The prediction of the critical factor of safety of homogeneous finite slopes subjected to earthquake forces using neural networks and multiple regressions

Yusuf Erzin* and T. Cetin a

Celal Bayar University, Faculty of Engineering, Department of Civil Engineering, 45140 Manisa, Turkey

(Received February 05, 2013, Revised July 18, 2013, Accepted August 15, 2013)

Abstract. In this study, artificial neural network (ANN) and multiple regression (MR) models were developed to predict the critical factor of safety ($F_s$) of the homogeneous finite slopes subjected to earthquake forces. To achieve this, the values of $F_s$ in 5184 nos. of homogeneous finite slopes having different slope, soil and earthquake parameters were calculated by using the Simplified Bishop method and the minimum (critical) $F_s$ for each of the case was determined and used in the development of the ANN and MR models. The results obtained from both the models were compared with those obtained from the calculations. It is found that the ANN model exhibits more reliable predictions than the MR model. Moreover, several performance indices such as the determination coefficient, variance account for, mean absolute error, root mean square error, and the scaled percent error were computed. Also, the receiver operating curves were drawn, and the areas under the curves (AUC) were calculated to assess the prediction capacity of the ANN and MR models developed. The performance level attained in the ANN model shows that the ANN model developed can be used for predicting the critical $F_s$ of the homogeneous finite slopes subjected to earthquake forces.

Keywords: artificial neural networks; critical factor of safety; homogeneous finite slope; pseudo-static approach; simplified Bishop method

1. Introduction

Slope failures are complex natural phenomena that constitute a serious natural hazard in many countries (Wang et al. 2005). Hence, it is very important to analysis the stability of slopes and that can be defined in terms of a factor of safety (Krishnamoorthy 2007). The analysis is mostly being performed under static loading conditions (Krishnamoorthy 2007). However, in a seismically active region, earthquakes are a major triggering force behind the instability of slopes (Hack et al. 2007). Therefore, in these regions, it is also necessary to perform seismic slope stability analysis (Krishnamoorthy 2007). The pseudo-static (PS) approach is the most common procedure employed for seismic slope stability evaluation even though more advanced and rigorous methods of analysis are currently available (Bandini et al. 2005). This approach has been implemented in various limit equilibrium methods in which earthquake effects are represented by an equivalent...
static force (Baker et al. 2006). The limit equilibrium methods, despite having several well-known disadvantages, are still commonly used to estimate the stability of slopes (Bakır and Akiş 2005). These methods satisfy either some or all of the equilibrium conditions that include: (1) some or all interslice forces (Fellenius 1936, Janbu 1954); (2) moment and/or some forces (Taylor 1940, Bishop 1955); and (3) moment and all forces (Morgenstern and Price 1965, Spencer 1967, Sarma 1979). The methods as proposed by Fellenious (1936), Taylor (1940) and Bishop (1955) can be utilized for circular slip surfaces while the others can be used for circular and non-circular slip surfaces.

Artificial neural networks (ANNs) are very sophisticated modeling techniques, capable of modeling extremely complex functions (Choobbasti et al. 2009). Therefore, ANNs, with their remarkable ability to derive a general solution from complicated and imprecise data, can be used to extract patterns and detect trends that are too complex to be noticed by either humans or other computer techniques (Yilmaz and Yuksek 2008). In this study, an ANN model, with respect to the above advantages, and a multiple regression (MR) model were developed to predict the critical factor of safety ($F_s$) of a homogeneous finite slope subjected to earthquake forces. Keeping this in view, a computer program with a user interface was developed in the Matlab programming environment (Cetin 2010). Two slope parameters (viz., height of the slope, $H$, and the cotangent of the slope angle, $\cot \alpha$), three soil properties (viz., cohesion, $c$, internal angle of friction, $\phi$, and bulk unit weight, $\gamma$) and two earthquake parameters (viz., magnitude of the earthquake, $M$, and the distance to epicenter, $R$) were considered as the varying parameters during the slope stability analyses. Then, the $F_s$ of the 5184 nos. of homogeneous finite slopes having different slope, soil and earthquake parameters were calculated by using the Simplified Bishop method (1955) for each trial failure surface. The minimum (critical) $F_s$ value for each case was then determined and used in the development of the ANN and MR models. The ANN and MR results were then compared with the results obtained from the Simplified Bishop method (1955) in order to assess the performance of the prediction capacity of the models. It was found that the ANN model exhibits more reliable predictions than the MR model.

