ESTIMATING THE INTEGRAL NON-LINEARITY OF AD-CONVERTERS VIA THE FREQUENCY DOMAIN

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Abstract

This paper proposes a method to estimate the Integral Non-Linearity of AD-converters from the lower order output Fourier coefficients of a sinusoidal input. In order to get a high quality estimate, the lower order Fourier coefficients have to be stabilized with respect to the rounding operation of the AD-converter. For this we use a computationally efficient and easily applied method named wobbling.

1. Introduction

The industrial testing of AD-converters is still largely specification based. The characteristics which are usually listed in the specification of an AD-converter can be divided into two groups.

The first group consists of characteristics which are a measure of the extent to which the transfer function of an AD-converter approximates a straight line. Examples of such characteristics are Integral Non-Linearity and Differential Non-Linearity.

The second group of characteristics is a measure of how well the sample set of an AD-converter represents the signal at its input. Examples of such characteristics are Total Harmonic Distortion and Signal to Noise and Distortion.

1. 1. Practical considerations

The most straightforward method of measuring Integral Non-Linearity is by applying a ramp to an AD-converter and by assessing how far the AD-converter's response is off from a straight line. The straight line can be determined either by a least-squares fit through the acquired sample set or by a straight line drawn through the AD-converter's end points.

It is, however, difficult to generate a high quality ramp on automated test equipment, especially if the number of bits of the AD-converter is large. The reason for this is that a ramp is a broad-band signal and sensitive to in-band noise or spurious signals. Also, Nyquist limitations of arbitrary waveform generators might make it impossible to reconstruct sufficient higher order harmonics.

Integral Non-Linearity can also be measured by using a sine wave as a stimulus. The advantage of using a sine wave is that it is a narrow-band signal so its quality can easily be improved by filtering. The disadvantage is that more samples are needed compared to a ramp because of the non-uniformity of the sine wave's amplitude density function.

Another problem is that the required number of samples can become unacceptably large if one wants to visit every code at least once for AD-converters with more than 12 bits. This is especially true in the presence of noise. Even present day automated test equipment has problems with the efficient handling and processing of such large sample sets.

In contrast to this, the measurement of Total Harmonic Distortion (THD) and Signal to Noise and Distortion (SINAD) is much easier. These characteristics are usually measured with a sinusoidal stimulus and the desired characteristics are computed after a Fast Fourier Transform of the acquired sample set. Relatively small sample sets are usually sufficient in order to get estimates with a good repeatability.

2. AD-converter model

For the moment we consider time-continuous signals and model the operation of an AD-converter as:

$$x \to \left[h(x) + \underline{n}\right]$$

where [] denotes the rounding operation, $\underline{n}$ is thermal noise and $h()$ is a smooth transfer function comprising the Integral Non-Linearity when present and which we model by a polynomial with degree $\leq L$, see Figure 1.

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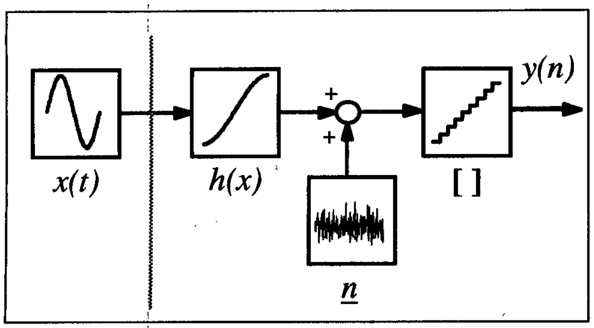

Fig. 1: AD-converter model

As an input stimulus we use a sinusoid $x(t)$:

$$x(t) = A\sin(2\pi\upsilon t) - c; \qquad A = \tilde{A} \times 2^{q-1},$$

where $\tilde{A}$ is the relative amplitude which is unity at full-scale, q is the number of bits of the converter, υ is the frequency which we set to unity, and c is the offset

The total harmonic distortion THD^2 of $x(t)$ is defined as:

$$\mathrm{THD}^2 = \sum_{n=2}^{L} \left|X(n)\right|^2 \Big/ \left|X(1)\right|^2,$$

where $X(n)$ is the n^{th} Fourier coefficient of $x(t)$ and L is an integer in the range 5,...,10.

