical Model Improvement (AMI) approach [8, 9, 10, 11]. The reduced order mass and stiffness matrices are used as the original seed matrices that are updated using the common solution for the AMI process [9]. The experimentally measured frequency and mode shapes are referred to as the targets. Another very important feature of the direct model updating process is that the target modes will be captured exactly in the updated mass and stiffness matrices; they are not approximate as in all of the sensitivity based techniques typically utilized in most commercially available software packages.

The improved mass and stiffness matrices can be calculated as

\[
\begin{align*}
\begin{bmatrix} M_1 \end{bmatrix} &= \begin{bmatrix} M_s \end{bmatrix} + \begin{bmatrix} V \end{bmatrix}^T \begin{bmatrix} I - M_s \end{bmatrix} \begin{bmatrix} V \end{bmatrix} \\
\begin{bmatrix} K_1 \end{bmatrix} &= \begin{bmatrix} K_s \end{bmatrix} + \begin{bmatrix} V \end{bmatrix}^T \begin{bmatrix} \omega^2 + K_s \end{bmatrix} \begin{bmatrix} V \end{bmatrix} - \begin{bmatrix} \omega^2 \end{bmatrix} \begin{bmatrix} E \end{bmatrix} \begin{bmatrix} V \end{bmatrix} - \begin{bmatrix} \omega^2 \end{bmatrix} \begin{bmatrix} E \end{bmatrix} \begin{bmatrix} V \end{bmatrix}^T
\end{align*}
\]

where

\[
\begin{bmatrix} V \end{bmatrix} = \begin{bmatrix} E \end{bmatrix}^T \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} E \end{bmatrix}^{-1} \begin{bmatrix} E \end{bmatrix}^T \begin{bmatrix} M \end{bmatrix} = \begin{bmatrix} \bar{M} \end{bmatrix}^{-1} \begin{bmatrix} E \end{bmatrix}^T \begin{bmatrix} M \end{bmatrix}
\]

Using this technique, the system described by the updated mass and stiffness matrices will have mode shapes and frequencies exactly matching the targets. However, the updated mass and stiffness matrices will not have the typical model topology that is seen in standard element types such as beams, plates, etc. These matrices will generally be fully populated.
Correlation Tools

For assessment of the models developed and results obtained, three tools were employed. The Modal Assurance Criteria (MAC), the Pseudo Orthogonality Check (POC) and the Time Response Assurance Criteria (TRAC) are identified below.

**Modal Assurance Criteria (MAC):**

For Modal Assurance Criteria [12] is widely used as a vector correlation tool and is given as

\[
MAC_{ij} = \frac{\left(\{u_i\}^T \{e_j\}\right)^2}{\left(\{u_i\}^T \{u_i\}\right) \left(\{e_j\}^T \{e_j\}\right)}
\]

(15)

The MAC values close to 1.0 indicate similarity whereas values close to 0.0 indicate no similarity.

**Pseudo Orthogonality Check (POC):**

For Pseudo Orthogonality Check [13] is widely used as a vector correlation tool and is given as

\[
POC = \left[U_a\right]^T \left[M_a I E_a\right] = \left[U_n\right]^T \left[M_n I E_n\right] = \left[U_a\right]^T \left[E_a\right]
\]

(16)

The POC is similar to the MAC but includes mass scaling which enhances the vector correlation process. When shapes are scaled to unit modal mass, the POC values will range between 0.0 and 1.0 with similar interpretation to the MAC.

**Time Response Assurance Criteria (TRAC):**

For assessment of the time response of the reduced order models compared to the full reference solutions, a time response assurance criteria (TRAC) was formulated as

\[
TRAC = \frac{\left(\{t_1\}^T \{t_2\}\right)^2}{\left(\{t_1\}^T \{t_1\}\right) \left(\{t_2\}^T \{t_2\}\right)}
\]

(17)

The TRAC is a variant of the MAC and is used for assessing similarity of time response traces. TRAC values close to 1.0 indicate similar time responses and TRAC values close to 0.0 indicate no similarity.
APPROACHES FOR REDUCING MODEL AND IDENTIFYING TARGETS FOR ADJUSTING MODELS

In order to provide an equivalent but simpler approach for the development of the models used in the multibody dynamic simulation, both model reduction, impedance modeling, modal modeling and direct model updating are employed. Several scenarios are presented here utilizing an assembled system as well as component representations. Each technique is described in the following sections.

For all cases, either a system model or component model is developed. These models are then reduced as either system model or component model. The experimental data is then used to perform a direct updating on the system model or component models. The system flexibility is then computed and compared to the actual system flexibility as a measure of the adequacy of the model. The system response is then computed due to two different forcing functions with different frequency bandwidth excitation ranges and compared to the actual system response as a measure of the adequacy of the model.

