Linear active disturbance rejection control of underactuated systems: The case of the Furuta pendulum

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Abstract

An Active Disturbance Rejection Control (ADRC) scheme is proposed for a trajectory tracking problem defined on a nonfeedback linearizable Furuta Pendulum example. A desired rest to rest angular position reference trajectory is to be tracked by the horizontal arm while the unactuated vertical arm (actuated arm) stays around its unstable vertical position without falling down during the entire maneuver and long after it concludes. A linear observer-based C-based controller of the ADRC type is designed on the basis of the flat tangent linearization of the system around an arbitrary equilibrium. The advantageous combination of flatness and the ADRC method makes it possible to on-line estimate and cancel the undesirable effects of the higher order nonlinearities disregarded by the linearization. These effects are triggered by fast horizontal arm tracking maneuvers driving the pendulum substantially away from the initial equilibrium point. Convincing experimental results, including a comparative test with a sliding mode controller, are presented.

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1. Introduction

The control of underactuated systems represents a difficult and challenging problem, especially when experimental implementations of synthesized control solutions are required. This is due, in some measure, to the existence of unmodeled dynamics and external forces, to the associated restrictions in the behavior of the non directly actuated variables [27] and the natural obstacle to linearity exhibited by a large subclass of these systems. Underactuated systems are becoming popular in many sophisticated control applications, such as spacecraft, aerial robotic systems, underwater vehicles, locomotive systems, flexible robots, etc. Some possible advantages associated to such systems are cost reduction, lighter structures, smaller dimensions, among others (see [24] for a comprehensive treatment of this class of systems).

The Furuta pendulum [12], also called the rotational pendulum, is one of the most popular underactuated systems in academic laboratories around the world. The system is provided with one control input and it has two mechanical degrees of freedom. It consists of an actuated arm, which rotates in the horizontal plane; the actuated arm is joined to a non actuated pendulum which rotates loosely in a vertical plane perpendicular at the tip of the horizontal rotating arm. The system is quite nonlinear due to the gravitational forces, the Coriolis and centripetal forces [4] and the acceleration couplings. In addition, it is nonfeedback linearizable and it exhibits a lack of controllability in certain configurations [7]. The system represents a suitable platform for testing diverse linear and nonlinear control laws.

Traditional control problems associated with the Furuta pendulum are mainly of two kinds: (1) the problem of balancing up the vertical pendulum to the upper, unstable, position (swinging up) and (2) the stabilization around this position. Several methodologies have been proposed to solve the problem of swinging up and balancing the Furuta pendulum, these include the energy based swinging up control [2], passivity-based control [26], adaptive attractive ellipsoid methods [25], friction compensation controllers [34], and extended state observer-based controllers [3], among others. In the study reported in [1], some different controllers for the Furuta pendulum were tested and compared to point out the principal advantages and drawbacks of the diverse control schemes. The study also considered the main physical limitations associated with the control of the pendulum, where among others, the possibility of control input saturations was specifically treated. Most of the

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stabilizing schemes rely on the tangent linearization around the unstable equilibrium point, and demand robust linear stabilizing schemes [25].

The linearized model of the Furuta pendulum is differentially flat (i.e., it is controllable) with a physically measurable flat output. Thus, the problem of stabilization and tracking can be tackled from a combined perspective of flatness and Active Disturbance Rejection Control. Here, we propose the use of a linear decoupled extended observer, motivated by the structure revealed by flatness, in an active disturbance canceling scheme of the ADRC class. In general, ADRC dates back to the French engineer Poncelet (see [14]). Seminal work about a closely related technique, called Disturbance Accommodation Control, is due to Johnson [16]. Other variants of the ADRC idea are found as the control of simplified purely phenomenological plant models using algebraic estimation techniques [8,9]. The idea of a controller with the capacity of lumped compensation of endogenous and exogenous disturbances by means of an observer based control was proposed by Han, introducing the concept of Active Disturbance Rejection Control (ADRC) [15]. ADRC controllers have led to a new paradigmatic view of traditional nonlinear control problems where disturbances, internal and external, are actively estimated and rejected. Experimental results have been reported in diverse examples of systems (see [6,13,14,36,38]). As mentioned in [23], linearized observer based control of nonlinear systems has produced successful implementations in disturbance canceling schemes. For the case of underactuated dynamical systems, observer based ADRC plus linearized flatness takes one further step into the ADRC control of nonlinearizable systems. Efforts on underactuated systems control have also been recently advanced with promising results (see [19,21,37]).

