

Received: 09 October 2018 / Accepted: 29 November 2018 / Published online: 20 December 2018

*automatic transmissions gearboxes, Hsu graph,
optimization, game-tree structures,
computer analysis*

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APPLICATION OF COMPLEX GAME-TREE STRUCTURES FOR THE HSU GRAPH IN THE ANALYSIS OF AUTOMATIC TRANSMISSION GEARBOXES

In the article was discussed the possibility of structures and information systems complex game trees for the analysis of automatic gearboxes. The purpose of modelling an automatic gearbox with graphs can be versatile, namely: determining the transmission ratio of individual gears, analysing the speed and acceleration of individual rotating elements. In a further step, logic tree-decision methods can be used to analyse functional schemes of selected transmission gears. Instead, for graphs that are models of transmission, parametrically acting tree structures can be used. This allows for the generalization and extension of the algorithmic approach, furthermore in the future it will allow further analyses and syntheses, such as checking the isomorphism of the proposed solutions, determining the validity of construction and / or operating parameters of the analysed gears. The game tree structure describes a space of possible solutions in order to find optimum objective functions. There is the connection with other graphical structures which can be graphs in another sense, or even decision trees with node and/or branch coding.

1. INTRODUCTION

Analysis and synthesis of mechanisms can be performed by means of versatile methods. These tasks can suffer from human errors. So, it is reasonable to have some alternative methods which allow for comparison of results and for detection of almost unavoidable mistakes. The graph-based methods deliver such alternative approaches for modelling of a wide class of mechanical systems. There were also some other attempts to model planetary gears via diagrams e.g. Wolf's pictograms [1], but these methods did not further evolve due to lack of generalization and lack of connections with other branches of [2, 3] mathematics. Also, the method based on signal flow graph theory for modelling of gears [4, 5] has not been too frequently used. On the contrary, the graph based methods have been independent, intensively developed for several recent years all over the world, see e.g. [4–6]. Engineering practice requires a correct evaluation of the mathematical model describing a given system with some variables. A proper mathematical model contains

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<https://doi.org/10.5604/01.3001.0012.7713>

a group of functions joining different variables and describing connections between quantities in the system. Decision tables [6–9] and logical functions [7], can be applied in the simulation of machine systems, for example, described by ordinary or partial differential equations. It results from the fact that the occurring nonlinear elements can be divided into the finite number of linear elements (parts), and in a consequence, we obtain some linear systems as for simulation from the primary single nonlinear system. Analysis and synthesis of mechanisms can be performed by means of versatile methods.

Graph theoretical ideas are highly utilized by computer science applications. Especially in research areas of computer science data mining, clustering, images capturing, network etc. Graph theoretical concepts are widely used to study and model various application, in the different area. For example, the traveling salesman problem, the shortest spanning tree in a weighted graph, obtaining an optimal match of jobs and men and locating the shortest path between two vertices in a graph. In the last dozen or so years there has been an extremely rapid pattern of applications of graph theory, decision trees and networks in various fields of science and technology. We can mention here: the theory of operational research, genetics, linguistics, econometrics, electronics, and others. Thus, the need for further development of this theory and its methods increased. From a mathematical point of view, the theory of graphs and decision trees does not bring anything new. The advantage of these methods, however, is the adaptation of mathematical methods and apparatus to the convenient modelling of phenomena, which are increasingly devoted to the contemporary world. Graphs, networks, and trees are a convenient formal apparatus for system modelling of machine, hydraulic and transmission systems [10–12].

The article is another part concerning the analysis of an automatic gearbox modeled with the Hsu graph using multi-valued logic trees and parametric graphs. Paper [13] discusses the use of decision logical trees and dependency graphs. Multi-valued logical trees were used and the Hsu graph was separated from the selected yoke into parametrically acting structures.

The work [14] describes the application of decision logical trees and predominant logical variables information systems. In turn, the work [15] additionally includes the use of information systems. The current article indicates the need to include complex game-tree structures in the analysis of automatic gearboxes.

2. THEORY- GRAPHIC MODELS OF TRANSMISSIONS

The objectives of modelling the gears with graphs were varied – among others: dynamic analysis, kinematic analysis [16], synthesis [17, 18] structure analysis [19] enumeration, optimization of gear sequences and automatic design based on the so-called graph grammars. The advantage of modelling gears with graphs is that issues considered using graph models can be solved in an algorithmic manner, which allows the use of computer programs and widely understood integrated decision-making systems in a simple manner. The graph in the sense of graph theory is associated with many other algebraic structures as, for example, arrays, matroids, structural numbers, linear spaces of cutoffs. These objects enable the coding of the transmission structure, which allows

the use of advanced algorithms of artificial intelligence: evolutionary, formic, genetic or immunological. An important advantage of modelling mechanical systems with graphs is, among others, that some considerations can be carried out in parallel in the field of mechanics and graph theory [16]. The relevance of the results is based on the transformation of knowledge between these two areas. At present, there is considerable interest in graphical methods in optimization, and especially in the modelling of gears, hydraulic systems, all mechanisms, trusses, and frames. Among the methods of analysis of planetary gears, one can distinguish among others [19–21].

The contour graph method used for the analysis of mechanical systems was discussed, among others in [21–23]. It is particularly useful for considering mechanisms of various types (so-called planar, crossheads, etc.). In particular, it can be used in the analysis of planetary gears. The idea of this method is based on distinguishing a series of subsequent rigid links of mechanisms that form a closed loop - the so-called contour. Unlike graphs, dendrite-tree structures do not have cycles, but there may be a different number of initial vertices. Therefore, a different approach has been developed as a translation of a directed graph of dependence, among others for parametrically acting structures. This approach is different from the previous literature on parametric automation machines and their applications, related to control systems, operating systems, importance analysis of construction and / or operating parameters, and analysis of gears previously modeled using other types of graphs. For example, in [22], the structures that parametrically used for the contour graph were used as a further step in the analysis of planetary gears. In works [13, 14, 15] multivalent logical trees and structures that are parametrically used in the analysis of automatic gearboxes, previously modeled with the Hsu graph [1, 19].

3. THE DEPENDENCE GRAPH FOR TREE GAME STRUCTURES

The equations of dynamics can be used for determination of mutual connections of all the functions dependent on time. As a result of notation and decomposition of the dependence graph of those functions, we obtain the groups of distribution which describe properties of successive subsystems of the considered machine system and a set of suitable constructional and service parameters. An oriented graph can be defined as an ordered pair of sets. The graph vertices are included in the first one, whereas graph edges, that is an ordered pair of vertices, are included in the second one (Fig. 1) [6].

The oriented game graph is shown in Fig. 1 is composed of a set of vertices Q :

$$Q = \{q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8\} \quad (1)$$

and of a set of edges Z , that is an ordered pair of vertices:

$$Z = \{z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8, z_9, z_{10}, z_{11}, z_{12}, z_{13}\} \quad (2)$$

As a result of a graph distribution from the chosen vertex, a tree structure with cycles is obtained in the first step and then, a general game tree structure is obtained. Each of them has an appropriate analytical formulation G_i^+ and G_i^{++} .