In the ideal case, where we could ignore both the thermal noise and the rounding operation, we could compute $h(x)$ from the same Fourier coefficients according to:

$$\tilde{h}(x) = X(0) + 2\sum_{n=1}^{L} i^n X(n) T_n(x/A), \qquad (1)$$

where T_n is the n^{th} Chebyshev polynomial of the first kind.

In general, thermal noise is not a big problem as long as the order N of the DFT which is used to compute $X(n)$ is large compared to L. The rounding operation, however, poses a far more serious problem.

3. Wobbling

Wobbling was introduced by De Vries and Janssen as a method to stabilize AD-converter characteristics [1].

It was shown that small variations in amplitude or offset of a sinusoidal stimulus could cause relatively large fluctuations in the lower order Fourier coefficients of the AD-converters response. These fluctuations could be explained by an interaction between the small amplitude and offset variations and the rounding operation of the AD-converter.

De Vries and Janssen suggested to add a DC-free ramp with a span of 1 LSB covering an integral number M of sine periods to the sinusoidal stimulus in order to stabilize the AD-converter characteristics. This ramp can be added to the sinusoid in the memory of the arbitrary waveform generator, where one has full control over the signals.

After sampling and quantization the ramp can be subtracted from the sine wave before computation of the desired characteristics. The whole procedure is shown in Figure 2.

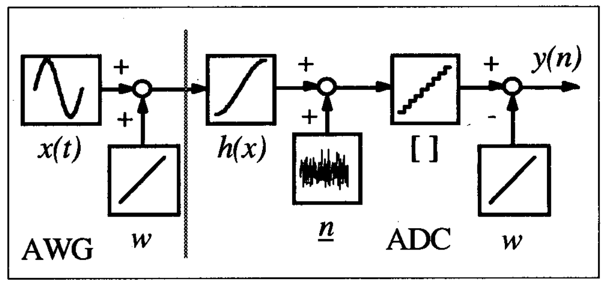

Fig. 2: Wobbling at work

What is achieved by using the above mentioned procedure, which is called wobbling, is that all AD-converter characteristics are averaged over 1 LSB variation in amplitude and offset. The extent to which this averaging occurs depends on M, the number of sine periods covered by the ramp. Loosely speaking, one may say that wobbling with M sine periods amounts to replacing the actual quantizer one has by a quantizer having a quantization step a factor M smaller. This point was already made [1], Eq. 11, and is elaborated in detail in [2].

The characteristic which is the most sensitive to amplitude or offset variations is THD because it is based only on the lower order Fourier coefficients. The average THD for a perfectly linear AD-converter ($h(x) = x$) when no wobbling is used is given by:

$$\overline{\mathrm{THD}^2} = 10^{10}\log\left[0.01234(L-1)A^{-3}\right]$$
$$= -9.54 - 30^{10}\log A \qquad [dB],$$

where L = 10. The maximal THD^2, on the other hand, is equal to:

$$\mathrm{THD}^2_{max} = 10^{10}\log\left[0.07006(L-1)A^{-3}\right]$$
$$= -2.00 - 30^{10}\log A \qquad [dB],$$

again for $L = 10$. Notice the large difference between the constants -9.54 and -2.00 in the formulas for the average and maximal THD^2, which explains to a large extent the strong sensitivity of the THD^2 to amplitude and offset variations.

