Channel Allocation in OFDMA based Wireless Ad Hoc Networks

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ABSTRACT

Wireless ad hoc networks find great utility in situations where there is no wired base station infrastructure. For large networks wireless channel access needs to be performed in a distributed manner. Orthogonal Frequency Division Multiple Access (OFDMA) is an emerging multiple access technique that is used in several new networking technologies. In this paper we study the use of OFDMA for ad hoc networks. Each point-to-point link does not transmit over the entire band and uses a subset of the total number of available subcarriers. This enables flexible network resource management with the ability to vary the output link capacity depending on the traffic load. Specifically, we address the problem of subcarrier allocation for point-to-point links of ad hoc networks. We present a distributed algorithm for subcarrier allocation and compare its performance to a centralized graph-theoretic heuristic. The simulation results also show that our protocol is robust and scalable.

1. INTRODUCTION

Over the past few years, a lot of interest has focussed on wireless ad hoc networks as they allow flexible topology organization. Apart from the growing demand for broadband wireless Internet access there is also a need to connect multiple networks together via wireless IP in situations where wired connections are not available or are costly to install. This requires the development and deployment of fast wireless backbone routers.

While the exact system and deployment scenarios that would emerge from the convergence of the Internet and the wireless communications realm are still evolving, a key enabling technology is going to be high-speed wireless routers [1][2]. Since these would be used as nodes in multihop wireless backbones, they would require extremely fast point-to-point links. Such wireless backbones can be treated as a cloud with multiple access points. It would essentially provide the service of transporting IP traffic among its multiple access points with diverse traffic patterns and rate requirements.

The various point-to-point links would need to support extremely high data rates since they form part of core network. This is likely to place an extremely high premium on the limited radio spectrum. This necessitates efficient channel assignment mechanisms that involve intensive spectrum reuse for the achievement of high data transmission rates. Spectrally efficient modulation schemes like Orthogonal Frequency Division Multiplexing (OFDM) [3][4] can be employed. As the network would be operating in multi-path fading environments, the major problem is that of inter-symbol interference (ISI). Again, multicarrier modulation techniques such as OFDM have been shown to reduce ISI and multipath delay spread considerably, thus enabling transmission with high symbol rates.

OFDM is becoming an increasingly popular modulation technique and has been standardized for Digital Audio Broadcast (DAB) and Digital Video Broadcast (DVB). OFDM was also adopted in the new 802.11a standard for Wireless Local Area Networks (WLANs) [5]. Flarion Technologies has proposed a variant, called Flash OFDM, for 3G systems [6]. Cisco is considering vectored OFDM for digital microwave communications [7].

Figure 1 illustrates the basic operation of an OFDM modulator. A high data rate stream is split into *N* lower rate streams, which are modulated separately on orthogonal subcarriers. In most systems, each subcarrier is modulated with *M*-ary Quadrature Amplitude Modulation (QAM) [8]. The collection of symbols on the *N* subcarriers is known as an OFDM symbol. The OFDM symbol length is *N* times that of the original symbol. This is what helps reduce the multipath delay spread.