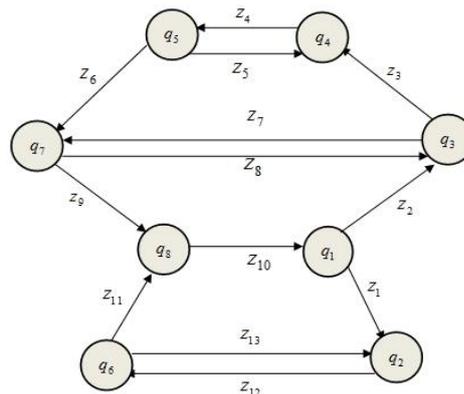


Fig. 1. An oriented game graph

A game tree structure is a part of the systematic searching method. A start vertex q_i is chosen in the first step. Acting in accordance with the algorithm and assuming that the start vertex is q_1 , it is possible to transform the oriented dependence graph presented in Fig. 1 into an analytical formulation $G_{q_1}^+$, and then we obtain the following formulation as a result of the operation (3).

$$G_{q_1}^+ = ({}^0q_1({}^1z_2q_3({}^2z_7q_7({}^3z_8q_3, z_9q_8({}^4z_{10}q_1)^4)^3), z_3q_4({}^3z_4q_5({}^4z_5q_4, z_6q_7)^4)^3)^2, z_1q_2({}^2z_{12}q_6({}^3z_{11}q_8, z_{13}q_2)^3)^2)^1)^0 \tag{3}$$

The next step is to obtain a tree structure that plays a parametrically defined expression.

$$G_{q_1}^{++} = ({}^0q_1({}^1z_2q_3({}^2z_7q_7({}^3z_8q_3^1, z_9q_8({}^4z_{10}q_1^1)^4)^3), z_3q_4({}^3z_4q_5({}^4z_5q_4^1, z_6q_7({}^5z_8q_3^2, z_9q_8({}^6z_{10}q_1^2)^6)^5)^4)^3)^2, z_9q_8({}^6z_{10}q_1^2)^6)^5)^4)^3)^2, z_1q_2({}^2z_{12}q_6({}^3z_{11}q_8({}^4z_{10}q_1^3)^4, z_{13}q_2^1)^2)^1)^0 \tag{4}$$

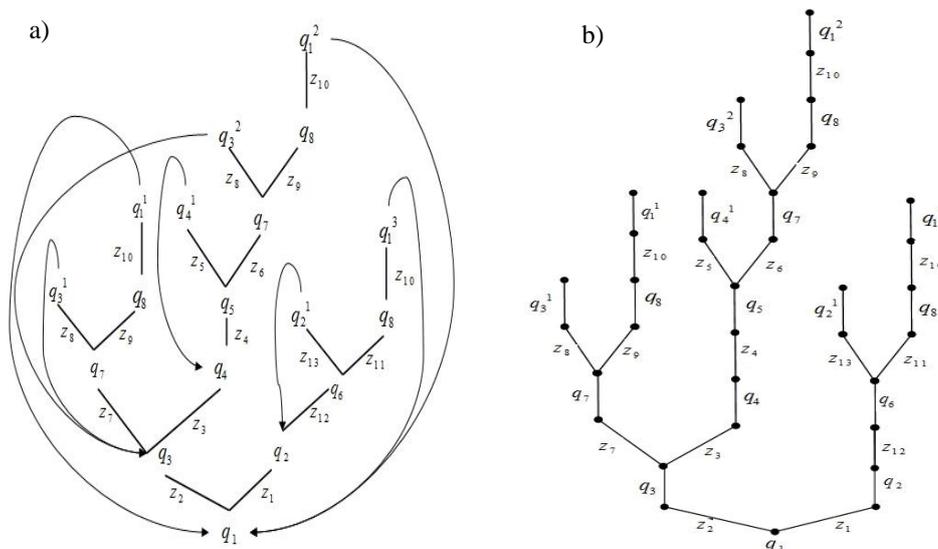


Fig. 2. The tree structure with cycles and the initial vertex q_1 (a) and game tree game structures from the initial vertex q_1 (b)

It is possible to return to an earlier vertex and even to a start vertex from an appropriate end vertex, so we obtain a game tree structure presented in Fig. 2.

The contour graph distribution from any vertex in the first stage leads to a tree structure with cycles, and next to a general tree game structure. Each structure has a proper analytic notation G_i^{++} - where i - is a vertex, from which the graph decomposition started determining a way of transition from the dependence graph to the tree structure.

3.1. A GRAPHICAL MODEL ON THE EXAMPLE OF A PLANETARY GEAR

The general idea of the graph-based modelling of mechanical systems consists in the following steps [1–3]:

– *discretization of a mechanical system*. This means that appropriate simplifications have to be made. Some aspects are omitted. Some structural elements are considered as essential and they are interpreted as graph vertices. Some connections or relationships between these elements are abstracted. They are represented via graph edges, usually, some system of weights can be assumed to the edges or vertices and edges,

– *assignment of the graph to the mechanism* (especially planetary gear) based upon special rules. There are several different rules depending on the object of modelling and problems solved via the graph based method,

– *derivation of special subgraphs, e.g. f-cycles or contours*. These subgraphs can be singled out based upon the graph-theoretical algorithms what causes that the approach is simple and algorithmic,

– *listing the codes of these graph elements*. The encoding rules are clearly defined what allows for avoidance of mistakes,

– *generation of a system of equations in an algorithmic way using the codes*. These codes allow for management (assignment) of the indicators of variables existed in the considered equations in a straightforward manner,

– *the solution of the obtained system* in a chosen algebraic way to obtain needed angular velocities, ratios, forces, accelerations etc.

The similar routine can be formulated for the reverse order of activities, i.e. synthesis: going from the graph generation towards the creation of a gear functional structure [1, 3].

The contour method of modelling of a mechanical system consists in the creation of a special graph enclosing the contours, i.e. closed circles built of arrows connecting the vertices representing the elements of the system.

Example: An exemplary planetary gear and its contour graph are shown in Fig. 3

For the considered gear, the following data for teeth numbers and the module are assumed: $z_1 = 15$; $z_2 = 24$; $z_3 = 63$ (-63); $z_4 = 18$; $z_5 = 21$ and $z_6 = 60$ (-60), $m = 2$ mm, where negative values of the teeth numbers are considered for the internal gearing in the case of the Willis method, and one common module for all meshings has been assumed.

For example, graph distribution from vertices O leads to analytical expressions (5):

$$G_{p0}^{++} = ({}^0 0({}^1 [II] \cdot h({}^2 [II] \cdot 2({}^3 [I] \cdot h^1, [III \wedge IV] \cdot [6/3]({}^4 [II] \cdot 0^1)^4)^3, [I] \cdot 0^2)^2, [I \wedge III] \cdot [1/4]({}^2 [I \wedge IV] \cdot 2({}^3 [I] \cdot h^2, [III \wedge IV] \cdot [6/3]({}^4 [II] \cdot 0^3)^4)^3, [III \wedge IV] \cdot 5({}^3 [III] \cdot H({}^4 [III] \cdot 0^4)^4, [IV] \cdot [6/3]({}^4 [II] \cdot 0^5)^4)^3)^2)^1)^0 \quad (5)$$

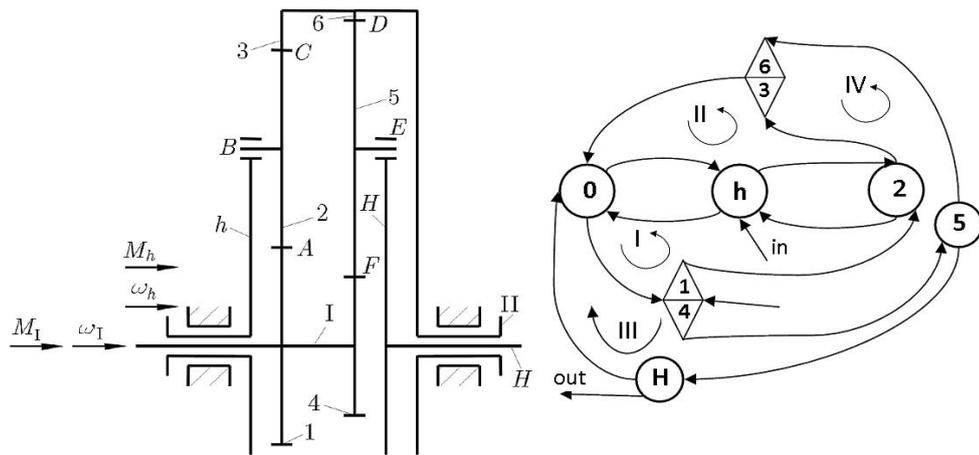


Fig. 3. Functional scheme of an exemplary planetary gear and its contour graph: 1, 2, . . . , 6 sun wheels, wheels with internal toothings and planets; h, H – arms; A, B, \dots, F – characteristic points, M – angular velocities and torques [21–23]

Figure 4 shows the game structures G_0^{++} from the initial vertex 0.