The system is considered as either an assemblage or as individual components. For the evaluations presented herein, a simple upright support structure is used to represent the helicopter fuselage and a horizontal wing member is used to represent the helicopter wing arrangement. These are shown in Figure 4. The conceptualization of the model reduction and the corresponding experimental test DOF are shown schematically in Figure 5 for the individual components; the full system configuration is not shown.

Figure 4. CAD model of assembled structure, and FEM of separate components.

Figure 5. Model reduction of support structure and wing.

The three approaches are presented next.
Approach 1 – Complete System Model Reduction with Reduced Model Updating:
(Modal Based System Response – MBSR)

The full system model is used to develop a highly reduced order model. Test data is acquired at these reduced order DOF to obtain target mode shapes for the AMI process. AMI is performed to update the system model to reflect the measured characteristics of the assembled system. This reduced order, updated model is used for response analysis. The model development is schematically shown in Figure 6 and is referred to as the Modal Based System Response (MBSR).

Figure 6. Model reduction and updating of complete structure.
Modal Based System Response (MBSR)
**Approach 2 – Reduced Component Model Updating from Assembled System Measured Characteristics:**

*(Modal Based Component Response – MBCR)*

The wing component model alone is used to develop a reduced order model. Test data is acquired at these reduced order DOF but with the wing component assembled to the upright structure to obtain target mode shapes for the AMI process. AMI is performed to update the model to reflect the measured characteristics of the assembled system. This reduced order, updated model of the wing component alone is used for response analysis. The model development is schematically shown in Figure 7 and is referred to as the *Modal Based Component Response (MBCR)*.

![Image of model development process](image)

*Figure 7. Reduction and updating of wing-only model. Modal Based Component Response (MBCR)*
Approach 3 – Reduced Component Model Updating from Impedance Developed Model for System Characteristics: (Frequency Based Component Response – FBCR)

The wing component model alone is used to develop a reduced order model (as in Approach 2). However, test data is acquired at these reduced order DOF with the wing component disassembled from the upright structure. Impedance modeling techniques are applied to dynamically couple the wing to the upright to obtain target mode shapes for the AMI process. AMI is performed on the wing component alone to update the model to reflect the measured characteristics of the assembled system from the impedance model approach. This reduced order, updated model of the wing component alone is used for response analysis. The model development is schematically shown in Figure 8 and is referred to as the Frequency Based Component Response (FBCR).

Figure 8. Reduction and updating of wing-only model from impedance/modal targets. Frequency Based Component Response (FBCR)
REFERENCE MODEL AND SOLUTION

The reference model used for this study is described herein. All models described herein were developed using FEMAP [14], solved and reduced using FEMtools [15], and subjected to updating and additional processing using MATLAB [16] with the MATSAP toolbox [17]. Experimental data collection as well as impedance based models were developed using LMS CADA-X [18].

**Full Finite Element Model**

A full finite element model of the system was created for the structure. The model consists of over 82000 DOF with primarily plate elements describing the system with constraint elements fixing the brackets to the plates. The entire underside of the base is supported on soft springs, and the four corners of the base are fixed. This model of the assembled structure is shown in Figure 9. The first nine mode shapes are shown for reference in Figure 10.

This reference model provided the simulated experimental data used as targets for updating. *(NOTE: Simulated data was used for all experimental data sets presented in this work in order to validate the proposed approaches; use of actual test data for the structure is currently in progress and will be the subject of future papers.)* The reference solutions were obtained from this model for the overall system flexibility and for the two forced response cases shown. Details of the flexibility analysis was reported earlier [1,2] and only summarizing statements and results are mentioned in this paper. The forced response solutions obtained are described in more detail in the following sections. Both static and dynamic response are needed to confirm that the updated matrices provide physical characteristics.

**Flexibility Analysis:**

In order to confirm the stiffness of the updated model, many static load cases were evaluated. Only one case with a tip static load is presented here for brevity. More extensive discussion on flexibility analysis and results is presented in [1,2]. Several typical static load cases evaluated are shown in Figure 11.

**Forced Response Analysis:**

A force pulse was applied to the structure to compute the response. The first force pulse only applied significant spectral energy up to 200 Hz which causes participation of primarily the first eight modes of the system. The second pulse excites a much higher frequency range to 2000 Hz which involves the participation of primarily the first 67 modes of the system. The force is applied at the tip of the wing structure at one corner as shown in Figure 12. Also shown in the figure are the two force pulses applied along with their corresponding input spectra. The response was calculated in ABAQUS using Hilber-Hughes-Taylor numerical technique whereas the Newmark method was used for all MATLAB processing (due to the similarity to the HHT technique).
Figure 9. Full finite element model.