It has been shown that the use of approximate disturbance estimation, via extended Luenberger observers, known as Generalized Proportional Integral (GPI) observers [32], constitutes an effective manner of integrating ADRC schemes. The GPI observer naturally includes a self-updating, lumped, time-polynomial model of the nonlinear state-dependent perturbation (Ref. [18] studies another interesting approach of time-polynomial disturbance estimation technique). The GPI estimates the perturbations and delivers a time signal to the controller for on-line cancelation of the effect of unknown nonlinearities and foreign perturbations while, simultaneously, estimating the phase variables related to the measured output. The scheme achieves accurate on-line estimations of the joint effect of all unknown disturbances (state dependent or non state dependent). Some applications have been reported in the recent literature (see [30–33]). The main features of this control scheme lie in the fact that both, exogenous unstructured perturbation inputs and state-dependent perturbation inputs, appearing in the input-output model, are all lumped into a simplifying time-varying signal that needs to be linearly estimated. The control scheme takes advantage of the natural possibilities of differentially flat systems, allowing the use of linear disturbance estimation and linear output feedback control with disturbance cancelation (see [10] and the books [20,29] for a comprehensive treatment on differentially flat systems). The main difference between traditional flatness based controllers and the ADRC scheme for flat systems is the fact that traditionally flatness based controls need perfect knowledge of the plant while ARDC schemes for flat systems may largely ignore unknown nonlinearities and exogenous additive perturbation inputs in the input-to-flat output dynamics. Needless to say, flat systems do not have any zero dynamics, thus avoiding the problem of internal stability after feedback.

In this article, an ADRC scheme is proposed for a trajectory tracking problem associated with the Furuta pendulum. The tracking problem is that of having the horizontal arm follow a rest to rest angular position reference trajectory, while the unactuated pendulum is to remain around its unstable vertical position, without falling, during the entire tracking maneuver and long after it ceases. The control scheme assumes an important lack of knowledge of the system parameters, nonlinearities and exogenous disturbance signals. Using a tangent linearization model of the Furuta pendulum around an arbitrary equilibrium, we show that fast excursions from the unstable equilibrium point, triggering adverse effects of nonlinearities, are still feasible while maintaining the pendulum around its unstable vertical position. The scheme not only accurately estimates the effects of the excited nonlinearities, but it also reduces the tracking control problem to that defined on a chain of integrators after on-line active disturbance cancelations. The control scheme is tested on an experimental prototype, showing excellent results for the tracking error and the estimation of lumped state dependent and external disturbances.

Section 2 briefly considers the nonlinear model of the Furuta Pendulum and its tangent linearization around an arbitrary equilibrium point. In this section the flatness (controllability) of the linearized model is exploited to obtain a natural decoupled two stage observer design. The method is extendable to some other underactuated systems, i.e. the ball and beam, inverted pendulum on a cart, gantry crane systems, etc. Section 3 proposes a high gain extended linear observer based ADRC tracking scheme for the tangent linearized model of the system. High gain extended observers of the Luenberger type receive here the name of Generalized Proportional Integral (GPI) observers due to their dual relationship with robust linear GPI controllers, introduced by Flies et al. [11]. Section 4 special emphasis is placed on a careful pole placement for the linear observer estimation dynamics, avoiding the traditional “peaking” phenomenon, appearing in high gain controlled systems. Section 5 is devoted to present the details of the experimental setup and discusses the obtained closed loop performance of the system; besides, the section illustrates the controller behavior by means of an experimental comparison against a sliding mode controller, under the same control task. The conclusions and suggestions for further research constitute the topic of the last section.