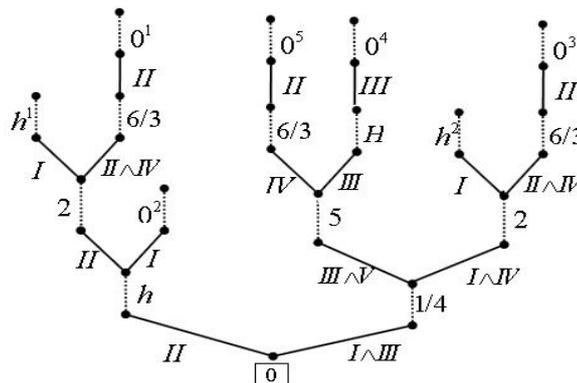


Fig. 4. Tree game structures from the initial vertex 0

4. ANALYSIS OF THE AUTOMATIC TRANSMISSION GEARBOXES INCLUDING COMPLEX GAME-TREE STRUCTURES

Among the methods of analysis of planetary gears, one can distinguish among others [17, 18, 19]. In the case of Hsu rules [24], the graph is built according to the following rules: geometrical dimensions are omitted and kinematic pairs are considered: rotary, “planet – yoke” and meshing. In particular, it can be used in the analysis of planetary gears. The idea of this method is based on distinguishing a series of subsequent rigid links of mechanisms that form a closed loop – the so-called contour. Unlike graphs, dendrite-tree structures do not have cycles, but there may be a different number of initial vertices. Therefore, a different approach has been developed as a translation of a directed graph of dependence, among others for parametrically acting structures.

In the case of analysis of car gearboxes, the graph-based methodology is also efficient using additionally transformations of a basic graph according to a particular gear drive. An introductory comparison of the graph and contour methods for analysis of gears was done by the authors in [1, 13, 14].

The analysis of automatic gearboxes is similar to the analysis of a single planetary gear [25] (Fig. 5).

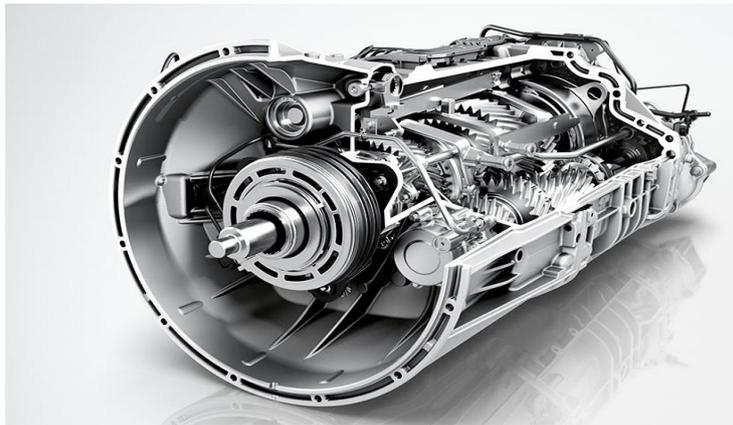


Fig. 5. Model drawing of an exemplary automatic gearboxes [25]

The analysis is carried out for each run separately by introducing some transformations of the respective graphs. A novelty proposed by Zawislak [1] is the modification of the Hsu graph by introducing a path from the entrance to the exit. This path is formed by the corresponding edges of the gear graph. Input and output are marked additionally. This path allows the analysis of the sequence of transmission of rotational motion by subsequent elements of the transmission. In addition, it allows the detection of so-called redundant elements for a given gear currently under consideration. The consequence of this approach is the idea of transforming the graph proposed in the work. The results obtained in subsequent decisions depend on the initial decisions, which allows the creation of dynamic models. In the case of automatic gearboxes modeled with the Hsu graph, the following aspects of artificial intelligence can be distinguished in graphical modelling of mechanical systems: – automatic generation of systems of equations describing certain dynamic properties of systems, – transfer of knowledge in the field of mechanics to the field of graph theory and reverse direction, – electromechanical analogies (network generalization). The transformation of knowledge related to automatic gearboxes in the field of mechanics consists in transformation, i.e. expression of relations, laws in the field of mechanics with the help of graph concepts. Then, from the Hsu graph after the transformation back to equations, you can get the desired transmission value. Fundamental cycles can be determined on the basis of an algorithm referring to a special matrix representing the Hsu graph. On the parametric trees, parametrically from each of the top of the graph, the decision process and the space of possible states of the analyzed system are described. Each parametric structure that is obtained requires the calculation of a complex complexity index. In tree structures in terms of the increase in entropy (information) as a heuristic for the choice of a decision parameter, it is important to have

branches as far away as possible from the root of the structure. Then, the information string is kept for as long as possible and only at the branch, the selection of decision variables is appropriately separated into new nodes (subtree). Therefore, the value of complex complexity coefficients of trees in which branching occurs on the top floors is the smallest. The use of parametric trees is to allow to obtain the desired transmission value directly from the Hsu graph – after it has been transformed into a dependency graph.

The review analysis of the graph application issues in the transmission analysis can include: Kinematic analysis, torque analysis, power flow analysis and mechanical efficiency analysis. The velocity equation for each basic epicyclic gear train can be written as follows:

$$\omega_i - \omega_j N_{ij} + \omega_k (N_{ij} - 1) = 0 \quad (6)$$

where: $\omega_i, \omega_j, \omega_k$ denote the angular velocities of links i, j, k and

$$N_{ji} = \pm \frac{z_j}{z_i} (z_i, z_i) \quad (7)$$

denote the number of teeth on gears i, j , respectively, and the ratio is positive or negative [1].

The torque analysis and mechanical analysis are described, among others, in the works [26–32]. During the process of kinematic structure enumeration using graph theory, isomorphism identification of graphs is an important and complicated problem. There are many approaches to detect isomorphic graphs, and these approaches have been largely algorithmic. The following will present an isomorphism identification approach applying the concept of structural code – which was proposed by Hsu [33–35].

4.1. ANALYSIS OF AN EXEMPLARY AUTOMATIC GEARBOX INCLUDING GAME-TREE STRUCTURES

Figure 6 shows an example of an automatic transmission gearbox performing four gears.

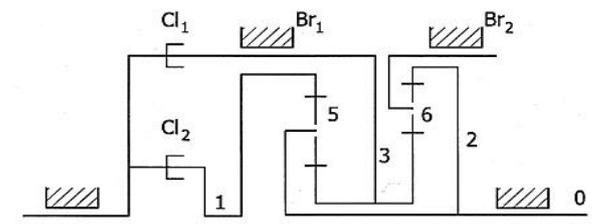


Fig. 6. Functional diagram of an exemplary automatic gearbox, where: Cl – clutch, Br – brake automatic gearboxes [1]

The automatic clutch and brake control system makes it possible to achieve next gears, hence table 1 lists the corresponding sequences of control settings. In the work of the gear unit, it is assumed that the clutch Cl and the brake Br can take two states:

1 and 0 (1 – active, 0 – passive). For the gears from figure 5 there are 4 decision variables: Cl_1, Cl_2, Br_1, Br_2 – divalent (Table 1) [1–3].