(a) Mode 1  
13.25 Hz

(b) Mode 2  
27.46 Hz

(c) Mode 3  
28.22 Hz

(d) Mode 4  
43.61 Hz

(e) Mode 5  
64.46 Hz

(f) Mode 6  
72.10 Hz

(g) Mode 7  
140.19 Hz

(h) Mode 8  
189.44 Hz

(i) Mode 9  
222.76 Hz

Figure 10. First nine mode shapes and frequencies of reference model.
(a) Case 1: +50 at corner  
(b) Case 2: -20 on side  
(c) Case 3: +100 in center, -20 at corner

Figure 11. Three static load cases applied.

(a) Force Load Point  
(a) Time domain  
(b) Frequency domain

Wide (low frequency) time force pulse.

(a) Force Load Point  
(a) Time domain  
(b) Frequency domain

Narrow (high frequency) time force pulse.

Figure 12. Applied Load Cases for Response Computation.
A perturbed model of the full structure was generated by reducing the thickness of the brackets by 50%. This was done to simulate an erroneously developed model that has some significant effect on the overall flexibility of the system. Table 1 compares the natural frequencies of this perturbed model to those of the reference model (which is the simulated experimental data). Table 2 compares the flexibility of the perturbed model and the reference model for load case 3. Note that there is a significant difference, especially in the lower order modes, which greatly influences the overall flexibility of the system. Clearly, the perturbation has a significant effect on both the static stiffness as well as the frequencies and mode shapes.

### Table 1. Comparison between resonant frequencies of perturbed model and reference model.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Reference model Frequency</th>
<th>Perturbed model Frequency</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.0</td>
<td>11.3</td>
<td>-13.55%</td>
</tr>
<tr>
<td>2</td>
<td>27.0</td>
<td>21.8</td>
<td>-19.36%</td>
</tr>
<tr>
<td>3</td>
<td>27.1</td>
<td>25.8</td>
<td>-4.81%</td>
</tr>
<tr>
<td>4</td>
<td>43.9</td>
<td>43.4</td>
<td>-1.15%</td>
</tr>
<tr>
<td>5</td>
<td>64.2</td>
<td>65.4</td>
<td>1.86%</td>
</tr>
<tr>
<td>6</td>
<td>71.8</td>
<td>68.5</td>
<td>-4.58%</td>
</tr>
<tr>
<td>7</td>
<td>137.3</td>
<td>131.0</td>
<td>-4.58%</td>
</tr>
<tr>
<td>8</td>
<td>185.6</td>
<td>162.2</td>
<td>-12.63%</td>
</tr>
<tr>
<td>9</td>
<td>221.8</td>
<td>219.6</td>
<td>-0.98%</td>
</tr>
<tr>
<td>10</td>
<td>239.5</td>
<td>220.5</td>
<td>-7.92%</td>
</tr>
</tbody>
</table>

### Table 2. Comparison between flexibility of perturbed model and reference model.

<table>
<thead>
<tr>
<th>Node</th>
<th>Reference model displacement</th>
<th>Perturbed model displacement</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>0.0430</td>
<td>0.0658</td>
<td>53.15%</td>
</tr>
<tr>
<td>145</td>
<td>0.0502</td>
<td>0.0740</td>
<td>47.48%</td>
</tr>
<tr>
<td>161</td>
<td>0.1106</td>
<td>0.1580</td>
<td>42.90%</td>
</tr>
<tr>
<td>207</td>
<td>0.0924</td>
<td>0.1387</td>
<td>49.99%</td>
</tr>
<tr>
<td>861</td>
<td>0.1685</td>
<td>0.2379</td>
<td>41.19%</td>
</tr>
<tr>
<td>873</td>
<td>0.1674</td>
<td>0.2369</td>
<td>41.50%</td>
</tr>
<tr>
<td>885</td>
<td>0.1629</td>
<td>0.2322</td>
<td>42.55%</td>
</tr>
<tr>
<td>897</td>
<td>0.1528</td>
<td>0.2216</td>
<td>45.06%</td>
</tr>
<tr>
<td>909</td>
<td>0.1389</td>
<td>0.2070</td>
<td>49.04%</td>
</tr>
<tr>
<td>1246</td>
<td>0.1060</td>
<td>0.1522</td>
<td>43.54%</td>
</tr>
<tr>
<td>1261</td>
<td>0.0522</td>
<td>0.0769</td>
<td>47.40%</td>
</tr>
<tr>
<td>38201</td>
<td>-0.0377</td>
<td>-0.0512</td>
<td>35.59%</td>
</tr>
<tr>
<td>38414</td>
<td>-0.0467</td>
<td>-0.0617</td>
<td>32.16%</td>
</tr>
<tr>
<td>38425</td>
<td>-0.0479</td>
<td>-0.0628</td>
<td>31.24%</td>
</tr>
<tr>
<td>38451</td>
<td>-0.0385</td>
<td>-0.0519</td>
<td>34.76%</td>
</tr>
</tbody>
</table>