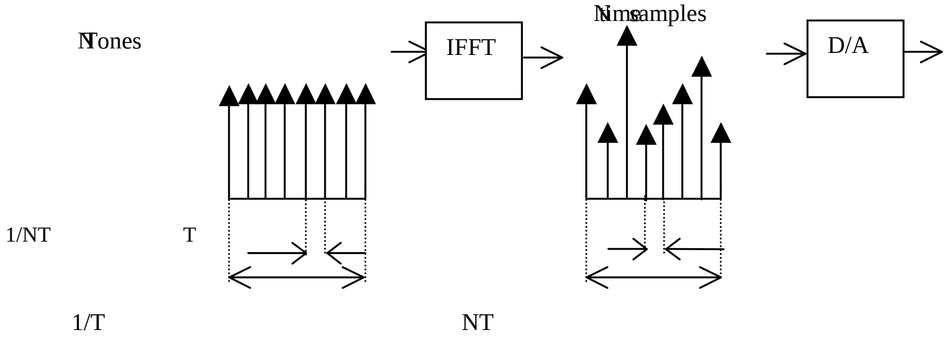

Fig.1 Schematic diagram of OFDM

In OFDMA each point-to-point link does not transmit over the entire band and uses a subset of the total number of available subcarriers. This enables flexible network resource management with the ability to vary the output link capacity depending on the traffic load. Specifically, we address the problem of subcarrier allocation for point-to-point links of ad hoc networks. We present a distributed algorithm for subcarrier allocation and compare its performance to a centralized graph-theoretic heuristic.

2. RELATED WORK

We now briefly review some prior work on channel allocation techniques for ad hoc networks. In [9] and [10] TDMA broadcast scheduling is considered, while in [11] and [12] the authors study TDMA link scheduling. An investigation into the complexity of scheduling problems is given in [13]. Distributed scheduling algorithms are presented in [12] and [14]. Many assignment problems have been shown to be equivalent to coloring the nodes or edges of an appropriately defined graph and hence are NP-complete [15], including TDMA broadcast scheduling, link scheduling, FDMA frequency assignment and CDMA code assignment. In [16] a unified framework and algorithm for all kinds of assignment problems is presented.

In [17] the authors propose an extension to the IEEE 802.11a wireless LAN standard [5]. Their model is a single network access point serving a number of terminals, which require varying data rates. They also use OFDMA so that different terminals are allocated different numbers of subcarriers depending on their data rate requirement. In this manner all terminals can transmit simultaneously without collisions. The use of an IFFT in an OFDM modulator greatly simplifies this because subcarriers that are not used can be simply zeroed out at the IFFT input. They also go on to discuss various timing and frequency synchronization issues. In our work we consider ad hoc networks that use OFDMA.

3. SYSTEM MODEL

The system under consideration is an ad hoc network of wireless routers as shown in figure 2. Each router node consists of multiple ports, each of which is equipped with an OFDM radio. A port on one node can communicate with a port on another node by forming a point-to-point wireless link. We assume that all radios are half-duplex. Every wireless point-to-point link transmits on a subset of the total number of subcarriers. In this way, transmissions along every link can take place concurrently without collisions. Subcarriers can be reused beyond a certain distance. Two links are said to be neighboring if they have a node in common. The spatial reuse condition is as follows: each link must use subcarriers that are different from the ones used by its neighbors as well as the ones used by the neighbors' neighbors. In other words, if

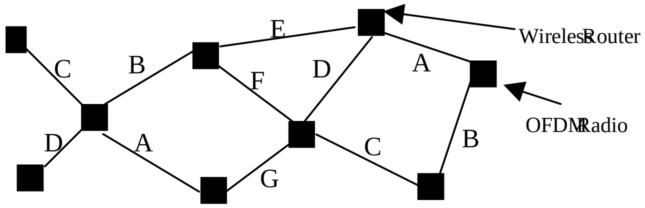

Fig. 2. OFDMA based wireless ad-hoc network

two links use the same subcarrier(s), then they must have at least two links between them to ensure collision-free transmission. This is equivalent to coloring the edges of the underlying graph. Under the above constraints, several centralized heuristics exist. Our objective is to develop distributed algorithm based on local exchange of control messages.

As an analogy, each link can be regarded as a "pipe" whose thickness depends on the number of subcarriers allocated to that link, which in turn depends on the data rate requirement of that link. OFDMA allows us to dynamically adjust the "thickness" of the pipe by adding or removing subcarriers. However, due to the limited spectrum, increasing the thickness of one pipe comes at the expense of reducing the thickness of some other pipe(s). In this work we are assuming that the channels for all subcarriers are identical and that all subcarriers carry the same number of bits per second.