Table 1. Sequences of control elements in the considered transmission [1]

Control Drive	Cl_1	Cl_2	Br_1	Br_2
I	0	1	0	1
II	0	1	1	0
III	1	0	1	0
Rev	1	0	0	1

In the case of Hsu rules, the graph is built according to the following rules: geometrical dimensions are omitted and kinematic pairs are considered: rotary, “planet – yoke” and meshing. A set of equations describing transmission kinematics is generated. Each equation is assigned to one f -cycle. The number of f -cycles is equal to the number of bar edges, and for each f -cycle the equation is met:

$$\omega_i - \omega_k = \pm N_{j,i}(\omega_j - \omega_k)$$

Figure 7 shows a graph representing the general structure of the transmission from Fig. 6. For graphs in the drawing, a new idea of graphing the transmission path of the rotational motion through the transmission was used [1, 13, 14].

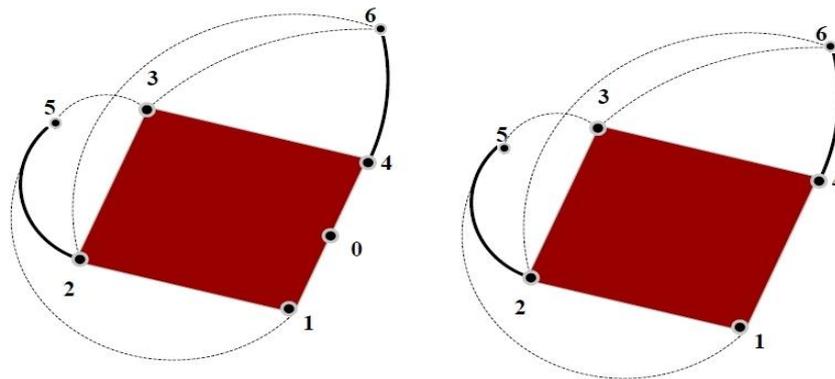


Fig. 7. Graphs for an exemplary automatic gearbox from Figure 6 [1, 13, 14]

The path starts from the input element and ends on the output element. In addition, incident edges to the braked element are drawn with double lines. Figure 8a shows a simplified diagram of the gear for 1st gear, while the graph for this gear is shown in Fig. 8b.

For a graph with a path from the entrance to the output from Fig. 6, it is possible to build a set D of parametrically acting structures

$$D = \{G_1^{++}, G_2^{++}, G_3^{++}, G_3^{++}, G_5^{++}, G_6^{++}\} \tag{8}$$

For the needs of algebraization, the speed indication is also assigned ω_i to the graph as indexes of the order of the read edges (Fig. 9).

Using the algorithm of graph distribution of the signal flow from the starting vertex q_1 , a structure with the cycles $G_{q_1}^+$ described by the expression (9) is obtained

$$G_{q_1}^+ = ({}^0q_1({}^1\omega_1q_5({}^2\omega_5q_3({}^3\omega_3q_6({}^4\omega_4\omega_6q_4({}^5\omega_4\omega_6q_6, kq_1, kq_3)^5, \omega_6q_2({}^5kq_3, kq_1, \omega_2\omega_5q_5)^5)^4, kq_4, kq_2)^3, \omega_2\omega_5q_2)^2, kq_4, kq_2)^1)^0 \quad (9)$$

Figure 10 shows the game structures $G_{q_1}^+$ with cycles and without cycles from the initial vertex.

Next, the Hsu graph was decomposed from the other three vertices q_2, q_3, q_4, q_5, q_6 forming the k -click, receiving structures $G_{q_2}^+, G_{q_3}^+, G_{q_4}^+, G_{q_5}^+, G_{q_6}^+$.

$$G_{q_2}^+ = ({}^0q_2({}^1kq_3({}^2\omega_3q_6({}^3\omega_4\omega_6q_4({}^4\omega_4\omega_6q_6, kq_1({}^5\omega_1q_5({}^6\omega_5q_3, \omega_2\omega_5q_2)^6, kq_2)^5, kq_3)^4, \omega_6q_2)^3)^2, kq_1, \omega_2\omega_5q_5)^1)^0 \quad (10)$$

$$G_{q_3}^+ = ({}^0q_3({}^1\omega_3q_6({}^2\omega_4\omega_6q_4({}^3\omega_4\omega_6q_6, kq_1({}^4\omega_1q_5({}^5\omega_5q_3, \omega_2\omega_5q_2({}^6kq_3, kq_1)^6)^5, kq_2)^4, kq_3)^3, \omega_6q_2)^2, kq_2, kq_4)^1)^0 \quad (11)$$

$$G_{q_4}^+ = ({}^0q_4({}^1\omega_4\omega_6q_6({}^2\omega_4\omega_6q_4, \omega_6q_2({}^3kq_3({}^4\omega_3q_6, kq_4)^4, kq_1({}^4\omega_1q_5({}^5\omega_5q_3, \omega_2\omega_5q_2)^5, kq_2, kq_4)^4, \omega_2\omega_5q_2)^3)^2, kq_3, kq_1)^1 \quad (12)$$

$$G_{q_5}^+ = ({}^0q_5({}^1\omega_5q_3({}^3\omega_3q_6({}^4\omega_4\omega_6q_4({}^5kq_1({}^6\omega_1q_5, kq_2({}^7kq_3, \omega_2\omega_5q_5)^7, kq_4)^6, kq_3, \omega_4\omega_6q_6)^5, \omega_6q_2)^0 \quad (13)$$

$$G_{q_6}^+ = ({}^0q_6({}^1\omega_4\omega_6q_4({}^2\omega_4\omega_6q_6, kq_1({}^3\omega_1q_5({}^4\omega_5q_3({}^5\omega_3q_6, kq_4, kq_2({}^6\omega_2\omega_5q_5, kq_1)^6)^5, \omega_2\omega_5q_5)^4, kq_2)^3, kq_3)^2, \omega_6q_2)^1)^0 \quad (14)$$

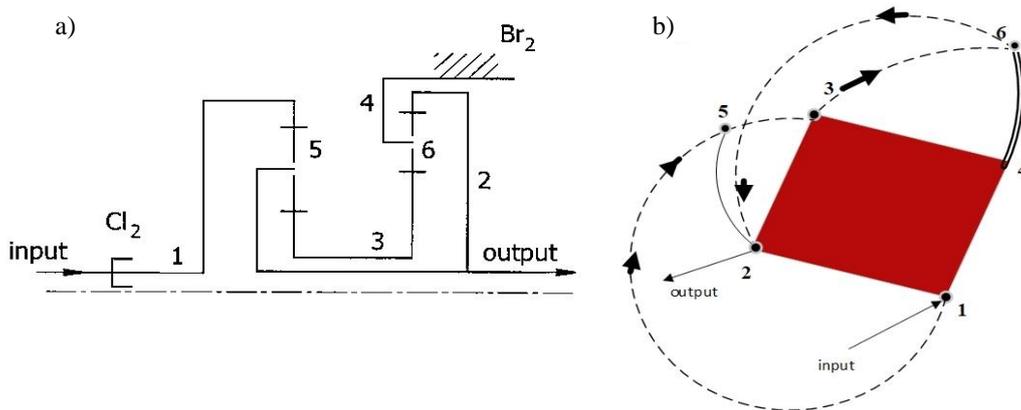


Fig. 8. Functional diagram of the gear for 1st gear (a), graph with the path from the entrance to the exit (b) [1, 13, 14]

Figures 11–12 show the structure from each of the vertices: q_2, q_3, q_4, q_5, q_6 of the graph from Fig. 9.

