

Chiral Restoration in the Early Universe: Pion Halo in the Sky

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1 Introduction

I am very pleased to have the opportunity to present to this distinguished audience some recent developments concerning chiral symmetry of the early universe. Chiral restoration is so taken for granted that it has not even been raised by others at this astroparticle workshop.

As you will see, there is indeed a chiral symmetry at high T , but this ‘restored’ chirality is a morphosis of the old zero temperature chirality. The original NJL vacuum undergoes an interesting *new phase transformation* such that $\langle \bar{\psi}\psi \rangle$ vanishes, but the vacuum continues to break our zero temperature chirality. The pion remains a Nambu-Goldstone boson, and actually acquires a halo while propagating through the early universe.

The pion has always played a ubiquitous role in strong interaction physics. In the conventional scenario, however, it has not been given any role at high T but is ignominiously dismissed in the early universe, and condemned to dissociate in the early alphabet soup. The results reported here correctly restore the pion to its rightful place in the early universe.

The pion is a messenger of an underlying broken symmetry of the universe, *viz.* that of chirality, under the transformation $\psi(\vec{x}, t) \rightarrow e^{i\alpha\gamma_5} \psi(\vec{x}, t)$. The chiral charge, Q_5 , which generates this transformation

$$Q_5 = \int d^3x \psi^\dagger(\vec{x}, t) \gamma_5 \psi(\vec{x}, t) \quad (1)$$

does not annihilate the vacuum. Instead, acting on the NJL vacuum[1], it generates, up to a normalization factor, the state for a *zero momentum pion*, $Q_5|vac\rangle \propto |\vec{\pi}(\vec{p}=0)\rangle$, where ($s = \mp 1$ for L, R helicities)

$$|vac\rangle = \prod_{p,s} \left(\cos\theta_p - s \sin\theta_p a_{p,s}^\dagger b_{-p,s}^\dagger \right) |0\rangle \quad (2)$$

Using the fact that Q_5 is a constant of motion, it is easy to show directly that this zero momentum pion, $Q_5|vac\rangle$, has zero energy, thus confirming the status of the pion as a QCD Nambu-Goldstone boson[2].

A signature of this dynamical symmetry breaking is the familiar order parameter, $\langle \bar{\psi} \psi \rangle$. For $T > T_c$, however, it is well known that $\langle \bar{\psi} \psi \rangle$ vanishes. Chiral symmetry is said to be restored at T_c , but is it the *same old chiral symmetry we knew at $T = 0$* ?

2 High Temperature Effective Action

At high temperatures, lattice work as well as continuum field theory calculations show that the effective action indeed exhibits a manifest chiral symmetry. In thermal field theory, there is the famous BPFTW action[3] that describes the propagation of a QCD fermion through a hot medium ($T'^2 \equiv \frac{g_r^2}{3} T^2$, while the angular brackets denote an average over the orientation \hat{n})

$$\mathcal{L}_{\text{eff}} = -\bar{\psi} \gamma_\mu \partial^\mu \psi - \frac{T'^2}{2} \bar{\psi} \left\langle \frac{\gamma_o - \vec{\gamma} \cdot \hat{n}}{D_o + \hat{n} \cdot \vec{D}} \right\rangle \psi \quad (3)$$

and we see the global chiral symmetry of the action. But the *nonlocality* of the action implies that the Noether charge for this new chirality is not the same as that in eq.(1).

The fermion propagator that results from this action shows a pseudo-Lorentz invariant particle pole of mass T' (the so-called thermal mass). But, in addition, there is a pair of conjugate *spacelike* plasmon cuts in the p_o -plane that run just above and below the real axis[4], from $p_o = -p$ to $p_o = p$. As a result, for $t > 0$, say, the propagator function takes the form

$$\langle T(\psi(x)\bar{\psi}(0)) \rangle_\beta = \langle \psi(x)\bar{\psi}(0) \rangle \quad (4)$$

$$= \int \frac{d^3 p}{(2\pi)^3} e^{i\vec{p} \cdot \vec{x}} \left\{ Z_p \frac{-i\vec{\gamma} \cdot \vec{p} + i\gamma_o \omega}{2\omega} e^{-i\omega t} \right. \quad (5)$$

$$\left. - \frac{T'^2}{8} \int_{-p}^p \frac{dp'_o}{p^3} \frac{i\vec{\gamma} \cdot \vec{p} p'_o - i\gamma_o p^2}{p^2 - p_o'^2 + T'^2} e^{-ip'_o t} \right\} + O(T'^4) \quad (6)$$

In a recent study of the spacetime quantization of the BPFTW action[5], I have shown that the spacelike cuts dictate a new thermal vacuum of the type

$$|vac' \rangle = \prod_{p,s} \left(\cos \theta_p - i s \sin \theta_p a_{p,s}^\dagger b_{-p,s}^\dagger \right) |0 \rangle \quad (7)$$

The 90° phase here in the generalized NJL vacuum is the reason why $\langle \bar{\psi} \psi \rangle$ vanishes for $T \geq T_c$.