4. ALGORITHM

In this section, we describe our distributed channel allocation algorithm. Our system model is as described in Section 3. We consider a network of high-speed wireless routers, which form a gigabit wireless cloud transporting traffic among multiple access points to the cloud with diverse traffic patterns and bandwidth requirements. Even before addressing the problem of how to assign channels to the various links in the wireless cloud, it is necessary for us to determine how much to allocate to each link, which is dictated by the parameters of the system model. In this work, we do not address the problem of `how much' to allocate to each link. We assume that each link has pre-specified requirement in terms of number of sub-channels. Given this requirement, we propose a distributed algorithm to efficiently perform the actual task of channel allocation while honoring the various constraints.

The pseudo-code for our algorithm is given in Figure 3. Our algorithm combines the concepts of hierarchical clustering and polling to enable efficient channel allocation. The algorithm proceeds as follows. The ad-hoc network is first divided into clusters based on the well known lowest ID clustering algorithm [18]. This results in a few nodes being elected as clusterheads. The next step involves another level of clustering among these clusterheads. Thus, the clusterheads are grouped into super-clusters based on the same lowest ID clustering algorithm [18], which is applied this time only to the clusterheads. This results in the entire network being organized into super-clusters, with each super-cluster having a master and comprising of one or more clusters, which in turn have their own clusterheads.

This completes the clustering phase of our algorithm. The polling and actual allocation of frequencies starts after this. Note that the spectral re-use criterion effectively means that any two links that are in different super-clusters can use the same frequency bands (*i.e.*, sub-channels). This means that sub-channel assignment can start in parallel in all the super-clusters independent of each other. The polling phase starts with each master first allocating sub-channels to the links it shares with ordinary neighbors, as well as the links that are shared by two of its ordinary neighbors. After this, it polls each clusterhead under its jurisdiction, instructing the clusterhead to allocate frequencies to the links in its cluster. To prevent multiple clusterheads from picking the same sub-channels, the master polls the clusterheads in order of their ID, waiting for an acknowledgement from the polled clusterhead that the frequency assignment is complete. This is illustrated in Figure 4(a). Further, when the master polls a clusterhead, it also passes on the restriction that the clusterhead has to obey while allocating frequencies. The restriction is in the form of

```
Procedure CHANNEL_ALLOCATE
Inputs: Network_Topology N, Bandwidth_Requirements R, Available_Spectrum F
Outputs: Allocation_List A
SET C          Set of clusterheads
SET M          Set of masters
{
1. C ← Form_clusters(N);
2. M ← Form_Super-clusters(C, N);
3. For every node i ∈ M {
        Assign_frequencies_to_own_neighbors(N, R, A);
        Poll_each_clusterhead_and_get_acknowledgement(N, R, A);
   }
}
```

Fig. 3 Pseudo-code of the channel allocation algorithm

list of forbidden frequencies. The clusterhead then allocates the required number of sub-channels to the various links in its cluster and reports back to the master, passing a list of the allocated frequencies to it. This is illustrated in Figure 4(b). The master adds these frequencies to the forbidden list and proceeds to poll the next clusterhead in its super-cluster. The process continues till all the clusterheads are polled and allocate the required subchannels to their links. Our algorithm also handles the case where a clusterhead might belong to more than one super-cluster, in which case the clusterhead waits for go-ahead signal from all the masters whose super-cluster it is a part of, before attempting to allocate frequencies. Finally, we handle the case where the two endpoints of a link are in different super clusters through the use of a simple negotiation protocol, wherein the endpoint with the lower ID requests its clusterhead to allocate the required number of subchannels for the link in question.