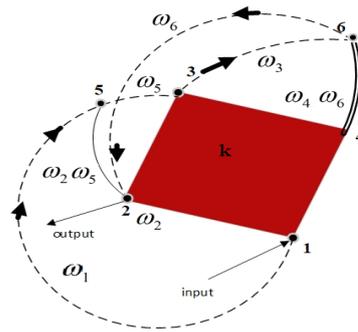


Fig. 9. Functional Hsu graph with the path from the input to the output for the transmission in terms of signal flow graph [1, 13–15]

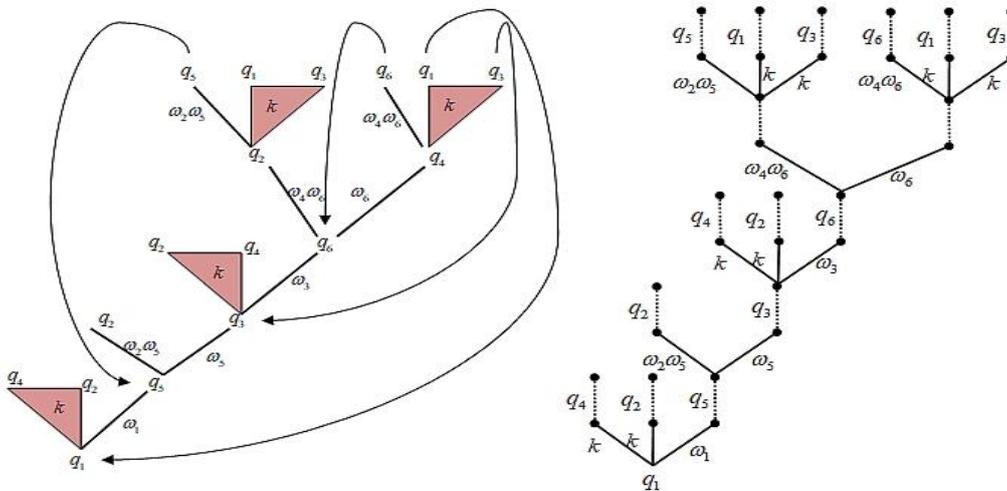


Fig. 10. Tree game structures from the initial vertex q_1 [13]

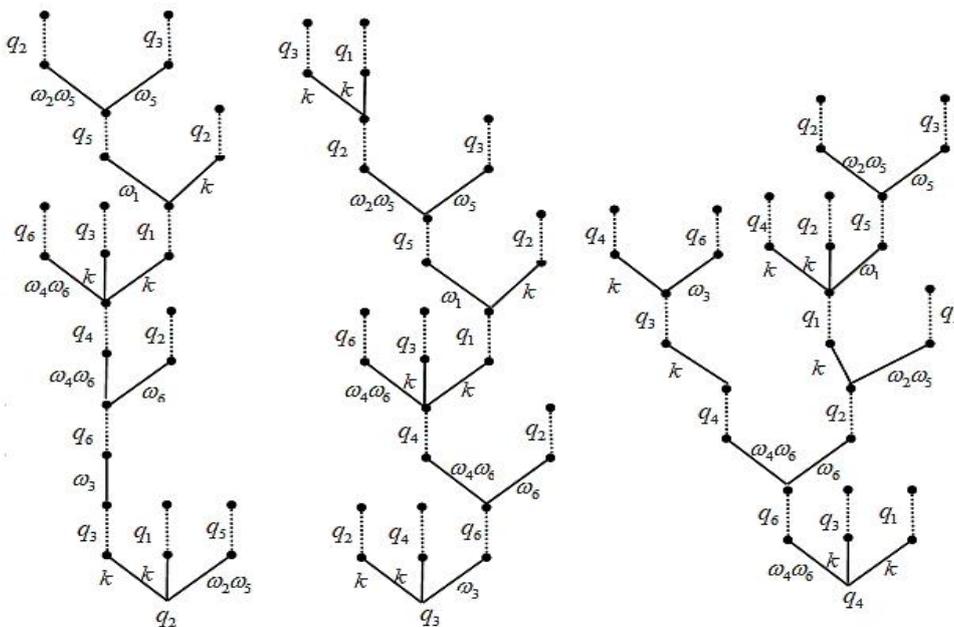


Fig. 11. Parametric tree structures with initial vertices: q_2, q_3, q_4

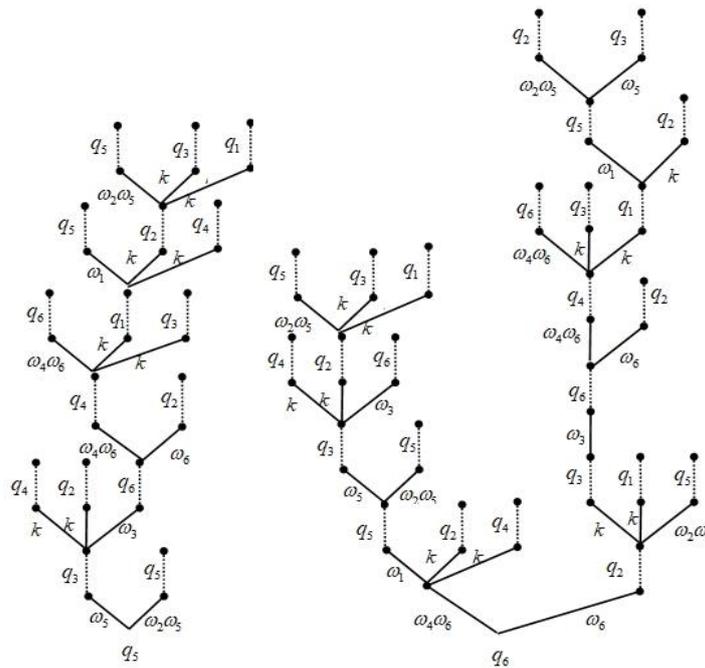


Fig. 12. Parametric tree structures with initial vertices: q_5, q_6

The following set of f -cycles can be distinguished: (1,5) 2; (3,5) 2; (2,6) 4; and (3,6) 4. Then, the f -cycles equations can be assigned to them. Condition of inhibition:

$$\omega_4 = 0 \tag{15}$$

Other equations describing kinematics have the form:

$$\begin{cases} \omega_1 - \omega_2 = +N_{51}(\omega_5 - \omega_2) \\ \omega_3 - \omega_2 = -N_{53}(\omega_5 - \omega_2) \\ \omega_2 - \omega_4 = +N_{62}(\omega_6 - \omega_4) \\ \omega_3 - \omega_4 = -N_{63}(\omega_6 - \omega_4) \end{cases} \tag{16}$$

Searched gear:

$$\frac{\omega_2}{\omega_1} = \frac{N_{63}N_{53}}{N_{53}N_{63} + N_{51}(N_{63} + N_{62})} \tag{17}$$

4.2. APPLICATION OF COMPLEX PARAMETRIC STRUCTURES

Complex decision trees allow to join construction and/or exploitation parameters of similar properties as well as of the same adopted discretization of interval values. It makes it possible to decrease the calculation complexity in order to specify the most important subgroups of the whole set or system. In case of joining decision variables of separate properties and functions that they have in the system, a subanalysis of a given set or system is obtained.

