On the design of multivariable PID controllers via LMI approach

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In the past decades, we have witnessed the developments of various kinds of modern or postmodern control theories, such as LQG or LQR optimal control, \(H_\infty\) control, and \(\mu\) analysis and synthesis, from their emergence to maturity and applications in some practical systems. Despite of the success of the modern or postmodern control theory, we have to admit that the majority of the controllers used in industry are of classical proportional-plus-integral-derivative (PID) type. This is mainly due to its relatively simple structure, ready-in-hand implementation, and perhaps, being easily understood. Therefore it is often the case that one first considers PID controllers in practical applications unless evidence shows that they are insufficient to meet specifications.

Because of the popularity of PID controllers in the industrial world, many approaches have been developed to determine the parameters of PID controllers. The first systematic tuning method for PID parameters was proposed by Ziegler and Nichols in the early 1940s. Then came the well-known formulae such as Cohen-Coon method, integral absolute error (IAE) optimum method, integral time-weighted absolute error (ITAE) optimum method, internal model control (IMC) method and relay auto-tuning method, etc. Recently, many modified tuning methods have also been proposed associated with different performance specifications or different methods used accordingly. Notice that the aforementioned tuning methods are only suitable to single-input single-output (SISO) systems. Compared to the voluminous references on tuning PID controllers for SISO systems, only a few results have been reported in the literature on PID controller tuning for multi-input multi-output (MIMO) systems. In this area, a relatively widely used approach was proposed by Luyben using biggest-log modulus tuning (BLT) method. Wang developed an auto-tuning method for multivariable PID controllers using their decoupling approach. Generally speaking, the tuning of PID controllers for MIMO systems remains to be an open problem.

Most of the approaches mentioned above use a common assumption that the plant can be modelled as a first- or second-order system plus a dead-time. Indeed, many industrial processes such as petroleum refining, chemical reaction, and motor servo etc. can be approximated sufficiently well, for the control purpose, by the first- or second-order systems. But there do exist many other kinds of systems which are of MIMO and cannot be well approximated by the first- or second-order systems. For these kinds of systems, we cannot apply most of the aforementioned methods to tune PID controllers, since many design specifications, even the most basic one, i.e., the stability of closed-loop systems, cannot be guaranteed by these methods. Therefore, it is highly desirable to develop effective methods to determine the parameters of PID controllers for MIMO high-order systems. This paper aims at to make an initial step towards solving this problem.

Specifically, we study the design problem of multivariable PID controllers which guarantee the stability of the closed loop systems, \(H_2\) or \(H_\infty\) performance specifications, or maximum output control requirement, respectively. The basic idea of our method is to transform the problem of PID controller design to that of static output feedback controller design. Algorithms based on iterative linear matrix inequality technique have been developed to find the feedback matrices of PID controllers corresponding to the four cases. A key point in our approach for solving MOC problem is that an optimal estimate for a parameter, namely \(\tau_2\), in the matrix inequalities associated with MOC problem is found, which makes it possible to solve MOC problem in a reasonable computational burden. A numerical example on the design of PID controllers for aircraft has been presented to illustrate the feasibility of the proposed method.

Compared to most of the prevalent PID controller design methods, advantages of our methods are: (i) Stability of closed loop systems is guaranteed; (ii) No specific requirement on either system structure or system order is imposed. The disadvantages of our methods are: (i) System parameters must be exactly known; (ii) The iterative algorithms developed here are based on sufficient criteria for the corresponding problems.