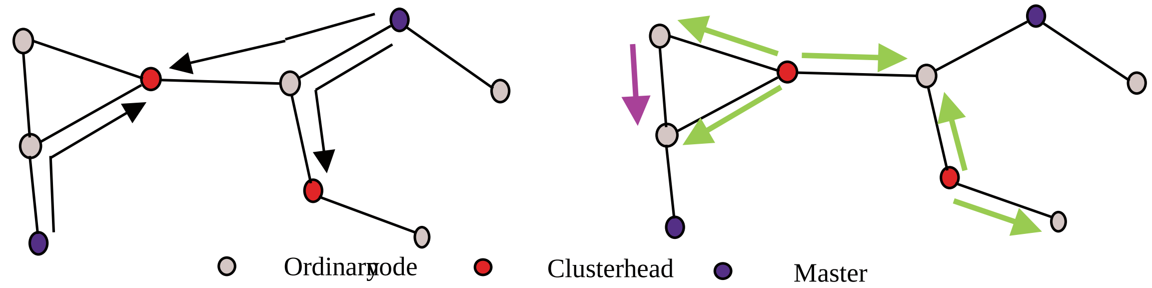

Fig. 4 (a) Masters poll clusterheads sequentially, and (b) Each clusterhead allocates subchannels

5. ANALYSIS

We now present some analysis of clustering. We assume a network in which N nodes are randomly distributed over a square field of side L and each node has a transmission radius R. As a consequence, a node can send and receive messages from other nodes that are located within a circular disc of area πR^2 and centered at that node. The average node density, D, is defined as:

$$D = \frac{N}{L^2} \qquad \ldots(1)$$

The average number of nodes within such a disc, or in other words the average degree of a node is given by

$$\lambda = D \cdot \pi R^2 \qquad \ldots(2)$$

For a uniform node distribution, this single parameter identifies the average connectivity of the network. We now calculate the distribution of the degree of a node. The probability *P(n)* of having degree *n* is given by the binomial distribution:

$$P(n) = \binom{N}{n} P^n \, (1-P)^{N-n} \qquad \ldots(3)$$

where P is given by

$$P = \frac{\pi R^2}{L^2} \qquad \ldots(4)$$

For large values of λ, the binomial distribution converges to Poisson distribution and taking into account that $\lambda = NP$, we get

$$P(n) = e^{-\lambda} \, \frac{\lambda^n}{n!}$$

We use this result to analyze the least id based clustering algorithm proposed by the authors of [18]. Specifically, for given N and λ, we calculate the average number of clusterheads in the network. When a node has n neighbors, the probability of its being clusterhead is $(1/(n+1))$ since the nodes as well as their ids are uniformly randomly distributed. Therefore, the probability that any given node is clusterhead is given by

$$\begin{aligned} &\sum_{n=0}^{\infty} \frac{1}{n+1} P(n) \\ &= \sum_{n=0}^{\infty} \frac{1}{n+1} e^{-\lambda} \, \frac{\lambda^n}{n!} \\ &= \frac{1}{\lambda} \sum_{n=0}^{\infty} e^{-\lambda} \, \frac{\lambda^{n+1}}{(n+1)!} \\ &= \frac{1-e^{-\lambda}}{\lambda} \qquad \ldots(6) \end{aligned}$$

Thus, the average number of clusterheads is given by

$$N_1 = \frac{N(1-e^{-\lambda})}{\lambda} \qquad \ldots(7)$$

We see that the average number of clusterheads is directly proportional to the total number of nodes and (approximately) inversely proportional to the average degree of a node. Approximately, one out of λ nodes, on an average, becomes a clusterhead. The clusterheads further select their leader, referred to as "master". Two clusters are said to be neighboring if there exist an edge between any two nodes, one from each cluster. Each cluster can now be considered to be a supernode and two supernodes are neighbors if the corresponding clusters are neighboring. We now analyze the graph with supernodes as the vertices. The goal is now to calculate the average degree of this new graph. We note that there is just one edge between two neighboring supernodes whether there are one or more links connecting the corresponding clusters.

For large values of λ, and even for relatively small values (≥ 4), N_s is approximately equal to N/λ. N_s is the average number of supernodes. To calculate the average degree of the new graph we consider the incidence matrices of the two graphs as shown in Figure 5. Since the original graph is undirected, the incidence matrix is symmetric and the new graph is also undirected. On an average when λ nodes are clustered to form one supernode, $\lambda \times \lambda$ order submatrices from the original incidence matrix map onto a single element of the new incidence matrix. The submatrices that are symmetric about the diagonal map onto the diagonal zeros in the new matrix. In total there are $N/\lambda \times N/\lambda$ submatrices of $\lambda \times \lambda$ order and hence the new incidence matrix is of order $N/\lambda \times N/\lambda$.