Distribution graph of the dependency of Fig. 9 gives a set D of trees:

$$D = \{G_{\{q1\}}^{++}, G_{\{q2\}}^{++}, G_{\{q3\}}^{++}, G_{\{q4\}}^{++}, G_{\{q5\}}^{++}, G_{\{q6\}}^{++}\} \tag{18}$$

In a complex parametric tree structure, by imposing all parametric game structures, from each vertex to the game structure from the predetermined vertex, we obtain a set S of total complex parametric structures:

$$S = \{S_{G\{1\}}, S_{G\{2\}}, S_{G\{3\}}, S_{G\{4\}}, S_{G\{5\}}, S_{G\{6\}}\} \tag{19}$$

where: $S_{G\{i\}}$ - a complex structure with all the tree structures from the D set superimposed on the structure that is parametrically defined from the predetermined top

Example

To build a complex structure $S_{G\{i\}}$, it is necessary to transform the structure G_{q6}^+ (14) into G_{q6}^{++} (20). For this purpose, decisive decomposition is used:

$$G_{q6}^{++} = ({}^0q_6({}^1\omega_4\omega_6q_4({}^2\omega_4\omega_6q_6, kq_1({}^3\omega_1q_5({}^4\omega_5q_3({}^5\omega_3q_6^1, kq_4^1, kq_2({}^6\omega_2\omega_5q_5^1, kq_1^1)^6)^5, \omega_2\omega_5q_5^2)^4, kq_2({}^4\omega_2\omega_5q_5({}^5\omega_3q_3({}^6\omega_3q_6^2, kq_4^2, kq_2({}^7\omega_2\omega_5q_5^3, kq_1^2)^7)^6), \omega_2\omega_5q_5^4)^4, kq_1^3)^3, kq_3({}^5\omega_3q_6^1, kq_4^1, kq_2({}^6\omega_2\omega_5q_5^1, kq_1^1)^6)^5)^2 \tag{20}$$

In the next step, we perform the tacking operation. It means 'joining' on the base structure another structure that is parametrically at the vertex of the initial.

In Figure 13, a complex parametrically $S_{G\{6\}}$ defined structure is shown.

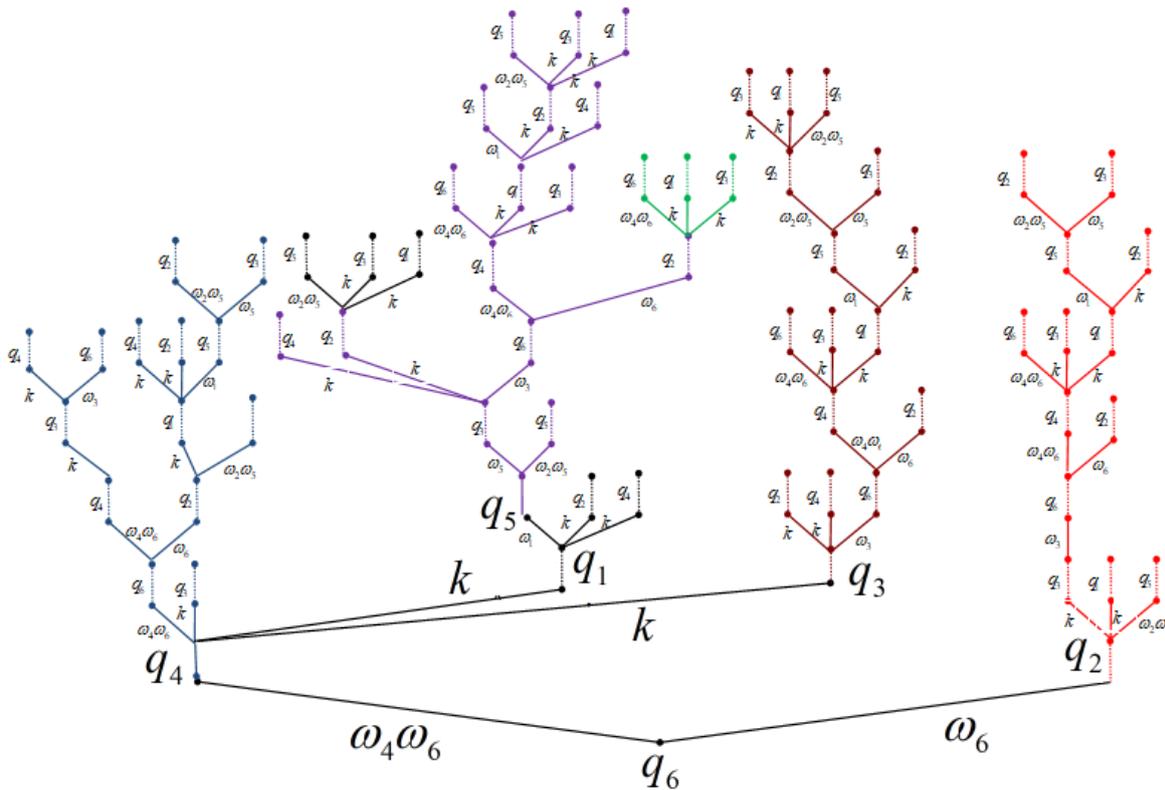


Fig. 13. Complex parametric tree structures with $S_{G\{6\}}$

Individual elements of the complex structure $S_{G_{\{6\}}}$ are elements of individual elements of parametric game trees structures $G_{\{q1\}}^+, G_{\{q2\}}^+, G_{\{q3\}}^+, G_{\{q4\}}^+, G_{\{q5\}}^+$.

All transformations are suitable for the contour graph of the analysed planetary gear. Because the complex structures can be n complete nodes γ_n , therefore, there exists a family B of sets of full nodes from all the sets of the set S:

$$B \subseteq \left\{ \left\{ \gamma_1, \dots, \gamma_n \right\} : \gamma_1, \dots, \gamma_n \in G_{\{1\}}^{++} \wedge G_{\{4\}}^{++} \wedge G_{\{5\}}^{++} \wedge G_{\{6\}}^{++} \right\} \left\{ \gamma_1 \neq \gamma_2 \dots \neq \gamma_n \right\} \quad (21)$$

Figure 14 shows a schematic representation of the complex structure $S_{G_{\{i\}}}$ from the vertex G_{q6}^+ .

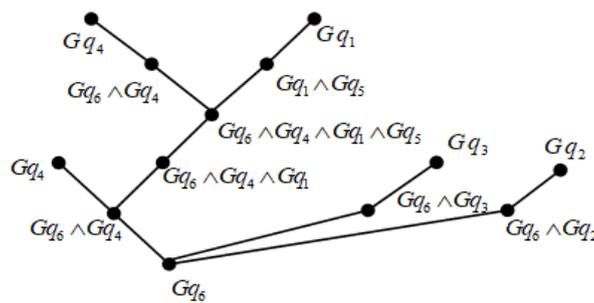


Fig. 14. A schematic representation of the parametric complex structure $S_{G_{\{6\}}}$

In the work [13], the authors presented a new method including game trees structures in the analysis of the automatic gearbox. Currently, the authors have used complex game tree structures in the work. This means that structures built from each vertex (Figs 11-12) should be combined into one comprehensive structure. (Fig. 13). The structure describes a complex systematic search method. Newly determined analytical expressions for $G_{\{q1\}}^+, G_{\{q2\}}^+, G_{\{q3\}}^+, G_{\{q4\}}^+, G_{\{q5\}}^+$ allow to build the complex structure shown in Fig. 14.

Here the analysis does not end. There is a possibility of introducing further generalizations and modifications, in particular, the development of an optimization method for parametrically acting structures and direct generation of systems of equations with their solutions. In addition, parametric structures allow for future analyses and syntheses, such as checking the isomorphism of designed gears, analysing the range of transmission applications by generating the optimal set of ratios on individual gears. The traditional graph of dependence from the distribution relative to the different vertices only evaluates the importance of vertices importance to each other according to the taxonomic grouping:

- vertices associated with a large number of connections should be in a fixed group;
- different groups in relation to each other should be associated with a small number of connections;
- distribution from the not very important vertex leads to receiving a large number of low-cardinality groups;
- distribution from an important peak leads to a small number of groups of large numbers.