2. The Furuta pendulum

2.1. The nonlinear Furuta pendulum model

The Furuta pendulum system consisting of an unactuated pendulum attached to the end of a horizontal rotating arm (see Fig. 1). The pendulum is free to move on a plane perpendicular to the horizontal arm which is driven by a DC motor. The nonlinear
model of the mechanical part of the system, which can be derived from either Newton equations or from the Euler–Lagrange formalism, is given by [22]

\[
\begin{align*}
(l_0 + m_1l_0^2 + l_1^2 \sin^2(\phi_0))\ddot{\phi} - m_1l_0 \cos(\phi)\dot{\phi}^2 + 2m_1l_1^2 \dot{\phi}^2 \sin(\phi) \cos(\phi) + m_1l_0 \dot{\phi}^2 \sin(\phi) + m_1l_0 \dot{\phi}^2 - m_1gl_1 \sin(\phi) &= 0
\end{align*}
\]

where \( \phi \) is the angular displacement of the pendulum with respect to the vertical line passing through the joint of the pendulum with the horizontal arm, \( \theta \) is the angle of the horizontal arm, measured with respect to an arbitrary but fixed direction in the \((x, y)\) plane, \( m_1 \) denotes the mass of the pendulum, \( l_1 \) stands for the pendulum inertia; \( l_0 \) represents the inertia of the horizontal arm; \( l_0 \) and \( l_1 \) are, respectively, the lengths of the horizontal arm and the distance between the center of mass of the pendulum and the joint with the horizontal arm; \( \tau \) is the control input directly applied by the DC motor to the horizontal arm. We specifically assume that only the angular positions, \( \theta \) and \( \phi \), are measurable. The model (1) and (2) is not feedback linearizable, i.e., it is non-flat.

Considered the tangent linearization of the system around the following arbitrary unstable equilibrium point:

\[
\begin{align*}
\theta &= 0, & \phi &= 0, & \tau &= 0 \\
\dot{\theta} &= 0, & \dot{\phi} &= 0
\end{align*}
\]

One readily obtains

\[
\begin{align*}
[l_0 + m_1l_0^2] \ddot{\phi} - m_1l_0 \dot{\phi}^2 &= \tau_\delta \\
[l_1 + m_1l_1^2] \dot{\theta} - m_1l_1 \dot{\phi}^2 &= 0
\end{align*}
\]

where \( \phi_\delta = \phi - 0 = \phi \), \( \theta_\delta = \theta - 0 = \theta \), and \( \tau_\delta = \tau - 0 = \tau \) are the incremental states of the linearized system. Notice that the linearization can be performed for any fixed value of \( \theta \), say \( \theta, \theta = 0 \) was selected for simplicity. In order to simplify the notation, let us define

\[
\begin{align*}
e &= \frac{l_0}{l_1}, & \alpha^2 &= \frac{l_0}{m_1l_1^2}, & \beta &= \frac{g}{l_1} \\
\gamma &= \frac{1}{m_1l_1^2}, & \eta &= \frac{l_1}{m_1l_1^2} + 1
\end{align*}
\]

One is lead to the following implicit incremental description of the system:

\[
\begin{align*}
\frac{\alpha^2 + \varepsilon^2}{\varepsilon \beta} \ddot{x}_\delta - \varepsilon \dot{y}_\delta &= \gamma \tau_\delta \\
\eta \ddot{y}_\delta - \varepsilon \dot{y}_\delta &= \beta \phi_\delta
\end{align*}
\]

Thus

\[
\beta \phi_\delta = \ddot{y}_\delta
\]

2.2. Flatness of the linearized Furuta pendulum model

The linear model (6) is differentially flat, with incremental flat output, denoted by \( F_\delta \), given in this case by the following expression:

\[
F_\delta = \eta \dot{y}_\delta - \varepsilon \dot{y}_\delta
\]