In the original matrix, any off-diagonal element is 1 (or in other words, an edge between the corresponding nodes exists) with probability P and 0 with probability $(1 - P)$, where P is as defined in equation (4). These events are independent. An off-diagonal element of the new incidence matrix is 0 if all the elements in the corresponding $\lambda \times \lambda$ order submatrix are zeros. Thus, the probability for an off-diagonal element of the new incidence matrix being zero is

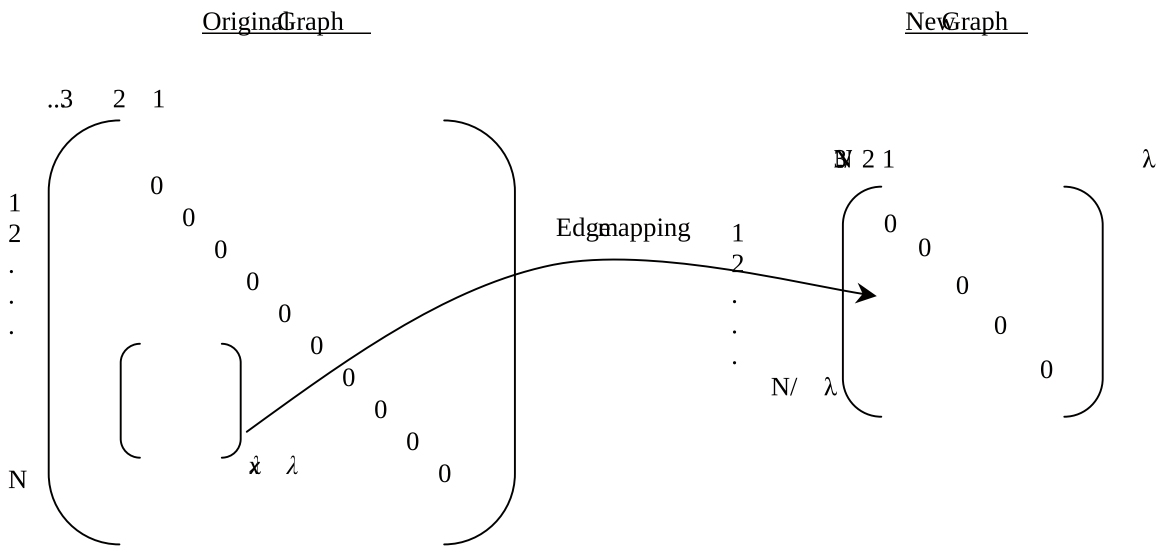

Fig. 5. Supernode transformation

In the original matrix, any off-diagonal element is 1 (or in other words, an edge between the corresponding nodes exists) with probability P and 0 with probability $(1 - P)$, where P is as defined in equation (4). These events are independent. An off-diagonal element of the new incidence matrix is 0 if all the elements in the corresponding $\lambda \times \lambda$ order submatrix are zeros. Thus, the probability for an off-diagonal element of the new incidence matrix being zero is

$$(1 - P)^{\lambda^2}$$

Hence, the probability of a non-zero element (or an edge between the corresponding supernodes) is equal to

$$\overline{P} = 1 - (1 - P)^{\lambda^2} \qquad \ldots(8)$$

We can now calculate the expected value of the total number of edges in the new graph. We note that the new incidence matrix is symmetric and need to sum over only the lower triangular elements.

$$Total\ number\ of\ edges = \overline{P}\left[\left(\frac{N}{\lambda} - 1\right) + \left(\frac{N}{\lambda} - 2\right) + \ldots + 1\right]$$

$$= \frac{1}{2}\frac{N}{\lambda}\left(\frac{N}{\lambda} - 1\right)\overline{P} \qquad \ldots(9)$$