2. Artificial neural networks

Artificial neural networks (ANNs), perhaps the most popular intelligent computational paradigms (Tsompanakis et al. 2009), are the form of artificial intelligence which are based on the function of human brain and nervous system (Shahin et al. 2001). An ANN consists basically of simple highly interconnected processing elements called neurons that are typically arranged in layers. An ANNs architecture consists of three or more layers, which contain an input layer, one or more hidden layers, and an output layer. The neurons in the input layer receive input from the external environment (Choobasti et al. 2009). This layer does not perform any computations (Choobasti et al. 2009). Hidden layer, which receives inputs from the input layer, performs computation and provides the outputs to output layer (Choobasti et al. 2009). Output layer consists of neurons that communicate the output of system to the user of external environment (Guo and Uhrig 1992). Each neuron in a given layer is connected to all the neurons in the next layer by means of weighted connections. This ANN architecture is commonly referred to as a fully interconnected feed-forward multi-layer perceptron (MLP) (Goktepe et al. 2004).

The usage of a number of hidden layers in the ANN depends on the degree of complexity in the pattern recognition problem, and one or two hidden layers are found to be quite useful for most
The prediction of the critical factor of safety of homogeneous finite slopes (Goh 1994, Orbanić and Fajdiga 2003, Sonmez et al. 2005). The number of neurons in the hidden layers also depends on the nature of the problem and plays an important role in ANN modeling. If it is too large, the ANN will get an overfit, i.e., the ANN will have a problem in generalization (Choo basti et al. 2009). Various methods have been employed by several researchers to determine this number (i.e., Hecht-Nielsen 1987, Hush 1989, Kaastra and Boyd 1996, Kanellopoulas and Wilkinson 1997, Grima and Babuska 1999, Haque and Sudhakar 2002). However, these methods present guidelines only for selection of an adequate number of neurons (Erzin et al. 2008).

Learning in a MLP is an unconstrained optimization problem, which is subjected to the minimization of a global error function depending on the synaptic weights of the network (Goktepe et al. 2004). For a given training data consisting of input-output vectors, values of synaptic weights in a MLP are iteratively updated by a learning algorithm to approximate the target behavior (Goktepe et al. 2004). This update process is usually performed by back-propagating the error signal layer by layer and adapting synaptic weights with respect to the magnitude of error signal (Goktepe et al. 2004). Several learning algorithms have been developed. The back-propagation learning algorithm is the most commonly used neural network algorithm (Singh et al. 2006) and has been applied with great success to model many phenomena in the field of geotechnical engineering (Shahin et al. 2001). It is most appropriate for training MLP (Liang and Zhang 2010). Each hidden and output neuron processes its input(s) by multiplying each by its weight, summing the product, and then processing the sum using a non-linear transfer function, also named as activation function, to obtain the desired result (Erzin et al. 2008). The most common transfer function implemented in the literature is the sigmoid function. The neural network “learns” by modifying the weights of the neurons in response to the errors between the actual output and the target output values (Erzin et al. 2008). This is performed through gradient descent on the sum of the squares of the errors for all the training patterns (Rumelhart and McClelland 1986, Goh 1995). The changes in the weights are proportional to the negative of the derivative of the error term (Erzin et al. 2008). One pass through the set of training patterns, together with the associated updating of the weights, is called a cycle or an epoch (Erzin et al. 2008). Training is carried out by repeatedly presenting the entire set of training patterns (updating the weights at the end of each epoch) until the average sum squared error over all the training patterns is minimal and within the tolerance, specified for the problem (Erzin et al. 2008).

At the end of the training phase, the neural network should correctly reproduce the target output values for training data; provided errors are minimal (i.e., convergence occurs) (Erzin et al. 2008). The associated trained weights of the neurons are then stored in the neural network memory (Erzin et al. 2008). In the next phase, the neural network is fed a separate set of data. In testing phase, the neural network predictions using the trained weights are compared to the target output values (Erzin et al. 2008). The performance of the overall ANN model can be assessed by several criteria. These criteria contain coefficient of determination, $R^2$, root mean squared error, mean absolute error, minimal absolute error, maximum absolute error and variance account for. A well trained model should result in an $R^2$ value close to 1 and small values of error terms.