Indeed, all system variables in the linear model, i.e., states and the control input are expressible as differential functions of the incremental flat output. In other words, they are expressible as functions of the flat output \( F_\delta \) and a finite number of its time derivatives:

\[
\begin{align*}
\phi_\delta &= \frac{F_\delta}{\beta}, & \dot{\phi}_\delta &= \frac{F_\delta^{(3)}}{\beta} \\
\theta_\delta &= \frac{1}{\varepsilon} \left[ \frac{F_\delta}{\beta} - \tilde{F}_\delta \right], & \dot{\theta}_\delta &= \frac{1}{\varepsilon} \left[ \frac{F_\delta^{(3)}}{\beta} - \tilde{F}_\delta \right] \\
\tau_\delta &= \frac{(\alpha^2 \eta - \varepsilon \beta)}{\varepsilon \beta} \ddot{F}_\delta - \frac{(\alpha^2 + \varepsilon^2)}{\varepsilon \beta} \frac{\ddot{F}_\delta}{\varepsilon}
\end{align*}
\]

The linearized system is clearly equivalent to the following input–output model:

\[
F_\delta^{(4)} = \frac{\gamma}{\alpha^2 \eta - \varepsilon \beta} \ddot{\tau}_\delta + \frac{(\alpha^2 + \varepsilon^2)}{\varepsilon \beta} \ddot{F}_\delta
\]

where \( \alpha^2 \eta / \varepsilon \beta - \varepsilon / \beta \neq 0 \).

2.3. A useful decoupling property of the tangent model

It is immediate from the differential parametrization (9) that the tangent system naturally decomposes into a cascade connection of two independent blocks, the first one controlled by the torque input \( \tau_\delta \) with the corresponding output given by the flat output incremental acceleration, \( \ddot{F}_\delta \). This output coincides, modulo
a constant factor $\beta$, with the vertical arm incremental angular position, $\phi_{fz}$, i.e., $\bar{F}_z = \beta \phi_{fz}$. The signal $\beta \phi_{fz}$ acts then as an auxiliary known (i.e., measurable) input to the second block, which consists of a chain of two integrators rendering the phase variables $\bar{F}_z$ and $F_z$. The last variable $F_\delta$ being the output of the second block and the output to be controlled for the overall system (see Fig. 2).

This cascading property simplifies and decouples the observer design task in the Flatness based ADRC scheme to be presented next.

3. A GPI observer-based active disturbance rejection controller

3.1. Problem formulation

On the basis of (10), we adopt the following simplified perturbed model for the underlying nonlinear Furuta pendulum system (10):

$$F_{\delta}^{eq} = \frac{\gamma}{(\frac{\alpha_n}{\eta} - \eta)} \bar{F}_\delta + \bar{\xi}(t) \quad (11)$$

where $\bar{\xi}(t)$ represents state dependent expressions, all the higher order terms (h.o.t) neglected by the linearization, the possibly unmodeled dynamics, and external unknown disturbances affecting the system. We lump all this uncertain terms into a single time-varying function model represented by $\bar{\xi}(t)$, which in our case is of the form

$$\bar{\xi}(t) = (e^{2} + \varepsilon^{2}) F_{\delta} + h.o.t \quad (12)$$

Suppose it is desired to transfer the horizontal arm from the initial position $\theta_{i}(t) = 0$ towards the final position $\theta_{f}(t) = \Theta$ in a finite, prespecified, time interval $[0, t_f]$. The maneuver is to be carried out while the pendulum evolves closely around the unstable position (i.e., $\phi_0(t) = 0$). It is desired to accomplish this maneuver without losing the vertical unstable position of the pendulum arm $\phi_2(t) = 0$ at the end of, and long thereafter of, the prescribed time interval.