The formulas above are valid for time-continuous signals. In the case of discrete-time signals:

$$y_M\left(\frac{k}{N}\right) = \left[A\sin 2\pi M\frac{k}{N}\right], \quad k = 0,\ldots,N-1,$$

where N is the number of samples and M is the number of periods of the sinusoid, one has to distinguish between three cases. The oversampled case, which is characterized by:

$$2\pi MA < N,$$

the undersampled case:

$$2\pi MA > N,$$

and the so-called critical case:

$$2\pi MA \approx N.$$

For the oversampled case, the observations and formulas for average and maximal THD^2 as given above hold. For the undersampled case, the average THD^2 is given by:

$$\overline{\mathrm{THD}}^2 = 10^{10}\log\frac{M(L-1)}{3NA^2} \quad [dB],$$

while the THD^2 itself shows only modest variations with amplitude and offset variations. The critical case should be avoided because the THD^2 behaves irregularly. For details we refer to [2].

3. 1. Reduction of rounding THD

During our investigations we observed that wobbling does not only stabilize the THD but also the involved lower order Fourier coefficients themselves with respect to the rounding operation.

The observation made for the time-continuous case that wobbling amounts to decreasing the quantizer step of the quantizer by a factor M (equivalently: adding $^2\log M$ bits to the quantizer) continues to hold when sampling comes into play, provided that the number of samples N is a multiple of M. This point is elaborated in [2]. Accordingly, in the case of wobbling one can obtain the ranges of oversampling, undersampling and critical sampling with their respective maximum and average THD^2 values from those of the previous case without wobbling by simply replacing all amplitudes A by MA. Thus one gets in all cases a substantial decrease of THD^2 values.

To illustrate the above, consider a perfectly linear 11-bits AD-converter. In the case that we measure THD^2 with a sinusoid with 32 periods in 2048 samples, we expect an average THD^2 due to rounding of -73 dB (no wobbling, undersampled case), which is confirmed by the simulation, see Figure 3.

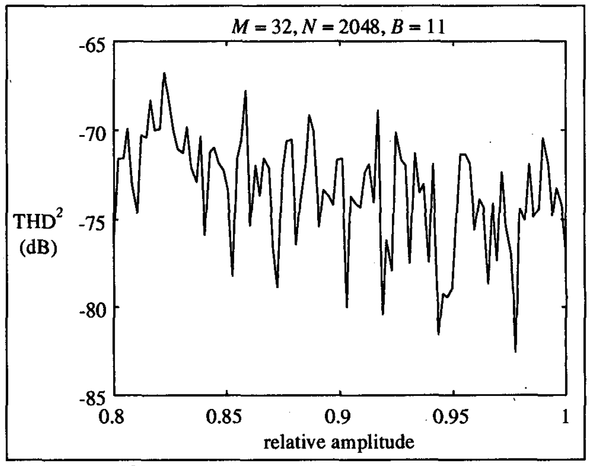

Fig. 3: THD^2 due to rounding

We can get the same quality with a 6-bits AD-converter when wobbling is used but with otherwise the same parameters, see Figure 4: by wobbling with $M = 32$ we gain $^2\log 32 = 5$ bits.

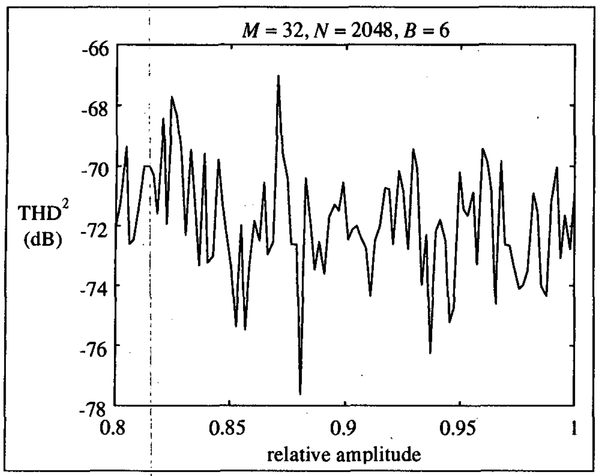

Fig. 4: Reduced THD^2 due to rounding

4. Estimating Integral Non-Linearity

By using wobbling we are able to reduce the contribution of the rounding operation to the THD^2 substantially and are now able to estimate the Integral Non-Linearity directly from equation (1).