3. Calculation of factor of safety value of the homogeneous finite slopes subjected to earthquake forces

A computer program with a user interface was developed in the Matlab programming
environment for estimating factor of safety, $F_s$, of homogeneous finite slopes subjected to earthquake forces (Cetin 2010). Among the limit equilibrium methods, the Simplified Bishop method (1955) was selected in this study due to its simplicity which makes it easier for this application. In the Simplified Bishop method (1955), it is assumed that the failure surface is represented by a circular arch, which has a center represented by $O$ and a radius represented by $R$ (Zhu 2008). The soil mass of the chosen failure surface is divided into $n$ vertical slices, as depicted in Fig. 1. For the $i$th slice, the width is $b_i$, the angle of base is $\alpha_i$, the weight is $W_i$, the horizontal interslice forces are $E_i$ and $E_{i+1}$, the vertical interslice forces are $X_i$ and $X_{i+1}$, the normal force that affects the middle of the slice is $N_i$, the tangential force that affects base of the slice is $T_i$ (Zhu 2008). Considering the vertical force equilibrium and the moment equilibrium with respect to the centre $O$ of circular slip surface, the factor of safety, $F_s$, is determined using the following equation

$$F_s = \frac{\sum_{i=1}^{n} \left( W_i + X_{i+1} - X_i - u_i b_i \right) \tan \phi' + c b_i \right) / m_{ai}}{\sum_{i=1}^{n} W_i \sin \alpha_i}$$

where $c$ is the cohesion, $\phi'$ is the angle of internal friction, $u$ is the pore water pressure at the base, and $m_{ai}$ is obtained from the following equation.

$$m_{ai} = \cos \alpha_i + \left( \sin \alpha_i \tan \phi_i \right) / F_s$$

The Simplified Bishop method (1955) assumes that the contribution of vertical interslice forces to the factor of safety is neglected. In this study, it was assumed that the ground water table is deep, and so the ground water does not have any influence on the slope stability. Then, $F_s$ values were determined using the following equation

$$F_s = \frac{\sum_{i=1}^{n} \left( W_i + X_{i+1} - X_i - u_i b_i \right) \tan \phi' + c b_i \right) / m_{ai}}{\sum_{i=1}^{n} W_i \sin \alpha_i}$$

where $c$ is the cohesion, $\phi'$ is the angle of internal friction, $u$ is the pore water pressure at the base, and $m_{ai}$ is obtained from the following equation.

$$m_{ai} = \cos \alpha_i + \left( \sin \alpha_i \tan \phi_i \right) / F_s$$

The Simplified Bishop method (1955) assumes that the contribution of vertical interslice forces to the factor of safety is neglected. In this study, it was assumed that the ground water table is deep, and so the ground water does not have any influence on the slope stability. Then, $F_s$ values were determined using the following equation

$$F_s = \frac{\sum_{i=1}^{n} \left( W_i + X_{i+1} - X_i - u_i b_i \right) \tan \phi' + c b_i \right) / m_{ai}}{\sum_{i=1}^{n} W_i \sin \alpha_i}$$

where $c$ is the cohesion, $\phi'$ is the angle of internal friction, $u$ is the pore water pressure at the base, and $m_{ai}$ is obtained from the following equation.

$$m_{ai} = \cos \alpha_i + \left( \sin \alpha_i \tan \phi_i \right) / F_s$$

The Simplified Bishop method (1955) assumes that the contribution of vertical interslice forces to the factor of safety is neglected. In this study, it was assumed that the ground water table is deep, and so the ground water does not have any influence on the slope stability. Then, $F_s$ values were determined using the following equation

$$F_s = \frac{\sum_{i=1}^{n} \left( W_i + X_{i+1} - X_i - u_i b_i \right) \tan \phi' + c b_i \right) / m_{ai}}{\sum_{i=1}^{n} W_i \sin \alpha_i}$$

where $c$ is the cohesion, $\phi'$ is the angle of internal friction, $u$ is the pore water pressure at the base, and $m_{ai}$ is obtained from the following equation.