Clearly, such a rest-to-rest maneuver is feasible via an adequate planning of the trajectory for the flat output $F_{\delta} = \eta \phi_2 - \varepsilon \theta_1$. The vertical arm is required to start and end at the unstable position, then $\phi_0(0) = \phi_0(t_f) = 0$. The horizontal arm orientation $\Theta$ starts at rest at $\theta_0(0) = 0$ and ends the maneuver at a different rest point, $\theta_0(t_f) = \Theta_i$. Hence, the initial value of the flat output is $F_{\delta}(0) = 0$, while its final value is $F_{\delta}(t_f) = -\varepsilon \Theta_i$. One could prescribe a smooth rest to rest trajectory for $F_{\delta}(t)$ using, for instance, a Bézier polynomial with sufficient time derivatives being zero at the initial and at the final instants.

The flat output trajectory tracking error $e_{F_\delta} = F_{\delta1} - F_{\delta2}^{eq}$ is seen to evolve according to

$$e_{F_{\delta}}^{eq} = \frac{\gamma}{(\frac{\alpha_n}{\eta} - \eta)} \bar{F}_{\delta} + \bar{\xi}(t) \quad (13)$$

where $\bar{\xi}(t)$ represents the previously defined unknown input term, complemented now by the effects arising from the prescribed nominal flat output trajectory and its various time derivatives. Let $e_{F_{\delta}}^{eq} = e_i$, i = 1,2,3,4. The flat output trajectory tracking error perturbed state space model is, given by,

$$\dot{e}_1 = e_2$$
$$\dot{e}_2 = e_3$$
$$\dot{e}_3 = e_4$$
$$\dot{e}_4 = \frac{\gamma}{(\frac{\alpha_n}{\eta} - \eta)} \bar{F}_{\delta} + \bar{\xi}(t) \quad (14)$$

At this point we make use of the cascading property and view the previous system as the connection of two subsystems. Note that

$$e_{\delta} = \bar{F}_{\delta} - \bar{F}_{\delta}^{eq}(t)$$

is the known input to the second order pure integration system:

$$\dot{e}_1 = e_2$$
$$\dot{e}_2 = e_3$$
$$\dot{e}_3 = e_4$$
$$\dot{e}_4 = \frac{\gamma}{(\frac{\alpha_n}{\eta} - \eta)} \bar{F}_{\delta} + \bar{\xi}(t) \quad (16)$$

3.2. A cascaded GPI observer-based ADRC for the Furuta pendulum

The perturbation term $\bar{\xi}(t)$ is algebraically observable according to the results of Diop and Fließ [5]. Then, one may propose an instantaneous virtual evolution model of a time-polynomial nature for such a time-varying function $\bar{\xi}(t)$, denoted by $\xi$, and adopt, say, the following fifth order time polynomial model for $\bar{\xi}(t)$, i.e., $\bar{\xi}(t) = \xi(t) = z_i(t)$, the flat output trajectory tracking error model (14) may be rewritten as follows:

$$\dot{e}_1 = e_2$$
$$\dot{e}_2 = e_3$$
$$\dot{e}_3 = e_4$$
$$\dot{e}_4 = \frac{\gamma}{(\frac{\alpha_n}{\eta} - \eta)} \bar{F}_{\delta} + \xi(t) \quad (17)$$

The observation error, $\bar{\xi}_3 = e_3 - \bar{\xi}_3$, of the incremental flat output tracking error, generates the following linear injected estimation error dynamics:

$$\dot{\bar{\xi}}_1 + \lambda_1 \dot{\bar{\xi}}_1 + \lambda_0 \bar{\xi}_1 = 0, \quad \lambda_0, \lambda_1 > 0 \quad (20)$$

An appropriate choice of the design coefficients: $(\lambda_1, \lambda_0)$, placing the roots of the corresponding characteristic polynomial deep into the left half of the complex plane, renders an asymptotically, exponentially decreasing estimation error state, $\bar{\xi}_1, \dot{\bar{\xi}}_1 = \bar{\xi}_2$. The tracking error velocity for the flat output $e_{F_\delta}$ is, thus, accurately estimated for feedback purposes.
In the same manner, consider the observation error, $\hat{e}_3 = e_3 - \hat{e}_3$, of the flat output acceleration tracking error. It generates the following dominantly linear reconstruction error dynamics:

$$\dot{\hat{e}}_3^{(b)} + \lambda_3 \dot{\hat{e}}_3^{(3)} + \lambda_6 \dot{\hat{e}}_3^{(6)} + \cdots + \lambda_{1} \dot{\hat{e}}_3 + \lambda_0 \hat{e}_3 = \xi(t)$$