To illustrate this, first consider an ideal 11-bits AD-converter ($h(x) = x$), where we have chosen $M = 16$ and $N = 2048$. If we do not wobble, we expect a THD^2 due to rounding of around -76 dB, see the dashed-dotted line in Figure 5.

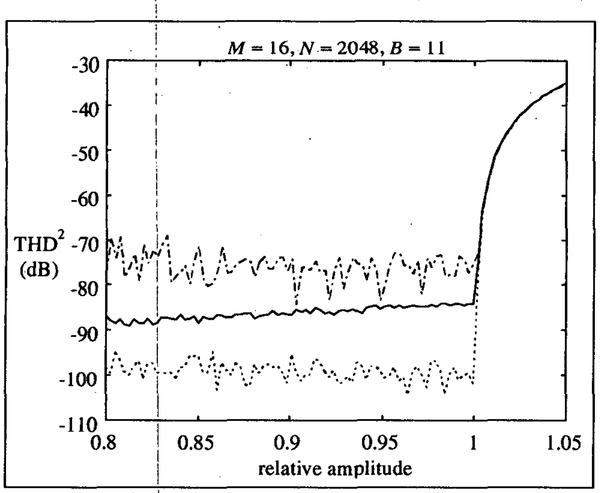

Fig. 5 THD^2 due to rounding with (dotted) and without (dashed-dotted) wobbling, THD^2 due to $h(x)$ (solid)

Consider a smooth transfer function $h(x)$ representing the Integral Non-Linearity, which we have modeled by:

$$h(x) = \alpha_5 x^5 + \alpha_4 x^4 + \alpha_3 x^3 + \alpha_2 x^2 + \alpha_1 x + \alpha_0,$$

where we have chosen:

$\alpha_5 = 5\text{x}10^{-5}$, $\alpha_4 = 2.5\text{x}10^{-5}$, $\alpha_3 = 1.5\text{x}10^{-4}$, $\alpha_2 = 3.5\text{x}10^{-5}$, $\alpha_1 = 1$, $\alpha_0 = 0$.

Then we see from Figure 5 (solid line, wobbling used) that $h(x)$ gives rise to a THD^2 of around -85 dB. It is clear that we cannot estimate the Integral Non-Linearity directly without wobbling.

By using wobbling, we can significantly reduce the THD^2 due to rounding to a level of around -100 dB, see Figure 5 (dotted line), again for an ideal AD-converter. We thus have reduced the THD^2 due to rounding to a level far below the level of the THD^2 due to the non-linearity.

This reduction is enough in order to get a high quality estimate of $h(x)$ by using equation (1), see Figure 6, where we have shown $\tilde{h}(x) - x$ and $h(x) - x$.

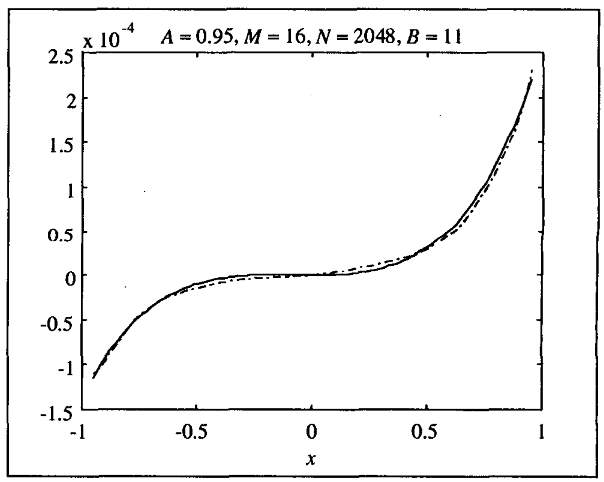

Fig. 6: $\tilde{h}(x) - x$ (dashed-dotted), $h(x) - x$ (solid).

In Figure 7, we also show $\tilde{h}(x) - x$ and $h(x) - x$ where we have now used equation (1) without wobbling.