Note that for any graph, the sum of the degrees of all nodes is equal to twice the number of edges. As a consequence, the average degree of supernodes is given by

$$\begin{aligned} \bar{\lambda} &= \left(\frac{N}{\lambda} - 1\right)\bar{P} \\ &= \left(\frac{N}{\lambda} - 1\right)\left[1 - (1-P)^{\lambda^2}\right] \qquad \ldots(10) \end{aligned}$$

For any given graph with known connectivity and number of nodes, equation (7) gives the average number of clusters. Applying the same result to the new graph with N_1 number of supernodes and average connectivity as given in equation (10), we can calculate the number of super-clusters or the number of masters, N_2.

$$\begin{aligned} N_2 &= \frac{N_1}{\bar{\lambda}}\left(1 - e^{-\bar{\lambda}}\right) \\ &\cong \frac{N_1}{\bar{\lambda}} \qquad \ldots(11) \end{aligned}$$

Since different super-clusters perform subcarrier allocation simultaneously, the computational complexity is greatly reduced.

6. SIMULATION RESULTS

We have implemented our algorithm in PARSEC [19], a parallel simulation language, and have evaluated its performance through simulations. There are two measures of effectiveness for such a channel assignment algorithm. First, the algorithm should minimize the total number of frequency sub-bands used. This will enable efficient use of the wireless spectrum to support more links. The second measure of performance is scalability of the algorithm (*i.e.*, how the algorithm performs as the network size increases). To evaluate its performance, we compare our algorithm with a centralized, graph theoretic approach to channel allocation. The channel allocation problem is similar to the graph-coloring problem with additional constraints. We developed a graph-theoretic heuristic for finding the minimum total number of frequency sub-bands needed, given a network topology and the bandwidth requirements along each link.

Figures 6 and 7 plot the total number of frequency sub-channels required against the network connectivity, and the network size, respectively. The red curve (the curve on the top) gives the number of frequency bands required using the proposed algorithm and the blue curve (the curve on the bottom) gives the minimum number of frequency sub-bands required using the graph-theoretic heuristic. As can be seen in the figure, our algorithm performs well and uses only a slightly higher number of frequency sub-channels, compared to the centralized algorithm. Figures 8 and 9 show the scalability of the system in terms of the number of control messages transmitted in the network during the allocation process, plotted against the network size, and the connectivity, respectively. This gives a measure of the communication overhead involved. As can be seen in the figures, the communication overhead scales linearly with both the number of nodes in the network as well as the network connectivity, showing that the protocol is scalable and can be efficiently applied to large, highly connected networks.