**Average % difference:** 42.50%
The first and more traditional method involves updating a reduced-order model of the entire structure using the Analytical Model Improvement (AMI) direct updating technique. The perturbed model (which simulates the starting FEM) was reduced to an evenly-distributed 33 DOF using Guyan reduction in FEMtools. While any of the reduction schemes could be employed, Guyan reduction is a very commonly used method and was utilized for these analyses. However, there are limitations to this approach, as mentioned in the latter part of this paper, that suggest alternate reduction methods may be better suited in certain instances. The perturbed model, with the selected ADOF, is shown in Figure 13. This reduced-order model provided the mass and stiffness matrices which were used as seeding matrices for the model updating. The static flexibility due to one of the load cases evaluated along with the forced response for the two different loading conditions are presented below.

Figure 13. Perturbed model with selected ADOF.

Results from Static Loading – Flexibility Analysis

The model was updated and the improved stiffness was used for a static load evaluation. The reference model was used to compute the actual displacements and then compared to the results of the updated stiffness flexibility results. In all cases studied, the average error was less than 1%. The results for load case 3 are shown in Table 3—the average error is only 0.75%. (This result was typical of all load cases studied for this configuration.)
Table 3. Approach 1, Load case 3

<table>
<thead>
<tr>
<th>Node</th>
<th>Reference model displacement</th>
<th>Guyan reduced perturbed model, updated with 24 modes</th>
<th>% difference from original</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>0.042033</td>
<td>0.04250</td>
<td>-1.12%</td>
</tr>
<tr>
<td>145</td>
<td>0.060091</td>
<td>0.04976</td>
<td>-0.87%</td>
</tr>
<tr>
<td>161</td>
<td>0.14323</td>
<td>0.10945</td>
<td>-1.02%</td>
</tr>
<tr>
<td>207</td>
<td>0.097895</td>
<td>0.09130</td>
<td>-1.24%</td>
</tr>
<tr>
<td>861</td>
<td>0.23286</td>
<td>0.16701</td>
<td>-0.90%</td>
</tr>
<tr>
<td>885</td>
<td>0.18987</td>
<td>0.16151</td>
<td>-0.86%</td>
</tr>
<tr>
<td>909</td>
<td>0.15886</td>
<td>0.13855</td>
<td>-0.27%</td>
</tr>
<tr>
<td>1246</td>
<td>0.11603</td>
<td>0.10470</td>
<td>-1.25%</td>
</tr>
<tr>
<td>1261</td>
<td>0.054418</td>
<td>0.05168</td>
<td>-0.99%</td>
</tr>
<tr>
<td>38201</td>
<td>-0.039446</td>
<td>-0.03761</td>
<td>-0.35%</td>
</tr>
<tr>
<td>38414</td>
<td>-0.048629</td>
<td>-0.04656</td>
<td>-0.29%</td>
</tr>
<tr>
<td>38425</td>
<td>-0.051554</td>
<td>-0.04776</td>
<td>-0.23%</td>
</tr>
<tr>
<td>38451</td>
<td>-0.041345</td>
<td>-0.03837</td>
<td>-0.35%</td>
</tr>
</tbody>
</table>

Average % difference: -0.75%

Results from Updated Model Forced Response – Low Frequency Pulse

The forced response due the low frequency loading applied is shown in Figure 14. The model was updated using only 10 of the lower order modes of the system. The overall response compares very well with the reference solution from the ABAQUS 82000 DOF model with a TRAC value of 0.9721. Additional modes were added to the updated model but essentially no difference in the response results. This is due to the low frequency excitation which only requires the lower order modes of the system to identify reasonable response.