$$m_{ai} = \cos \alpha_i + \left( \sin \alpha_i \tan \phi_i \right) / F_s$$

The Simplified Bishop method (1955) assumes that the contribution of vertical interslice forces to the factor of safety is neglected. In this study, it was assumed that the ground water table is deep, and so the ground water does not have any influence on the slope stability. Then, $F_s$ values were determined using the following equation

$$F_s = \frac{\sum_{i=1}^{n} \left( W_i + X_{i+1} - X_i - u_i b_i \right) \tan \phi' + c b_i \right) / m_{ai}}{\sum_{i=1}^{n} W_i \sin \alpha_i}$$

where $c$ is the cohesion, $\phi'$ is the angle of internal friction, $u$ is the pore water pressure at the base, and $m_{ai}$ is obtained from the following equation.

$$m_{ai} = \cos \alpha_i + \left( \sin \alpha_i \tan \phi_i \right) / F_s$$

The Simplified Bishop method (1955) assumes that the contribution of vertical interslice forces to the factor of safety is neglected. In this study, it was assumed that the ground water table is deep, and so the ground water does not have any influence on the slope stability. Then, $F_s$ values were determined using the following equation

$$F_s = \frac{\sum_{i=1}^{n} \left( W_i + X_{i+1} - X_i - u_i b_i \right) \tan \phi' + c b_i \right) / m_{ai}}{\sum_{i=1}^{n} W_i \sin \alpha_i}$$

where $c$ is the cohesion, $\phi'$ is the angle of internal friction, $u$ is the pore water pressure at the base, and $m_{ai}$ is obtained from the following equation.

$$m_{ai} = \cos \alpha_i + \left( \sin \alpha_i \tan \phi_i \right) / F_s$$

The Simplified Bishop method (1955) assumes that the contribution of vertical interslice forces to the factor of safety is neglected. In this study, it was assumed that the ground water table is deep, and so the ground water does not have any influence on the slope stability. Then, $F_s$ values were determined using the following equation

$$F_s = \frac{\sum_{i=1}^{n} \left( W_i + X_{i+1} - X_i - u_i b_i \right) \tan \phi' + c b_i \right) / m_{ai}}{\sum_{i=1}^{n} W_i \sin \alpha_i}$$

where $c$ is the cohesion, $\phi'$ is the angle of internal friction, $u$ is the pore water pressure at the base, and $m_{ai}$ is obtained from the following equation.

$$m_{ai} = \cos \alpha_i + \left( \sin \alpha_i \tan \phi_i \right) / F_s$$

The Simplified Bishop method (1955) assumes that the contribution of vertical interslice forces to the factor of safety is neglected. In this study, it was assumed that the ground water table is deep, and so the ground water does not have any influence on the slope stability. Then, $F_s$ values were determined using the following equation

$$F_s = \frac{\sum_{i=1}^{n} \left( W_i + X_{i+1} - X_i - u_i b_i \right) \tan \phi' + c b_i \right) / m_{ai}}{\sum_{i=1}^{n} W_i \sin \alpha_i}$$

where $c$ is the cohesion, $\phi'$ is the angle of internal friction, $u$ is the pore water pressure at the base, and $m_{ai}$ is obtained from the following equation.

$$m_{ai} = \cos \alpha_i + \left( \sin \alpha_i \tan \phi_i \right) / F_s$$
The prediction of the critical factor of safety of homogeneous finite slopes

The pseudo-static (PS) approach, apparently first introduced by Terzaghi (1950), is still the widely used in geotechnical engineering practice for the assessment of seismic slope stability due to ease of implementation and familiarity (Bakır and Akiş 2005). Therefore, in this study, the PS approach was used for considering the effects of an earthquake. This approach is a generalization of common limit equilibrium slope stability analysis (Baker et al. 2006). In this approach, the earthquake effects are represented by an equivalent static force, the magnitude of which is product of a seismic coefficient, $k$, and the weight, $W_i$, of the sliding mass (Baker et al. 2006). The PS approach has been implemented in Simplified Bishop method (1955) (Eq. (3)) and the factor of safety, $F_s$, values were then determined by using Eq. (4).