(21)

It may be proved that a necessary and sufficient condition for having the incremental flat output acceleration estimation error $\hat{e}_3$ and its associated phase variables, $\dot{\hat{e}}_3, \ddot{\hat{e}}_3, \ldots, \dot{\hat{e}}_3^{(b)}$, ultimately, uniformly, converge towards a small as desired neighborhood of the acceleration estimation error phase space is that $\hat{e}_3^{(b)}(t)$ be uniformly absolutely bounded. An appropriate choice of the design coefficients: $(\lambda_7, \ldots, \lambda_1, \lambda_0)$, placing the poles of the associated linear homogeneous system sufficiently far into the left half of the complex plane, renders a uniformly asymptotically convergent estimation error, $\hat{e}_3$, towards an arbitrary small vicinity of the origin along with a finite number of its time derivatives.

In our case, the observer gain parameters $\lambda_j$, for $j = 0, 1, 2, \ldots, 8$, are chosen using the methodology proposed by Kim et al. [17] based on the extensive use of the, so-called, characteristic ratios of the characteristic polynomial associated with the incremental flat output tracking error estimation dynamics. This methodology suitably mitigates the “peaking phenomenon” typical of linear high gain pole placement injected responses of the observer [35].

Consider a characteristic polynomial $p(s)$ of the form

$$a_0 s^n + a_{n-1} s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0, \quad a_i > 0$$

(22)

and let $a_i$ be the characteristic ratios of $p(s)$. It has been shown [17] that if the following two conditions are satisfied the polynomial (22) is Hurwitz

(A) $\alpha_1 > 2$;

(B) $\alpha_k = \frac{\sin \left( \frac{\pi k}{n} \right) + \sin \left( \frac{\pi k}{n} \right)}{2 \sin \left( \frac{\pi k}{n} \right)} \alpha_1$

for $k = 2, 3, \ldots, n - 1$. The construction of the all-pole stable characteristic polynomial involves only $\alpha_1$ which we require to be larger than 2. Thus, this result allows us to characterize the reference all-pole systems by adjusting a single parameter $\alpha_1$ to achieve the desired damping. The pole placement procedure is as follows:

For an arbitrary positive $a_0$ and $T > 0$

$$a_1 = \frac{T a_0}{a_{-1}}$$

Then, choose

$$\lambda_j = \frac{a_j}{a_0}$$

for $j = 0, 1, 3, \ldots, 8$

The smoothness of the error responses and noise rejection properties, achieved by this procedure, makes it a highly recommendable choice for pole placement in practical situations (see [17] for details).

4. The ADRC controller design

The control input may then be readily synthesized with an active disturbance canceling strategy for the uncertain input $\xi(t)$, in terms of its estimated value $\hat{\xi}(t)$, and the use, for feedback purposes, of the estimated time derivatives associated with the incremental flat output tracking errors: $e_1$ and the measurable incremental flat output acceleration error, $e_5$. We propose then

$$\tau_d = \frac{\alpha_1 g}{\gamma} \left[ \hat{e}_1 + k_3 \hat{e}_4 + k_2 e_3 + k_1 \hat{e}_2 + k_0 e_1 \right]$$

(23)

where, naturally, the tracking errors, $e_1$ and $e_3$, themselves are used instead of their redundant estimates. This is due to the fact that these variables are assumed to be measurable through the variables $\theta$ and $\phi$. Notice that the coefficients of the controller should be chosen in accordance with the fact that, asymptotically, the tracking error is being approximately governed by the differential equation:

$$\dot{e}_1^{(4)} + k_3 \dot{e}_1^{(3)} + k_2 \ddot{e}_1 + k_1 e_1 + k_0 e_1 = \xi(t) - \hat{\xi}_1$$

(24)

the set of design coefficients, $(k_3, \ldots, k_1, k_0)$ should render Hurwitz the underlying characteristic polynomial:

$$p(s) = s^4 + k_3 s^3 + k_2 s^2 + k_1 s + k_0$$

We propose $k_3 = 4 \alpha_1 \omega_1, k_2 = 2 \alpha_2 ^2 + 4 \alpha_1 ^2 \alpha_2 ^2, k_1 = 4 \alpha_1 ^2 \alpha_2 ^2, k_0 = \alpha_1 ^2$.