![Figure 14](image.png)

Figure 14. Response of Guyan-reduced model, updated with 10 modes, to wide pulse, compared to reference model.
Results from Updated Model Forced Response – High Frequency Pulse

The forced response due the loading applied is shown in Figure 15. The model was updated using only 10 of the lower order modes of the system. The overall response compares reasonably well with the reference solution from the ABAQUS 82000 DOF model with a TRAC value of 0.8336. Considering that only 10 modes were used and the excitation excites a much wider frequency range, these results are very good overall. The first 20% of the response is shown more clearly in Figure 16. From this plot it is clear that the low frequency trend is captured with 10 modes but that higher frequency response is not capture well; this is expected since only 10 modes were utilized. (NOTE: In order to address these higher frequencies, more modes could be added but is not the intent of the work described herein.)

Figure 15. Response of Guyan-reduced model, updated with 10 modes, to narrow pulse, compared to reference model.

Figure 16. Response of Guyan-reduced model, updated with 10 modes, to narrow pulse, compared to reference model.
The results shown in Figure 15 and 16 are due to model reductions considering the commonly used Guyan reduction process. Additional analyses were also performed using SEREP and Hybrid reduction schemes. These reduction schemes are known to have better overall representation of higher modes where the Guyan reduction process degrades in accuracy. While time responses are not shown in this paper, the TRAC values are shown in Table 4 to show the improvement in using the more accurate reduction schemes. (NOTE: Hybrid is not shown in the Table because those results are almost identical results to those obtained from the SEREP technique)

<table>
<thead>
<tr>
<th></th>
<th>TRAC values, compare to reference model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.05 seconds of response</td>
</tr>
<tr>
<td><strong>Wide (low frequency) pulse</strong></td>
<td></td>
</tr>
<tr>
<td>Guyan reduced model, updated with 10 modes</td>
<td>0.9947</td>
</tr>
<tr>
<td>Guyan reduced model, updated with 30 modes</td>
<td>0.9944</td>
</tr>
<tr>
<td>SEREP reduced model, updated with 10 modes</td>
<td>0.9944</td>
</tr>
<tr>
<td>SEREP reduced model, updated with 30 modes</td>
<td>0.9944</td>
</tr>
<tr>
<td><strong>Narrow (high frequency) pulse</strong></td>
<td></td>
</tr>
<tr>
<td>Guyan reduced model, updated with 10 modes</td>
<td>0.8610</td>
</tr>
<tr>
<td>Guyan reduced model, updated with 30 modes</td>
<td>0.9258</td>
</tr>
<tr>
<td>SEREP reduced model, updated with 10 modes</td>
<td>0.9100</td>
</tr>
<tr>
<td>SEREP reduced model, updated with 30 modes</td>
<td>0.9258</td>
</tr>
</tbody>
</table>
The second method is a further enhancement of Approach 1, in which only a subsystem (i.e., the wing alone) is used for the original model description, but the target modes are obtained from the modal test with the wing on the assemblage. In this case, the model of the wing can be used to represent the entire assembled structure. The wing part of the full reference model was used for this case. The wing alone was reduced to 15 DOF using Guyan reduction, implemented in FEMtools. Again, other reduction schemes might be advantageous, but Guyan was used here because it is commonly used and easily implemented in many software packages. The full wing model, along with the selected ADOF, are shown in Figure 17. The target mode shapes are obtained from the wing attached to the main structure but with only measurements made on the wing portion of the model corresponding to the ADOF as shown in Figure 18. This reduced-order model provided the mass and stiffness matrices which were used as seeding matrices for the model updating. The static flexibility due to one of the load cases evaluated along with the forced response for the two different loading conditions are presented below.

Figure 17. Model of wing with selected 15 ADOF.

Figure 18. Targets for updating of wing.
Results from Static Loading – Flexibility Analysis

The model was updated and the improved stiffness was used for a static load evaluation. The reference model was used to compute the actual displacements and then compared to the results of the updated stiffness flexibility results. In all cases studied, the average error was less than 1%. Table 5 shows the flexibility results of the reduced updated model. This model provides a good representation of the flexibility of the full structure, even though this is a model of only the wing—the average error is 0.69%. (This result was typical of all load cases studied for this configuration.)

Table 5. Approach 2, Load case 3.