$$F_s = \frac{\sum_{i=1}^{n} \left[W_i \tan \phi' + c_i b_i \right] m_{ai}}{\sum_{i=1}^{n} W_i \sin \alpha} \quad (3)$$

$$F_s = \frac{\sum_{i=1}^{n} \left[W_i \tan \phi' + c_i b_i \right] m_{ai}}{\sum_{i=1}^{n} W_i \sin \alpha + \sum_{i=1}^{n} k W_i} \quad (4)$$

In this study, the $k$ value in Eq. (4) was taken as $a_{max}/g$ in which $a_{max}$ is the peak ground acceleration and is calculated from the earthquake magnitudes ($M$) and distances to epicenter ($R$) by using the attenuation relationship (Eq. (5)) as proposed by Campbell (1981). All the coefficients in Eq. (5) were found to be statistically significant levels of confidence exceeding 99 per cent, based on empirical distributions of the coefficients developed using procedures set forth by Gallant (1975) (Campbell 1981).

$$a_{max} = 0.0159 \exp(0.868M)(R + 0.0606\exp(0.700M))^{1.09} \quad (5)$$

Two slope parameters (viz., height of the slope, $H$ and the cotangent of the slope angle, $\cot \alpha$, three soil properties (viz., cohesion, $c$, internal angle of friction, $\phi'$, and bulk unit weight, $\gamma$) and two earthquake parameters (viz., magnitude of the earthquake, $M$, and the distance to epicenter, $R$) were varied during the slope stability analysis. The values of the parameters as used in these analyses are given in Table 1. Then, the factor of safety ($F_s$) for 5182 nos. of homogeneous finite

<table>
<thead>
<tr>
<th>Parameters used</th>
<th>Values used</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$ (m)</td>
<td>6, 8, 10</td>
</tr>
<tr>
<td>$\cot \alpha$</td>
<td>0.333, 1, 2</td>
</tr>
<tr>
<td>$\gamma$ (kN/m$^3$)</td>
<td>16, 18, 20</td>
</tr>
<tr>
<td>$c$ (kPa)</td>
<td>0, 10, 20, 50</td>
</tr>
<tr>
<td>$\phi'$ (deg)</td>
<td>10, 20, 35, 50</td>
</tr>
<tr>
<td>$M$</td>
<td>4, 6, 8</td>
</tr>
<tr>
<td>$R$ (km)</td>
<td>5, 15, 25, 40</td>
</tr>
</tbody>
</table>
slope having different slope, soil and earthquake parameters were calculated by using Eq. (4) for each trial failure surface and the minimum (critical) $F_s$ value was then determined for each case by using the written program.

4. Artificial neural network model

An ANN model was developed to predict the critical factor of safety ($F_s$) value of the homogeneous finite slope subjected to earthquake forces. Two slope parameters (viz., height of the slope, $H$ and the cotangent of the slope angle, $\cot \alpha$), three soil properties (viz., cohesion, $c$, internal angle of friction, $\phi$, and bulk unit weight, $\gamma$) and two earthquake parameters (viz., magnitude of the earthquake, $M$ and the distance to epicenter, $R$) were used as the input parameters in the ANN model, whereas, the calculated $F_s$ was the output parameter. The input and output data were then scaled to lie between 0 and 1, by using Eq. (6). In Eq. (6), where $x_{\text{norm}}$ is the normalized value, $x$ is the actual value, $x_{\text{max}}$ is the maximum value and $x_{\text{min}}$ is the minimum value.

$$x_{\text{norm}} = \frac{x - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}}$$ (6)