The introduction of a replacement variable allows to reduce the decision-making complexity of calculations and comprehensively obtain the optimal gear operation for given gears in the analysis of the automatic gearbox.

In the optimization of systematic searching, it is necessary to build the remaining complex structures: $S_{G\{1\}}, S_{G\{2\}}, S_{G\{3\}}, S_{G\{4\}}, S_{G\{5\}}, S_{G\{6\}}$.

A practical example of calculations for automatic transmission gear using parametric parameters will be presented in the next publication (next publications).

In the first stage, we have to number parametric structures. Next, we build matrices describing parametric structures. The above procedure can be implemented on a computer using the adjacency matrix and a nested-do loops algorithm. For the above example, the corresponding matrix representations are shown in Fig. 15.

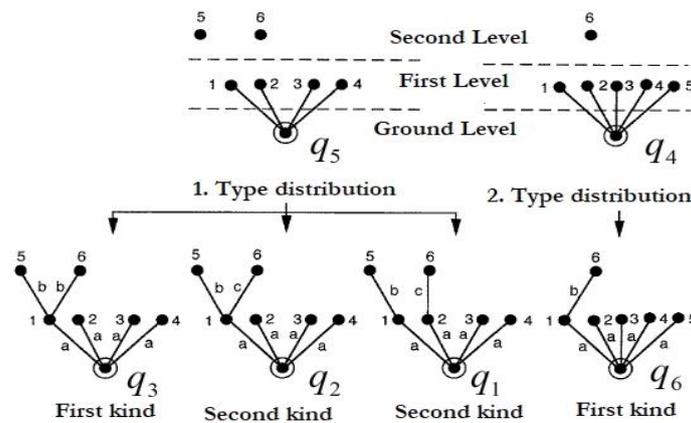


Fig. 15. Enumeration of decision game tree structures from the initial vertex q_6

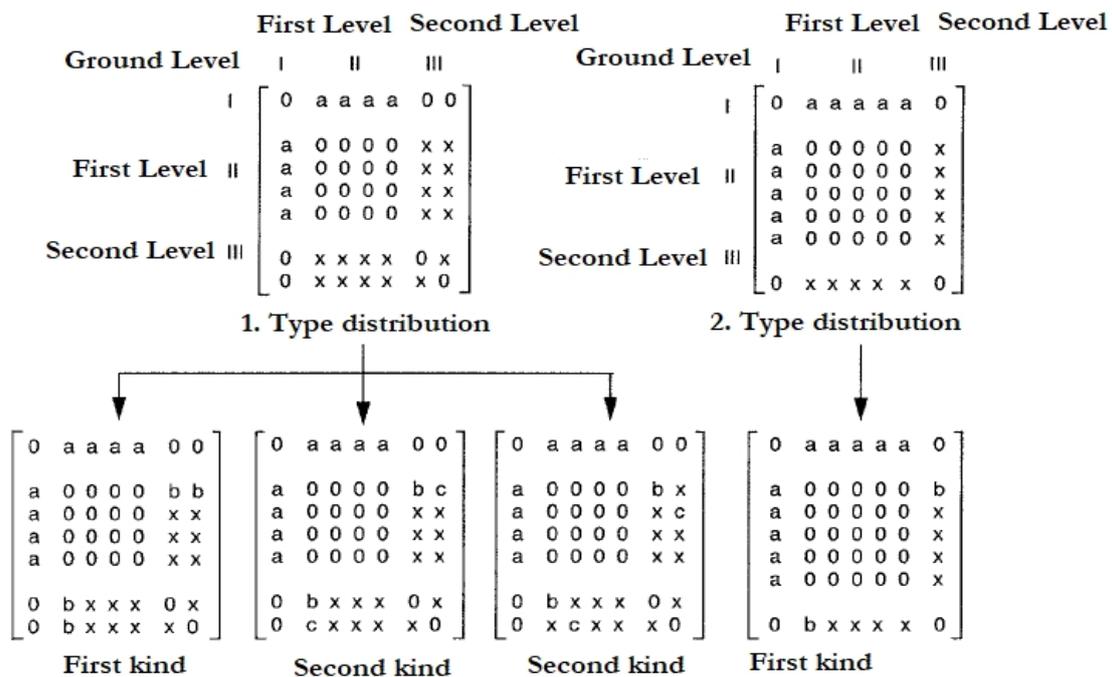


Fig. 16. Matrix representation of game tree structures from the initial vertex q_6

The matrix can be divided into nine submatrices. Since the root does not connect to itself, the element of the I-I submatrix is set to zero. Since the first level vertices are connected to the root by thin edges of the same label and not among themselves, all elements of the I-II and II-I submatrices are given by the same edge label “a”, whereas all elements of the II-II submatrix are set to zero.

Next we connect one second level vertex to one first level vertex by a geared edge. That is, the II-III (and III-II) submatrix of the adjacency matrix shown in Fig. 16 is under consideration. While adding geared edges, make sure that every second level vertex is incident by at least two geared edges. The Total number of geared edges to be added is equal to $n-3$ where n is the number of links, including the housing.

5. CONCLUSION / SUMMARY

Thanks to the construction of parametric structures from the primary contour graph, a better representation of the planetary gear can be obtained in two areas: structural analysis and the analysis of subordinate expressions. For a given structure, it is possible to predict whether a given sequence of decisions in the structure can lead to the determination of the complete path of the solution of the system of equations or to find the optimal value of the given parameter. An expression describing the degree of subtraction of a given component graph – that is, the parametric game structure – is parametrically marked with a pair of parentheses within which the expression being an analytical model is written. Each parametric game structure can describe an independent grouped relational preference system. Choosing the right parametric game structure requires a proper definition of a set of variable gears to determine strategies such as solving equation systems. For a given structure, it is possible to predict whether a given sequence of decisions in the structure can lead to the determination of the complete path of the solution of the system of equations or to find the optimal value of the given parameter.

Application of complex game trees does not change types and graphical shapes of such structures. They are more complex but they keep the given structural properties resulting from the initial dependence graph. Thus, a local role of decomposition can be distinguished. The introduced decision decomposition eliminates interaction of constructional and service parameters because a designer can make a decision about only single changes and observation at the successive stages. The algorithmic method of formation of game structures from the contour graph of the describes the optimization method of systematic search. The game structure describes a space of possible solutions in order to find optimum objective functions. There is the connection with other graphical structures which can be graphs in another sense, or even decision trees with node and/or branch coding. Such interpretation can lead to different types of logical tree structures on the basis of research results described in the article.

Most combinatorial enumeration procedures are done through the process of generating and testing. The procedure is thus divided into two parts: a generator of all possible solutions and a tester that selects only those solutions that meet the constraints. An important issue in using a generating and testing technique is the distribution of knowledge

between the generator and tester. The generator produces solutions satisfying some of the constraints. The tester then selects those solutions that satisfy the rest of the constraints. While this technique is valid for solving transmission design problems, it limits the solutions to the knowledge (information) contained in the generator and tester. This inevitably reduces the efficiency of this solution technique and needs complicated computer algorithm. Usually, putting more knowledge in the generator results in a more efficient procedure. However, the elimination of invalid clutching sequences was conducted by inspection. Identifying invalid clutching sequences by inspection is not always reliable. The paper presents the possibility of applying logical decision trees and information systems in the analysis of an exemplary gearbox. Here the analysis does not end. There is a possibility of introducing further generalizations and modifications, in particular, the development of an optimization method for parametrically acting structures and direct generation of systems of equations with their solutions. In addition, parametric structures allow for future analyses and syntheses, such as checking the isomorphism of designed gears, analysing the range of transmission applications by generating the optimal set of ratios on individual gears. The traditional graph of dependence from the distribution relative to the different vertices only evaluates the importance of vertices importance to each other according to the taxonomic grouping:

- vertices associated with a large number of connections should be in a fixed group;
- different groups in relation to each other should be associated with a small number of connections;
- distribution from the not very important vertex leads to receiving a large number of groups of small numbers;
- distribution from an important vertex leads to a small number of groups of large numbers.