<table>
<thead>
<tr>
<th>Node</th>
<th>Original full model displacement</th>
<th>Guyan reduced model of wing, updated with 15 modes</th>
<th>% difference from original</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>0.042983</td>
<td>0.042832</td>
<td>-0.35%</td>
</tr>
<tr>
<td>145</td>
<td>0.050203</td>
<td>0.049702</td>
<td>-1.00%</td>
</tr>
<tr>
<td>161</td>
<td>0.11058</td>
<td>0.1095</td>
<td>-0.98%</td>
</tr>
<tr>
<td>207</td>
<td>0.092448</td>
<td>0.091462</td>
<td>-1.07%</td>
</tr>
<tr>
<td>861</td>
<td>0.16852</td>
<td>0.16638</td>
<td>-1.27%</td>
</tr>
<tr>
<td>873</td>
<td>0.16743</td>
<td>0.16632</td>
<td>-0.66%</td>
</tr>
<tr>
<td>885</td>
<td>0.16291</td>
<td>0.16199</td>
<td>-0.56%</td>
</tr>
<tr>
<td>897</td>
<td>0.15275</td>
<td>0.15144</td>
<td>-0.86%</td>
</tr>
<tr>
<td>909</td>
<td>0.13892</td>
<td>0.13876</td>
<td>-0.12%</td>
</tr>
<tr>
<td>1246</td>
<td>0.10603</td>
<td>0.10443</td>
<td>-1.51%</td>
</tr>
<tr>
<td>1261</td>
<td>0.052191</td>
<td>0.051791</td>
<td>-0.77%</td>
</tr>
<tr>
<td>38201</td>
<td>-0.037743</td>
<td>-0.037635</td>
<td>-0.29%</td>
</tr>
<tr>
<td>38414</td>
<td>-0.04669</td>
<td>-0.046531</td>
<td>-0.34%</td>
</tr>
<tr>
<td>38425</td>
<td>-0.047863</td>
<td>-0.047712</td>
<td>-0.32%</td>
</tr>
<tr>
<td>38451</td>
<td>-0.038503</td>
<td>-0.038423</td>
<td>-0.21%</td>
</tr>
</tbody>
</table>

Average % difference: -0.69%

Results from Updated Model Forced Response – Low Frequency Pulse – Wing Alone

The forced response due the loading applied is shown in Figure 19. The model was updated using only 10 of the lower order modes of the system. The overall response compares very well with the reference solution from the ABAQUS 82000 DOF model with a TRAC value of 0.9721. Additional modes were added to the updated model but essentially no difference in the response results. This is due to the low frequency excitation which only requires the lower order modes of the system to identify reasonable response. Comparing these results to those previously computed, the reduced model of the wing alone (updated with modes with the wing attached to the assembly) are essentially the same. This is an extremely important result since the whole structure model need not be created to obtain accurate results. (In this case only modeling the wing and not including the whole helicopter fuselage offers tremendous advantages.)
The forced response due the high frequency loading applied is shown in Figure 20. The model was updated using only 10 of the lower order modes of the system. The overall response compares reasonably well with the reference solution from the ABAQUS 82000 DOF model with a TRAC value of 0.8173. As in the case of the full model, considering that only 10 modes were used to describe the wing component and the excitation excites a much wider frequency range, these results are very good overall. The first 20% of the response is shown more clearly in Figure 21. From this plot it is clear that the low frequency trend is captured with 10 modes but that higher frequency response is not capture well as expected using only 10 modes. With the addition of 5 extra modes, the updated model using 15 modes shows some slight improvement in the results shown in Figure 22 and 23. Since there are only 15 DOF on the wing, in order to improve the results more ADOF on the wing would be required so that additional modes could be added to the model. Because the intent is to address low order mode response, these cases were not studied. (However, there is no reason to expect that this technique could not easily be extended to higher frequencies with the addition of more modes.)
Figure 20. Response of Guyan-reduced model updated with 10 modes, to narrow pulse, compared to reference model.

Figure 21. Response of Guyan-reduced model updated with 10 modes, to narrow pulse, compared to reference model.
Figure 22. Response of Guyan-reduced model updated with 15 modes, to narrow pulse, compared to reference model.

Figure 23. Response of Guyan-reduced model updated with 15 modes, to narrow pulse, compared to reference model
The results shown in Figures 20 through 23 are due to model reductions considering the commonly used Guyan reduction process. Additional analyses were also performed using SEREP and Hybrid reduction schemes. These reduction schemes are known to have better overall representation of higher modes where the Guyan reduction process degrades in accuracy. While time responses are not shown in this paper, the TRAC values are shown in Table 6 to show the slight improvement in using the more accurate reduction schemes. (NOTE: Only SEREP is shown in the Table and almost identical results are expected from the Hybrid technique.) (Also note: When all 15 modes are used with 15 ADOF, the results of all the techniques are the same since an exact solution is obtained and is not dependent on the seed matrix used.