As mentioned earlier, over-fitting makes multi-layer perceptrons (MLPs) memorize training patterns in such a way that they cannot generalize well to new data (Twomey and Smith 1997, Choobbasti et al. 2009). Thus, the crossvalidation technique (Stone 1974), which is considered to be the most effective method to ensure over-fitting not to occur (Smith 1993), was utilized as the stopping criterion in this study. In this technique (Stone 1974), the database is divided into three subsets: training, validation and testing. The training set is used to adjust the connection weights (Shahin et al. 2004). The testing set is utilized to check the performance of the model at various stages of training, and to determine when to stop training to prevent over-fitting (Shahin et al. 2004). The validation set is used to predict the performance of the trained network in the deployed environment (Shahin et al. 2004). Shahin et al. (2004) obtained the optimal model performance when 20% of the data were utilized for validation and the rest data were divided into 70% for training and 30% for testing. Therefore, to avoid overfitting, the database was randomly divided into three sets: training, testing, and validation. In total, 56% of the data (i.e., 2903 data sets), 24% (i.e., 1244 data sets), and 20% (i.e., 1037 data sets) were randomly selected and used for training, testing, and validation sets, respectively, in the ANN model developed in this study.

The neural network toolbox of MATLAB7.0, a popular numerical computation and visualization software (Twomey and Smith 1997), was utilized for training, validation, and testing of MLPs in the ANN model. The Levenberg-Marquardt back-propagation learning algorithm (Demuth et al. 2006), was used in the training stage. One hidden layer with a sufficient number of hidden neurons is adequate approximating any continuous function (Hornik et al. 1989). Therefore, in this study, one hidden layer was chosen. Then, the optimum number of neurons in the hidden layer of the model was determined by varying their number starting with a minimum of 1 then increasing in steps by adding 1 neuron each time. Log-sigmoid transfer function, the most commonly used to construct the neural networks, was used in the ANN model to achieve the best performance in training as well as in testing. Two momentum factors, $\mu$, (equal to 0.01 and 0.001), were selected for the training process to search for the most efficient ANN architecture in each ANN model. The coefficient of determination, $R^2$, and the mean absolute error, $MAE$, were utilized
to evaluate the performance of each developed ANN model. The performance of the network during the training and testing processes was examined for each network size until no significant improvement occurred. The optimal ANNs performance was obtained with the model having 5 neurons in the hidden layer, and a 0.001 momentum factor.

5. Multiple regression model

Multiple regression (MR) is a statistical technique that allows us to predict someone’s score on one variable on the basis of their scores on several other variables (Auli et al. 2009). The purpose of MR is to learn more about the relationship between several independent or predictor variables and a dependent or criterion variable (Yılmaz and Yuksek 2008). MR equation takes the form \( y = b_1x_1 + b_2x_2 + \cdots + b_nx_n + c \) where \( \{b_1, b_2, \ldots, b_n\} \) are the regression coefficients, \( x_1, x_2, x_3, \ldots \) are the independent variables and \( c \) is y-intercept (Milton, 1997). MR is widely used in slope failure and landslides (i.e., Pradhan 2010a, 2010b, Erzin and Cetin 2012a, 2012b).

MR analysis was carried out by using SPSS 10.0 package to correlate the determined \( F_s \) value to two slope parameters (\( H \) and \( \cot \alpha \)), three soil parameters (\( \gamma, c, \) and \( \phi \)) and two earthquake parameters (\( R \) and \( M \)). The data used while developing the ANN model (i.e., 5184 nos. of data sets) were used in the development of the MR model. The MR model revealed the following correlation.

\[
F_s = 1.359 + 0.551 \cot \alpha - 0.088H - 0.039\gamma + 0.040c + 0.033\phi + 0.012R - 0.179M
\]

\[
R^2 = 0.853
\]

In Eq (7), \( H \) is in meters, \( \gamma \) in kN/m\(^3\), \( c \) in kN/m\(^2\), \( \alpha \) and \( \phi \) in degrees, and \( R \) in kilometers.