Fig. 3. Block diagram for the Furuta Pendulum control scheme.
5. Experimental results

Fig. 3 shows a diagram of the experimental platform used for the Furuta Pendulum. The experimental device (Fig. 4) consists of a Brushed servomotor from Moog, model C34L80W40, which drives the horizontal arm through a synchronous belt with a 4.5:1 ratio. The angles of the pendulum and arm (motor) can be measured with Incremental optical encoders of 2500 CPR. A Copley Controls digital amplifier model Junus 90, working in current mode, is in charge of driving the motor.

The Data acquisition is carried out through a data card from Quanser consulting, model QPIDe terminal board. This card reads signals from the optical incremental encoders and supplies control voltages to the power amplifiers. The control strategy was implemented in the Matlab-Simulink platform. Finally, the sampling time was set to be 0.0005 s. The Furuta Pendulum parameters were \( l_0 = 0.33 \text{ m}, \ l_1 = 0.275 \text{ m}, \ m_0 = 1.64 \text{ kg}, \ m_1 = 0.141 \text{ kg}, \ I_0 = 0.0481 \text{ kg m}^2, \ I_1 = 0.0036 \text{ kg m}^2 \), which are used in (18), (21) to obtain \( \beta, \eta, \alpha, \varepsilon, \) and \( \gamma \), respectively.

The initial conditions for the joint variables in the system were \( \phi = 0, \theta = 0 \). The observer gain parameters for the observation error \( \tilde{e}_1 \), were set to be \( \zeta_1 = 2, \ \omega_1 = 15 \). The observer gain parameters for observation error \( \tilde{e}_3 \) were set to be as follows: \( n = 8, \ T = 6, \ a_0 = 16, \ \alpha = 4 \). The controller design parameters were specified to be: \( \zeta_c = 1, \ \omega_c = 12 \).

Fig. 5 shows the Furuta Pendulum experimental platform performance; while the horizontal arm is rotating, pendulum’s vertically lies at the unstable vertical position \( \phi = 0 \). Fig. 6 shows the performance of the horizontal arm from the initial position \( \theta(2) = 0 \) towards the final position \( \theta(5.4) = 2\pi \) during a time interval \( t \in [2, 5.4] \text{ s} \). The produced control torque is depicted in Fig. 7, notice that the lumped disturbance estimation (see Fig. 8) determines the form of the control inputs, which tend to cancel out the additive disturbance inputs. Fig. 9 shows the behavior of

![Fig. 4. Furuta pendulum system prototype.](image)

![Fig. 5. Vertical angle closed loop behavior.](image)

![Fig. 6. Horizontal arm angular reference trajectory tracking behavior.](image)

![Fig. 7. Torque input trajectory.](image)
the flat output \( F_\delta = \eta \Phi_\delta - \varepsilon \Theta_\delta \) from the initial value \( F_\delta(2) = 0 \) towards the final value: \( F_\delta(5.4) = -2\pi \). Fig. 10 shows the tracking error which remains restricted to a bounded zone centered at the origin of the error phase space.