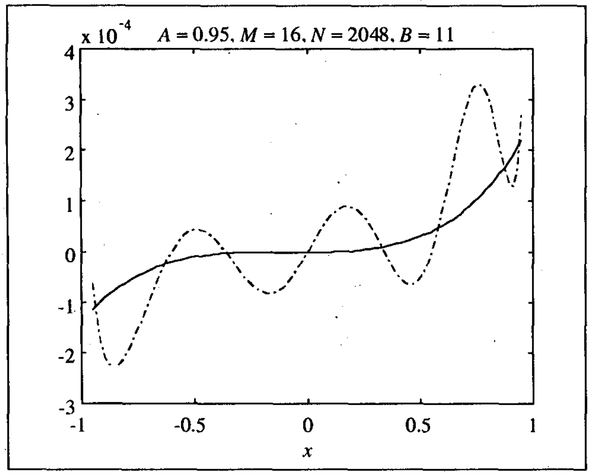

Fig. 7: $\tilde{h}(x)-x$ (dashed-dotted), $h(x)-x$ (solid).

It is clear from Figures 6 and 7 that retrieval of the non-linear part of *h(x)* becomes possible only after wobbling has been applied.

As a reference, we have repeated the above simulations but used subtractive noise dithering instead of wobbling, see Figure 8. We have chosen subtractive noise dithering because in general it gives better results than non-subtractive noise dithering. Clearly, the reconstruction result in Figure 6 is again superior. Note that a near to perfect synchronization of test equipment is required in the case of subtractive noise dithering.

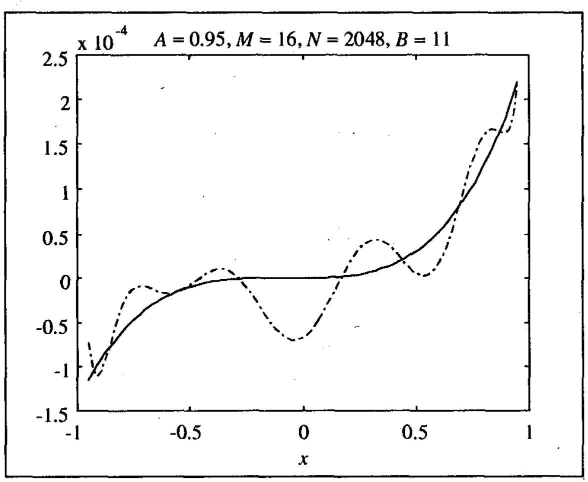

Fig. 8: $\tilde{h}(x)-x$ (dashed-dotted), $h(x)-x$ (solid).

5. Code visitation

With the introduction of wobbling, we have put forward the restriction that the number K:

$$K := N/M,$$

has to be an integer. This is against the common practice in DSP-based testing where M and N are chosen in such a way that their greatest common divisor (GCD) is 1 [3].

By making sure that GCD(M,N) = 1, one guarantees that the sine wave stimulus visits as many different codes in N samples as possible, see Figure 9 for a histogram of codes which are visited for $M = 5$ and $N = 512$ (not wobbled, 8-bits quantizer).

When we set M to 4 (GCD(M,N) = 4), then it is clear from the code visitation histogram that this is not an optimal choice for M if we want to visit as many codes as possible in N samples, see Figure 10 (not wobbled).

The code visitation properties of the wobbled sine wave can be improved by a slight extension of the wobbling technique. Instead of adding a ramp with a span of 1 LSB, a ramp with a span of Q LSB's can be added, where Q has to be chosen in such a manner that GCD(Q,M) = 1. A good choice for Q is often the nearest integer to $2\pi MA/N$ satisfying GCD(Q,M) = 1.