6. Results and discussion

The \( F_s \) values calculated from the Simplified Bishop method were compared with the \( F_s \) values predicted from the ANN model, as depicted in Figs. 2, 3, and 4 for training, validation, and testing samples, respectively. It can be noticed from the figures that the predicted values of \( F_s \) are quite close to the calculated values of \( F_s \), as their \( R^2 \) values are much close to unity. A paired t-test, a statistical test, utilizes the mean of the difference between the observations in one group and the matched observations in the other group. A paired t-test is carried out to determine if there is a significant difference between two observations. A paired t-test result can be expressed in terms of a \( p \)-value, which represents the weight of evidence for rejecting the null hypothesis (Ott and Longnecker 2001). The null hypothesis is the equality of mean of difference between comparisons (Ceylan et al. 2010). The null hypothesis can be rejected, that is, the mean of difference between comparisons are significantly different, if the \( p \)-value is less than the selected significance level (Ceylan et al. 2010). A significance level of 0.05 is used for all paired t-tests (Ceylan et al. 2010). Thus, \( p > 0.05 \) meant there was not a meaningful difference and \( p < 0.05 \) meant there was a meaningful difference (Tüysüz 2010). In this study, a paired t-test was performed by using the SPSS 10.0 package to look for a statistically significant difference between calculated and predicted \( F_s \) values. \( p \)-value was found as 0.329, indicating no significant difference in \( F_s \) between the calculated and predicted values. Therefore, the critical \( F_s \) value of the homogeneous finite
slope considered in this study could be predicted using trained ANN structures as quite easily and efficiently.

A comparison between the $F_s$ values as calculated from the Simplified Bishop method and the $F_s$ values as predicted from the MR model is shown in Fig. 5, for all the samples. It can be noted from the figure that the predicted $F_s$ values from the MR model are not in good agreement with the calculated $F_s$ values, as $R^2$ value is 0.8335. A paired-t test using the SPSS 10.0 package was also
The prediction of the critical factor of safety of homogeneous finite slopes

performed to determine whether there is a significant difference between calculated and predicted $F_s$ values. p-value was found as 0.000, indicating a meaningful difference between the calculated and predicted $F_s$ values. Therefore, the use of the MR model is not recommended at the preliminary stage of designing the homogeneous finite slope subjected to earthquake forces.

In fact, the coefficient of correlation between the measured and predicted values is a good indicator to assess the prediction performance of the any model developed. In this study, variance

![Fig. 4 The comparison of the calculated $F_s$ values with the predicted $F_s$ values from the ANN model for testing samples](image)

![Fig. 5 The comparison of the calculated $F_s$ values with the predicted $F_s$ values from the MR model for all samples](image)
VAF, given by Eq. (8), and the root mean square error RMSE, given by Eq. (9), were also computed to control the performance of the prediction capacity of predictive models developed in the study, as employed by previous researchers (Erzin 2007, Erzin et al. 2008, 2009, 2010, Erzin and Cetin 2012a, 2012b, Erzin an Gunes 2011).

\[
VAF = \left[1 - \frac{\text{var}(y - \hat{y})}{\text{var}(y)}\right] \times 100
\]  

and

\[
RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2}
\]

where \(\text{var}\) denotes the variance, \(y\) is the measured value, \(\hat{y}\) is the predicted value, and \(N\) is the number of the sample. If \(VAF\) is 100 % and \(RMSE\) is 0, the model is treated as excellent. The performance indices calculated for the ANN and MR models developed in this study are given in Table 2. As seen from Table 2, the ANN model has exhibited higher prediction performance than the MR model based on the computed performance indices.

In addition to the performance indices, a graph between the scaled percent error, SPE, (as given by Eq. (10) and employed by Kanibir et al. (2006) and Erzin et al. (2012)) and the cumulative frequency was also drawn as shown in Figs. 6 and 7 for the ANN and MR models, respectively, to show the performance of the models.

\[
SPE = \frac{(F_{sp} - F_{sc})}{(F_{sc})_{\text{max}} - (F_{sc})_{\text{min}}}
\]

where \(F_{sp}\) and \(F_{sc}\) are the predicted and the calculated factor of safety values; and \((F_{sc})_{\text{max}}\) and \((F_{sc})_{\text{min}}\) are the maximum and minimum calculated factor of safety values, respectively. It can be observed from Fig. 6 that about 99% of factor of safety values predicted from the ANN model developed fall into ± 5 % of the SPE indicating a perfect estimate for the \(F_s\) value of the slope. As depicted in Fig. 7, about 98% of factor of safety values predicted from the MR model developed fall into – 60% of the SPE indicating a poor estimate for the \(F_s\) value of the slope. From here, it can be concluded that the \(F_s\) value of the homogeneous infinite slope subjected to earthquake forces could be predicted from two slope parameters (\(H\) and \(\cot\alpha\)), three soil parameters (\(\gamma\), \(c\), and \(\phi\)) and two earthquake parameters (\(R\) and \(M\)) using trained ANNs values, with acceptable accuracy, at the preliminary stage of designing the homogeneous finite slope.