The introduction of a replacement variable allows to reduce the decision-making complexity of calculations and comprehensively obtain the optimal gear operation for given gears in the analysis of the automatic gearbox.

REFERENCES

- [1] ZAWISLAK S., 2012, *The Graph-based Methodology as an Artificial Intelligence Aid for Mechanical Engineering Design*, Wydawnictwo Akademii Techniczno-Humanistycznej Bielsko-Biała.
- [2] DREWNIAK J., ZAWISLAK S., 2010, *Graph methods in kinematical analysis of multi-speed epicyclic gears*, Inter. J. Appl. Mech. Eng., 17/3, 791–798.
- [3] DREWNIAK J., ZAWISLAK S., 2009, *Kinematical and dynamical analysis of closed kinematic chains using graphs and contour equations*, PAMM, 9/1, 547–548.
- [4] WOJNAROWSKI J., 1986, *Graphs and structural numbers as models of mechanical systems*, Gliwice, (in Polish).
- [5] UEMATSU S., 1997, *An application of graph theory to the kinematic analysis of planetary gear trains*, Inter. J. Japan Soc. Precis. Eng., 31, 141–146.
- [6] DEPTUŁA A., 2014, *Application of multi-valued weighting logical functions in the analysis of a degree of importance of construction parameters on the example of hydraulic valves*, Inter. J. Appl. Mech. Eng., 19/3, 539–548.
- [7] PARTYKA M.A., 1999, *Quine-McCluskey algorithm to minimize individual partial multi-valued logic functions*, St. i Monog., 109, Polit. Opol., Opole, (in Polish).
- [8] BERGHOFER S, REITER M, 2009, *Theorem Proving in Higher Order Logics*, LNCS, 5674, 147–163.

- [9] DEPTUŁA A., OSIŃSKI P., PARTYKA M.A., 2013, *Discrete optimization of a gear pump after tooth root undercutting by means of multi-valued logic trees*, Arch. Civ. Mech. Eng, 13/4. 422–431.
- [10] HARARY F., 1969, *Graph theory*, Addison-Wesley, Reading, Mass.
- [11] KORZAN B., 1978, *Elements of graph theory and networks – methods and applications*, WNT, Warszawa, (in Polish).
- [12] PIJLS W., DE BRUIN A., 2001, *Game tree algorithms and solution trees*, Theoretical Computer Science, 252/1-2, 197–215.
- [13] DEPTUŁA A., ZAWIŚLAK S., PARTYKA M.A., 2017, *Application of decision logic trees and dependence graphs in analysis of automatic transmission gearboxes*, Autobusy – Technika, Eksploatacja, Systemy Transportowe, 12, 808-814, (in Polish).
- [14] DEPTUŁA A., PARTYKA M.A., 2018, *Application of decision logical trees and predominant logical variables*, Proceedings of the 14th International Scientific Conference, Computer Aided Engineering, Polanica-Zdrój.
- [15] DEPTUŁA A., PARTYKA M.A., TISZBIEREK A., 2018, *Application of decision logic trees and information systems in analysis of automatic transmission gearboxes*, Inter. Conf. Przemysł 4.0, Komitet Budowy Maszyn PAN, Nowy Sącz, 21–24 May.
- [16] KRON G., 1930, *Generalized theory of electrical machinery*, AIEE Transactions, 49, 666-683.
- [17] HSU C.H., 1992, *Graph notation and kinematic equations of motion of planetary gear trains*, International J. of Vehicle Design, 13/3, 233–241.
- [18] FREUDENSTEIN F., 1971, *An application of Boolean algebra to the motion of epicyclic driver*, ASME Journal of Engineering for Industry, B, 93, 176–182.
- [19] HSU C.H., LAM K.T., LIN Y.L., 1994, *Automatic synthesis of displacement for planetary gear trains*, Math. Comput. Modelling, 19/11, 67– 81.
- [20] MARGHITU D., 2005, *Kinematic chains and machine components design*, Elsevier Amsterdam, San Diego, Academic Press, London.
- [21] DEPTUŁA A., DREWNIAK J., PARTYKA M.A., 2017, *Application of dependence graphs and game trees in analysis of a planetary gear modelled with a contour graph*, Machine Dynamics Research, 41/3.
- [22] DEPTUŁA A., DREWNIAK J., PARTYKA M.A., 2017, *Analysis of a planetary gear modeled with a contour graph taking into account the method of parametric play structures*, Mechanik, 7, 640–642, (in Polish).
- [23] DEPTUŁA A., DREWNIAK J., PARTYKA M.A., 2017, *Analysis of a planetary gear modelled with a contour graph considering the decision making complexity of game-tree structures*, II International Conference of Computational Methods in Engineering Science (CMES'17), Exploitation And Machine Building, ITM Web Conf. 15, 04002.
- [24] HSU C.H., 2002, *An analysis methodology for the kinematic synthesis of epicyclic gear mechanisms*, ASME Journal of Mechanical Design, 124, 574–576.
- [25] https://roadstars.mercedes-benz.com/en_GB/products/construction-transport/arocs/power/transmissions-gearshift.html
- [26] CERVANTES-SANCHEZ J.J., RICO-MARTINEZ J.M., PANDURO-CALVARIO C., 2012, *A general and systematic framework for the kinematic analysis of complex gear systems*, Meccanica, 47/1, 3–21.
- [27] TALPASANU I. SIMIONESCU P.A., 2012, *Kinematic analysis of epicyclic bevel gear trains with matroid method*, J. Mech. Des., 134/11, 1–8.
- [28] DING H.F., ZHAO J., HUANG Z., 2009, *Unified topological representation models of planar kinematic chains*, J. Mech. Des., 131/11, 1-6.
- [29] ESMAIL E.L., 2013, *Nomographs and feasibility graphs for enumeration of Ravigneaux-type automatic transmissions*, Adv. Mech. Eng., <http://dx.doi.org/10.1155/2013/120324>, 1–15.
- [30] SALGADO D.R., DEL CASTILLO J.M., 2014, *Analysis of the transmission ratio and efficiency ranges of the four-, five-, and six-link planetary gear trains*, Mech. Mach. Theory, 73, 218–243.
- [31] DEL PIO G., PENNASTRI E., VALENTINI P.P., 2013, *Kinematic and power-flow analysis of bevel gears planetary gear trains with gyroscopic complexity*, Mech. Mach. Theory, 70, 523–537.
- [32] CHEN C., 2013, *Power flow and efficiency analysis of epicyclic gear transmission with split power*, Mech. Mach. Theory, 59, 96–106.
- [33] HSU C.H., 1993, *Graph representation for the structural synthesis of geared kinematic chains*, Journal of the Franklin Institute, 330/1, 131–143.
- [34] XUE H.L., LIU G., YANG X.H., 2016, *A review of graph theory application research in gears*. Proc. IMechE., Part C, J. Mechanical Engineering Science, 230/10, 1697–1714.
- [35] HSU C.H., 2002, *An analytic methodology for the kinematic synthesis of epicyclic gear mechanisms*, J. Mech. Des., 124, 574–576.