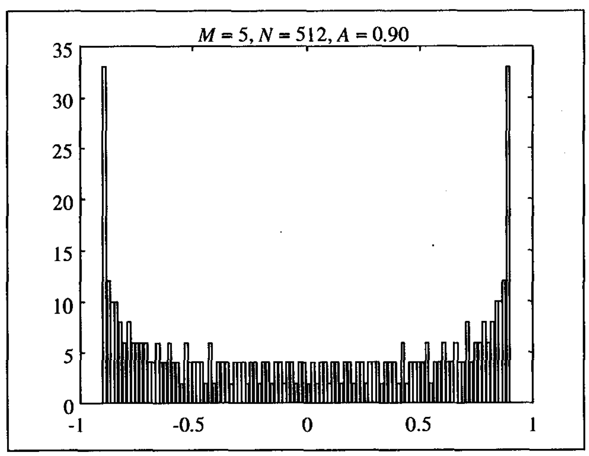

Fig. 9: Code visitation for $M = 5$, $N = 512$

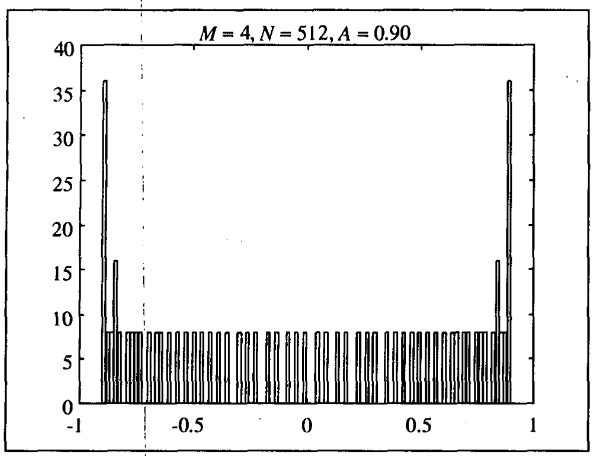

Fig. 10: Code visitation for $M = 4$, $N = 512$

As an illustration, consider the case above but now wobbled with a ramp with $Q = 7$. From the histogram in Figure 11, it is clear that the code visitation properties of the sinusoid have been improved considerably.

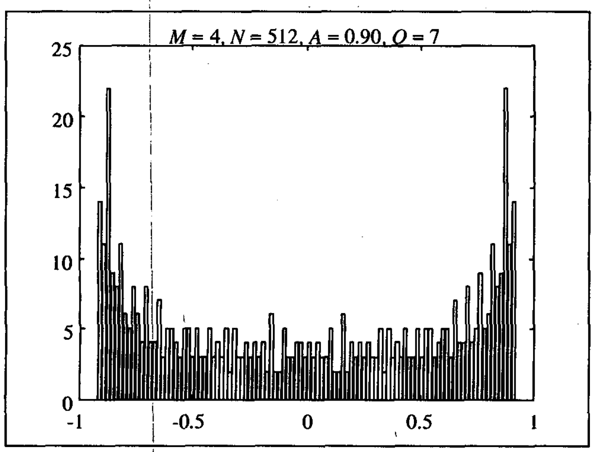

Fig. 11: Code visitation for $M = 4$, $N = 512$ and $Q = 7$

Another interesting feature of the ramp span parameter Q is that it can be used to increase the robustness of the method against relatively large variations in code width (Differential Non-Linearity). This can be explained by the fact that by increasing Q, the variation in code width is averaged over several neighboring codes.

6. Conclusion and outlook

A method has been presented which allows the derivation of the Integral Non-Linearity of mildly non-linear AD-converters from the lower order Fourier coefficients of a subtractively wobbled sinusoidal stimulus. Simulations have shown that the method is robust to variations in ramp span and alignment.

All of the signal processing is done in the digital domain and is not computationally intensive.

We are presently in the process of validating our method by using it for estimating the non-linearity of real AD-converters. A point of further investigations is incorporating non-ideal Differential Non-Linearities into the model of Sec. 2. This will complicate the mathematical analysis somewhat, but the model thus obtained provides a better connection with practical AD-converters.

References

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[3] Mahoney, M., 1987, "DSP-Based Testing of Analog and Mixed-Signal Circuits", Washington, D.C., USA, Computing Society of the IEEE