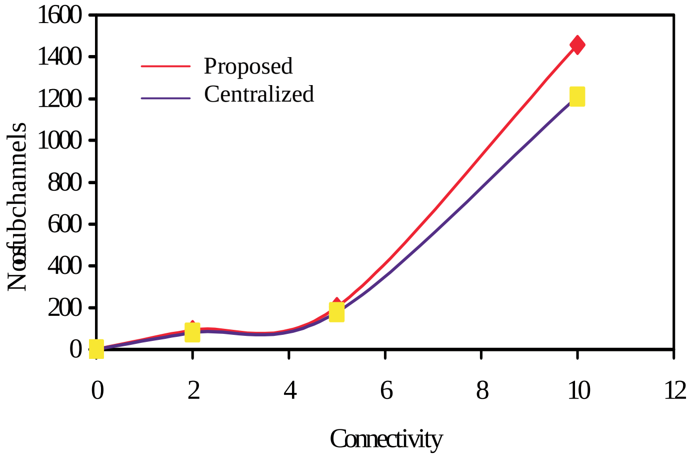

Fig. 6. Number of subchannels required ***vs.*** **network connectivity**

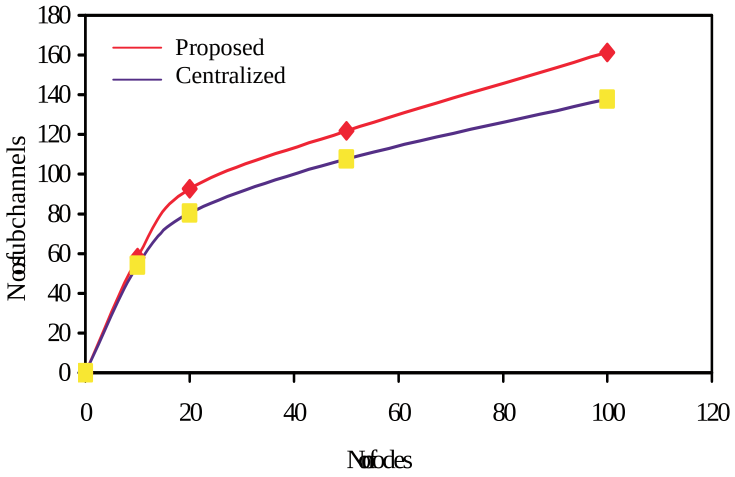

Fig. 7. Number of subchannels required ***vs.*** **network size**

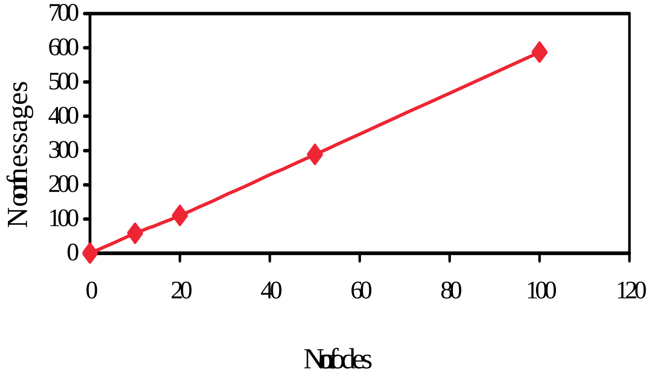

Fig. 8. Number of control messages ***vs.*** **network size**

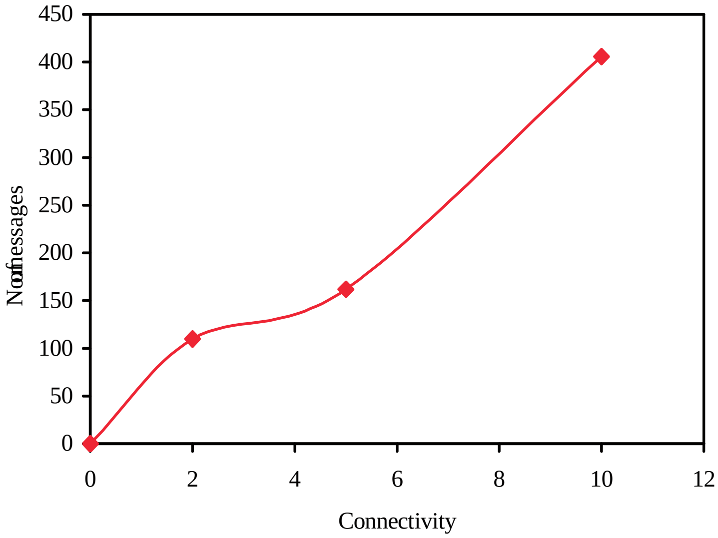

Fig. 9. Number of control messages *vs.* network connectivity

7. CONCLUSION

We have studied the use of OFDMA for ad hoc networks. The advantage of using OFDMA is that each point-to-point link can transmit simultaneously by using a subset of the total number of available subcarriers. This enables flexible network resource management with the ability to vary the output link capacity depending on the traffic load. We have presented distributed algorithm for subcarrier allocation and compared its performance to centralized graph-theoretic heuristic. The simulation results also show that our protocol is robust and scalable. The algorithm is based on hierarchical clustering and we presented some analytical results for the clustering scheme.

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