### 5.1. An experimental comparison test

In order to test the performance of the presented control scheme, we carried out a comparative analysis with respect to a sliding mode controller, which is a well-known robust control scheme. Taking advantage of the flatness of the linearized system, the implementation of a sliding mode (SM) based control scheme is feasible (see [28]). Define, for dynamics (14), the following sliding surface:

\[
\sigma = F_\delta^{(3)} - F_\delta^{(3)\ast} + \nu_1(F_\delta - F_\delta^{\ast}) + \nu_2(F_{\delta'} - F_{\delta'}^{\ast}) + \nu_3(F_\delta - F_\delta^{\ast})
\]  

(25)

where \( \{\nu_1, \nu_2, \nu_3\} \) are a set of real constants such that the polynomial \( s^3 + \nu_1 s^2 + \nu_2 s + \nu_3 \) is Hurwitz. Taking the time derivative of \( \sigma \) yields

\[
\dot{\sigma} = \gamma \left( \frac{\omega_p}{\omega_p^2 - \frac{\gamma}{p}} \right) \tau_\delta + \xi + \nu_1(F_\delta^{(3)} - F_\delta^{(3)\ast}) + \nu_2(F_{\delta'} - F_{\delta'}^{\ast}) + \nu_3(F_\delta - F_\delta^{\ast}).
\]

(26)

Thus, the sliding mode control is specified as follows:

\[
\tau_\delta = -\frac{\omega_p}{\omega_p^2 - \frac{\gamma}{p}} W \text{sign}(\sigma), \quad W > 0
\]

(27)

In order to reduce the chattering effect, the following control law is proposed:

\[
\tau_\delta = -\frac{\omega_p}{\omega_p^2 - \frac{\gamma}{p}} W \frac{\sigma}{\| \sigma \|^2 + \mu}
\]

(28)

where \( \mu = 0.0005 \) is a small positive scalar parameter and \( W \) must be selected such that \( W > \sup \| \xi(t) \| \), to guarantee that the tracking error tends to zero as time evolves. For the sliding surface \( \sigma = 0 \), the set of gains were chosen such that the following desired characteristic polynomial was matched: \( s^3 + \nu_1 s^2 + \nu_2 s + \nu_3 = (s + p) (s^2 + 2\zeta_c \omega_c s + \omega_c^2) \), with \( p = 5 \), \( \omega_c = 5 \), and \( \zeta_c = 1.7 \).

From Fig. 8, the value of \( W \) was set to be 2500, which is a reasonable experimental value for a good disturbance estimate from the observer-based results, \( W > \sup \| \xi(t) \| \). Fig. 11 shows the tracking error signal, which remains bounded for both controllers. Notice that the sliding mode controller (SM) presents a slight chattering due to high gain smoothing of the control input signal.
However, the GPI controller (GPI) offers a better steady state response. The flat output tracking errors are rather smooth and the GPI achieves the tracking task with a smaller error magnitude. Finally, Fig. 12 shows the sliding mode control input (SM). In order to compare the control efforts, a filtering of the sliding mode control input (SM LP) was carried out with a second order low pass filter of the Butterworth type with a cut-off frequency of 25 rad/s, with transfer function given by $LPF(s) = 625/(s^2 + 35.3533s + 625)$. The figure shows a comparison with respect to the GPI control input (GPI). Notice that both signals have a similar magnitude.

6. Conclusions and future work

The advantageous combination of flatness and active disturbance rejection control allows for the efficient solution of a challenging trajectory tracking problem in a popular underactuated, nonfeedback linearizable, mechanical system known as the Furuta Pendulum. The fact that the tangent linearization of this system, around an arbitrary equilibrium point, is controllable (hence, flat), allows the use of robust active disturbance rejection control in the efficient on-line estimation of the neglected nonlinearities and their active feedback cancelation. The flatness property reveals a cascading feature that immediately allows for a clear decoupling of the linear observer scheme. The solution, which is quite robust with respect to unmodeled disturbances and neglected nonlinearities, is entirely linear and it is based on a set of decoupled linear extended observers and a single linear output feedback controller with disturbance cancelation features. Experimental results reveal the effectiveness of the proposed design.

The scheme here advocated may be extended to an interesting class of underactuated systems, specifically, those which exhibit controllable tangent linearizations, i.e., the ball and beam system, the gantry crane, the double inverted pendulum, etc. These systems enjoy similar flatness and cascading properties in their linearized models. They will be the subject of further research in the upcoming future, in which a general solution methodology could be obtained.

References