The quantization of a nonlocal action is of course a technical matter. Suffice it here to say that the quantization has been formulated in terms of auxiliary fields so that the resulting action is local. In this context, the pseudo-Lorentz particle pole is described in terms of the massive canonical Dirac field, Ψ , and the spacelike cuts are associated with the auxiliary fields, which are functions of Ψ . This formulation allows for a systematic expansion of the ψ field in terms of the massive canonical Dirac field, Ψ . Let the $t = 0$ expansion for the original massless ψ field read

$$\psi(\vec{x}, 0) = \frac{1}{\sqrt{V}} \sum_p e^{i\vec{p} \cdot \vec{x}} \begin{pmatrix} \chi_{p,L} a_{p,L} + \chi_{p,R} b_{-p,R}^\dagger \\ \chi_{p,R} a_{p,R} - \chi_{p,L} b_{-p,L}^\dagger \end{pmatrix} \quad (8)$$

with a corresponding canonical expansion for the massive Ψ , then we find

$$a_{p,s} = A_{p,s} - i s \frac{T'}{2p} B_{-p,s}^\dagger + O(T'^2) \quad (9)$$

$$b_{p,s} = B_{p,s} + i s \frac{T'}{2p} A_{-p,s}^\dagger + O(T'^2) \quad (10)$$

The $O(T')$ terms in the Bogoliubov transformation imply the new thermal vacuum of eq.(7).

The chiral charge at high T is given by

$$Q_5^\beta = -\frac{1}{2} \sum_{p,s} s \left(A_{p,s}^\dagger A_{p,s} + B_{-p,s}^\dagger B_{p,s} \right) \quad (11)$$

so that it clearly annihilates the new thermal vacuum, in direct contrast with the $T = 0$ Noether charge

$$Q_5 = -\frac{1}{2} \sum_{p,s} s \left(a_{p,s}^\dagger a_{p,s} + b_{-p,s}^\dagger b_{-p,s} \right) \quad (12)$$

which clearly fails to annihilate the vacuum at high T .

3 $\langle \bar{\psi} \psi \rangle$ is an Incomplete Order Parameter

The traditional order parameter $\langle \bar{\psi} \psi \rangle$ cannot by itself give a full description of the nature of chiral symmetry breaking. The operator, $\bar{\psi} \psi$, belongs to a non-Abelian chirality algebra[6], $SU(2N_f)_p \otimes SU(2N_f)_p$. The original chiral broken ground state may be written as $|vac \rangle = \prod_p e^{iX_{2p}\theta_p} |0 \rangle$, where X_{2p} is an element of the algebra, while the new thermal vacuum is generated by a different element, Y_{2p} .

Our results here suggest the study of a new class of nonlocal order parameters,

$$-\frac{i}{\pi} \int d^3x \int_{-\infty}^{\infty} dt' \langle \bar{\psi}(\vec{x}, t) \psi(\vec{x}, t') \rangle + c.c. \quad (13)$$

which if nonvanishing would indicate the continued breaking of chiral symmetry. The integration over t' projects away the usual timelike spectrum of the operator ψ , and probes directly the properties of the spacelike cut. In our perturbative study here, this order parameter indeed is nonvanishing, being given by $-2 \sum_p \frac{T'}{p^2}$, analogous to the familiar expression for $\langle \bar{\psi} \psi \rangle$ at $T = 0$, given by $-2 \sum_p \frac{M}{\sqrt{p^2 + M^2}}$, where M is the mass gap parameter.

4 Pion halo in the Sky

The pion we know at zero temperature is not massless, but has a mass of 135 MeV . This is because of electroweak breakdown, giving rise to a primordial quark mass at the tree level. At very high T , when electroweak symmetry is restored, we have the interesting new possibility that the pion will fully manifest its Nambu-Goldstone nature and remain physically massless.[7]

The pion is described by an interpolating field operator, $\sim i\bar{\psi}\gamma_5 T^a \psi$, which does not know about temperature. It is the vacuum that depends on T . The state vector for a zero momentum pion at high T may be obtained from the thermal vacuum by the action $Q_5^a|vac' \rangle \propto |\pi^a(\vec{p}=0)\rangle$. This pion now has the property that even though it is massless, it can acquire a *screening mass* proportional to T . This is the pion mass that has been measured on the lattice at high T .

As a result, the pion propagates in the early universe with a halo. The retarded function for the pion shows that the signal propagates along the light cone, with an additional exponentially damped component coming from the past history of the source.

$$D_{\text{ret}}(\vec{x}, t) = \theta(-t) \left\{ \delta(t^2 - r^2) + \frac{T'}{r} \theta(t^2 - r^2) \left[e^{-T'|t-r|} + e^{-T'|t+r|} \right] \right\} \quad (14)$$

The screening mass leads to an accompanying modulator signal that ‘hugs’ the light cone, with a screening length $\propto 1/T$.

What are the cosmological consequences of a pion in the alphabet soup of the early universe?

I am not an expert, and part of my purpose in coming to this workshop is to learn from you. But one thing I know. In the usual scenario, the pion after chiral restoration will have acquired mass $\propto T$, and will quickly dissociate into constituent quark-antiquark pair. According to our new understanding, however, the Nambu-Goldstone theorem forces the pion to remain a strictly massless bound state at high T , and so the pion will contribute to the partition function of the early universe.

Fortunately, the pion does not contribute so many degrees of freedom as to upset the usual picture of the cooling of the universe. But I leave it to experts to help figure out the subtle changes there must surely be in the phase transitions of the early universe.

In the beginning there was light, and quarks, and gluons, to which we must now add the pions with halo.

References

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