<table>
<thead>
<tr>
<th>Wide (low frequency) pulse</th>
<th>TRAC values, compare to reference model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guyan reduced model, updated with 10 modes</td>
<td>0.9945</td>
</tr>
<tr>
<td>Guyan reduced model, updated with 15 modes</td>
<td>0.9934</td>
</tr>
<tr>
<td>SEREP reduced model, updated with 10 modes</td>
<td>0.9941</td>
</tr>
<tr>
<td>SEREP reduced model, updated with 15 modes</td>
<td>0.9934</td>
</tr>
</tbody>
</table>

| Narrow (high frequency) pulse | | |
|-----------------------------|------------------|
| Guyan reduced model, updated with 10 modes | 0.8445 | 0.8173 |
| Guyan reduced model, updated with 15 modes | 0.9311 | 0.8904 |
| SEREP reduced model, updated with 10 modes | 0.8940 | 0.8579 |
| SEREP reduced model, updated with 15 modes | 0.9311 | 0.8903 |
The third method is yet a further enhancement of the second approach. This approach is very similar to Approach 2 except that the targets are not obtained by assembly of the components to form the system. Rather, the FRFs of each of the unconnected components are used to compute an impedance model to form the system representation. The target mode shapes are then obtained from the impedance system model. Again the wing is modeled alone to form a reduced model as in Approach 2. But the target modes come from the impedance approach to develop the system targets. This is shown schematically in Figure 24.

The wing alone was reduced to 15 DOF using Guyan reduction, implemented in FEMtools. Again, other reduction schemes might be advantageous but are not used or needed here. The full wing configuration, along with the selected ADOF, are shown in Figure 25. The target mode shapes are obtained from the wing attached to the main structure but from an impedance modeling approach where the components are never physically assembled to determine the system characteristics as seen schematically in Figure 24. The reduced-order model provided the mass and stiffness matrices which were used as seeding matrices for the model updating. The results from this approach are essentially the same as that of Approach 2. (The flexibility results are not shown because they are essentially the same as the previous results.) The force response for the two different loading conditions are described below.
In order to develop the target mode shapes for the impedance model approach, the following steps were taken. FRFs were synthesized for both models at the connection DOF and the remaining response locations on the wing component. The synthesized FRFs were used to attach the wing component to the upright structure. This process is shown graphically in Figure 26. The resulting assembled system model then contains FRFs at all DOF describing the system. These FRFs are used to identify the target frequencies and mode shapes. Modal parameters were estimated from the system FRFs using LMS [18]. The extracted frequencies and mode shapes are then used to update the unmodified wing component using MATLAB [16].

\[
h_{Cj} = h_{Aj} - [H_{A}S_{j}] [H_{A}S_{j}]^{-1} [H_{A}]_{Sj}
\]

*Figure 26 – System modeling using impedance approach*

**Results from Updated Model Forced Response – Low Frequency Pulse – Wing Alone**

The forced response due the low frequency loading applied is shown in Figure 27. The model was updated using only 10 of the lower order modes of the system. The overall response compares very well with the reference solution from the ABAQUS 82000 DOF model with a TRAC value of 0.9714. Additional modes were added to the updated model but essentially no difference in the response results. This is due to the low frequency excitation which only requires the lower order modes of the system to identify reasonable response. Note that this approach offers great advantages over Approach 2 in that the wing need not be attached to the main structure to obtain targets – only the measured impedance at the attachment points are needed. Another important feature is that the wing dynamics can easily be addressed if the wing is attached to a variety of different helicopter airframes or various test fixtures for performance studies. This allows for significant flexibility in analyzing alternate configurations.

*Figure 27. Response of Guyan-reduced model updated with 10 modes, to wide pulse, compared to reference model*
Results from Updated Model Forced Response – High Frequency Pulse – Wing Alone

The forced response due the high frequency loading applied is shown in Figure 28. The model was updated using only 10 of the lower order modes of the system. The overall response compares reasonably well with the reference solution from the ABAQUS 82000 DOF model with a TRAC value of 0.8594. As in the case of the full model, considering that only 10 modes were used to describe the wing component and the excitation excites a much wider frequency range, these results are very good overall. The first 20% of the response is shown more clearly in Figure 29.

The wing model was also updated using 15 modes and the forced response results are shown in Figure 30. The response compares reasonably well with the reference solution from ABAQUS with a TRAC of 0.9530; the first 20% of the response is shown more clearly in Figure 31. The results of this case improve as more modes are added to the component description and updated component model.

Table 7 presents the results for the TRAC values for the cases presented here as well as for some cases not shown. The results are very good for the models developed.

![Figure 28](image)

Figure 28. Response of Guyan-reduced model updated with 10 modes, to narrow pulse, compared to reference model
Figure 29. Response of Guyan-reduced model updated with 10 modes, to narrow pulse, compared to reference model

Figure 30. Response of Guyan-reduced model updated with 15 modes, to narrow pulse, compared to reference model
**OBSERVATIONS**

Based on the work performed to date on this approach (and related work), several observations can be made regarding the development of these models. Also, some cautions that may be necessary to adequately utilize the techniques are described.