Table 2 Performance indices of the ANN and MR models developed

<table>
<thead>
<tr>
<th>Model</th>
<th>Data</th>
<th>(R^2) (%)</th>
<th>RMSE</th>
<th>MAE</th>
<th>VAF (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANN</td>
<td>Training set</td>
<td>98.60</td>
<td>0.13</td>
<td>0.10</td>
<td>98.67</td>
</tr>
<tr>
<td></td>
<td>Testing set</td>
<td>98.60</td>
<td>0.13</td>
<td>0.10</td>
<td>98.60</td>
</tr>
<tr>
<td></td>
<td>Validation set</td>
<td>98.50</td>
<td>0.13</td>
<td>0.11</td>
<td>98.50</td>
</tr>
<tr>
<td>MR</td>
<td>All set</td>
<td>83.30</td>
<td>0.43</td>
<td>0.33</td>
<td>85.28</td>
</tr>
</tbody>
</table>
The prediction of the critical factor of safety of homogeneous finite slopes

The effectiveness of the ANN and MR models developed was also checked by receiver operating characteristics (ROC), as employed by Pradhan et al. (2010), Oh and Pradhan (2011), Pradhan (2011), Erzin and Cetin (2012b). The ROC curve is a useful method of representing the quality of deterministic and probabilistic detection and forecast systems (Swets 1988). The ROC curve plots the false positive fraction (FPF = 1-specificity) on the X-axis and true positive fraction (TPF = sensitivity) on the Y-axis. It shows tradeoff between the two rates (Negnevitsky 2002). The
area under the ROC curve (AUC) characterizes the quality of a forecast system by describing the system’s ability to anticipate correctly the occurrence or nonoccurrence of predefined “events” (Negnevitsky 2002). The range of the AUC is 0.5 to 1.0. The result of test is considered perfect if $AUC = 1.0$, good if $AUC = 0.8$ to 1.0, moderate if $AUC = 0.6$ to 0.8, poor if $AUC = 0.5$ to 0.6, an AUC value of 0.5 reflects a random model (Metz 1986). The results of the ROC curve were obtained for validation samples for each model by using SPSS 10.0 package and shown in Fig. 8. The AUC values were obtained as 0.85 and 0.65 for the ANN and MR models, respectively. The results of the ANN model are considered as good and the results of the MR model are considered as moderate based on these AUC values. These indicate that ANN model has higher prediction performance than MR model, which indicates the usefulness of the ANN model.

7. Conclusions

In this study, the prediction of the critical $F_s$ value of the homogeneous finite slopes subjected to earthquake forces has been investigated by artificial neural network (ANN) and multiple regression (MR) models. To achieve this, with the written program, the $F_s$ values of 5184 nos. of homogeneous finite slopes having different slope, soil, and earthquake parameters were calculated by using the Simplified Bishop method and the minimum (critical) $F_s$ value for each case was determined. Two slope parameters ($H$ and $\cot \alpha$), three soil properties ($\gamma$, $c$, and $\phi$), and two earthquake parameters ($R$ and $M$) were used as input parameters in the ANN and MR models. The determined critical $F_s$ value was used as output parameter in both the models. The results as obtained from the ANN and MR models were compared vis-à-vis those obtained from the calculations. It is found that the ANN model exhibits more reliable predictions than the MR model. Therefore, the critical $F_s$ value of the homogeneous finite slope considered in this study could be predicted using trained ANN structures, as quite easily and efficiently.
To check the prediction performance of the ANN and MR models developed, several performance indices such as $R^2$, VAF, MAE, RMSE, and SPE were calculated. Also, ROC curves were drawn, and the AUC values were calculated. The ANN model has shown higher prediction performance than the MR model on the basis of performance indices and the AUC values. The performance level attained in the ANN model has shown that the ANN model is generalized enough and can be employed quite easily and accurately for estimating the critical $F_s$ value of the homogeneous finite slope subjected to earthquake forces.

References


The prediction of the critical factor of safety of homogeneous finite slopes