It was observed that the number of modes used as targets in the updating had only a small effect on the accuracy of the resulting model; between 10 and 33 target modes were utilized for the updating process. While this needs to be further investigated, the implication is that a large number of modes may not be necessary for the development of the reduced model for low frequency applications. Of course, this may only be applicable to the particular model used in this investigation. It may also be due to the fact that the majority of the flexibility is dominated by the lower order modes of this system. In fact, the lower order modes are the modes of significance for the overall system flexibility in multibody dynamic studies. This, however, is expected to be highly dependent on the particular model studied and use of this technique for general high frequency applications requires further study. In particular, rotational degrees of freedom may become more critical as the frequency increases.
For the models presented here, the Guyan reduction process was utilized. This is a commonly used approach and available in most finite element software packages. However, the Guyan reduction technique is highly sensitive to the selection of the ADOF used to generate the reduced model. If an inappropriate distribution of DOF is selected, then the reduced model will likely contain a distorted representation of one or more of the modes of the full system. When this is the case (as studied in several cases not reported herein), there was a very poor overall representation of the reduced modes and the updated model did not produce accurate results. Alternate reduction schemes such as SEREP and Hybrid may then be used to generate a more accurate reduced model. While some preliminary work has been done using these approaches, additional work needs to be completed in order to determine whether this will help the situation overall; this will be the subject of further research.

CONCLUSIONS

Modeling of multibody dynamics often requires dynamic flexibility of attachment/supporting structures in order to properly address the overall system dynamics. These attachment/supporting structures often require a detailed FEM, validated by test. This effort can be very time consuming and there are often difficulties in the model updating process.

Using direct model updating strategies with reduced substructure methodologies and impedance based system model techniques, drastically reduced models can be more readily adjusted using measured data and provide very accurate representations of the system flexibility. Data cases are presented to show the adjustment of a typical configuration to mimic a helicopter/wing configuration for missile launch dynamic response studies. Reduced models of the assemblage (wing and supporting structure) are used in a direct model updating scheme and provide accurate representations of the system as shown in the cases evaluated.

As an extension to the typical approach, a reduced model is developed of a subsystem of the assemblage but is updated with target modes from the full assemblage using a direct updating technique. These results also provide an accurate representation of the system for the cases studied. This is a significant advantage since the reduced model need not incorporate the entire assemblage to provide useful results. The target modes, however, must be obtained from the system assemblage in order to update the reduced model subsystem. The technique provides an extremely useful alternative to the traditional approach using full model representations of subsystems.

Another extension of this approach allows for the target modes to be identified using an impedance based modeling technique which does not require physical assembly of the system components to obtain target modes – these targets come from the results of the impedance based system model generated from component information. These results also provide very accurate response results for the cases studied. Again this is a significant advantage since a very reduced order model is used for the determination of the component response and provides yet another extremely useful technique.

In summary, several alternate techniques have been developed which provide very reduced order, test verified component and system models which can be identified using either assembled or unassembled component dynamic characteristics that were shown to be extremely accurate representations for flexible component descriptions for multibody dynamic simulations.

ACKNOWLEDGMENT

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NOTATION

**Matrix**

- $[M]$ analytical mass matrix
- $[K]$ analytical stiffness matrix
- $[U]$ analytical modal matrix
- $\tilde{[M]}$ diagonal modal mass matrix
- $\tilde{[K]}$ diagonal modal stiffness matrix
- $[T]$ transformation matrix
- $[E]$ experimental modal vectors
- $[I]$ identity matrix
- $[Ω^2]$ diagonal matrix of $ω^2$ values

**Vector**

- $\{X\}$ displacement

**Subscript**

- $n$ full set of finite element DOF
- $a$ tested set of experimental DOF
- $d$ deleted (omitted) set of DOF
- $G$ Guyan
- $U$ SEREP
- $H$ Hybrid
- $s$ seed matrix
- $I$ improved or updated matrix

**Superscript**

- $T$ transpose
- $g$ generalized inverse
- $-1$ standard inverse
- $A$ Component A
- $B$ Component B
- $C$ Assembled System
- $T$ transpose

**Acronyms**

- **DOF** Degrees of freedom
- **ADOF** Reduced degrees of freedom
- **AMI** Analytical Model Improvement method
- **SEREP** System Equivalent Reduction Expansion Process

\[
[V] = [E]^T [\tilde{M}]^{-1} [E]^T [M] = [\tilde{M}]^{-1} [E]^T [M] \quad \text{generalized inverse}
\]
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