Slow Resonance Ratio Control for Torsional Vibration Suppression

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Abstract
Vibration suppression and disturbance rejection control in torsional system is an important issue in the future motion control. Resonance ratio control is one of the most effective strategies, but the disturbance should be estimated much faster than the resonance frequency. Too fast disturbance observer causes implementation problem. To solve this problem, we propose a novel technique called "slow resonance ratio control".

1. Introduction
In recent industrial drive system like a steel rolling mill system, as the newly required speed response is very close to the resonant frequency, conventional techniques based on PI controller are not effective enough. To overcome the problems, various control strategies have been proposed mainly for controlling 2-inertia system, the simplest model of the flexible system.

A few years ago, we proposed and showed the excellent performance of "resonance ratio control" by simulation. In this method, by feeding back the torsional torque estimated by the disturbance observer, the virtual motor inertia moment can be changed to an arbitrarily value. However, the estimation speed of disturbance used in the resonance ratio control should be fast enough compared to the resonant frequency of the controlled object. Too fast disturbance observer causes implementation problem.

In this paper, we propose a novel technique, "slow resonance ratio control", where the optimal speed of the disturbance estimation is slower and is given by the explicit formula. We will confirm the effectiveness of this method by simulation and experiment.

2. Steel Rolling Mill and 2-Inertia Model
Fig.1 illustrates the typical configuration of steel rolling mill system. Usually it can be modeled using several inertia moments and springs. For example, 12 inertia moments are needed. 2-inertia system given by Fig.2 is the simplest model, but is a well justified starting point. Fig.3 gives its block diagram.

In Fig.2, we assume

\[ J_{M0} + J_L = 1, \quad K_s = 1 \]  

for comparative analysis. These equations mean that the total inertia moment of the motor and the load, and the spring coefficient are fixed to 1, respectively. Various 2-inertia systems with different inertia ratios will be investigated under this assumption.

The transfer function from \( T_M \) to \( \omega_M \) which is most important in the closed loop design, is given by

\[ G_{11}(s) = \frac{1}{s} \frac{J_s s^2 + K_s}{J_{M0} J_s s^2 + K_s (J_{M0} + J_L)} = \frac{1}{J_{M0} s^2 + \omega_0^2} \]  

This transfer function has two particular points: the resonant and anti-resonant frequencies given by

\[ \omega_0 = \sqrt{\frac{K_s}{J_s} \left( 1 + \frac{J_s}{J_{M0}} \right)} \]  

\[ \omega_a = \sqrt{\frac{K_s}{J_s}} \]  

where \( R_0 \) is the inertia ratio given by \( R_0 = J_L / J_{M0} \). At these frequencies, the phase characteristics change drastically.

![Fig.1 Typical configuration of steel rolling mill system.](image1)

![Fig.2 2-inertia system model.](image2)

![Fig.3 Block diagram of 2-inertia system.](image3)
3. Resonance Ratio Control

Fig. 4 depicts our new technique: the slow resonance ratio control. When \( T_q = 0 \), it gives the ideal "resonance ratio control" proposed a few years ago. In usual disturbance observer applications, 100% of the estimated disturbance is fed back to the motor torque. Here, 1-K of the estimated disturbance is fed back. We can change the virtual motor inertia moment to any value as

\[
J_M = J_{M0}/K
\]  

(6)

The inertia ratio can be changed to

\[
R = J_L/J_M = J_L/J_{M0}/K
\]  

(7)

The resonant frequency is then changed to

\[
\omega_r = \sqrt{\frac{K_\Sigma}{J_L} \left(1 + \frac{J_L}{J_M}\right)}
\]  

(8)

The anti-resonant frequency does not change. By setting the new resonance ratio \( H = \omega_0/\omega_a \) to be \( 2 - \sqrt{5} \), effective vibration control was achieved.

Fig. 4 Configuration of the slow resonance ratio control.

4. Slow Resonance Ratio Control

When the estimation speed of the observer is finite, i.e., \( T_q > 0 \), the ideal resonance ratio control is impossible. By using \( Q \) as the low-pass filter part: \( 1/(T_q s + 1) \) in Fig. 4, the following important transfer function is obtained.

\[
\frac{\omega_L}{\omega_a} = \frac{s}{s^2 + (1 - Q)R_0 + QR_0} \frac{\omega_a}{\omega_a}
\]  

(9)

\( Q \) is not so big when \( Q = 1 \). \( T_q = 0 \) when \( Q = 1/(T_q s + 1) \), eq.(10) is obtained by putting \( Q \approx 1 \).

\[
\frac{\omega_L}{\omega_a} = \frac{s}{s^2 + (1 + R)\omega_a} = \frac{1}{s} \frac{\omega_a}{s^2 + \omega_a^2}
\]  

(10)

When the observer is very fast, i.e., \( T_q = 0 \), eq.(12) is obtained by putting \( Q \approx 0 \).

\[
\frac{\omega_L}{\omega_a} = \frac{s}{s^2 + (1 + R)\omega_a} = \frac{1}{s} \frac{\omega_a}{s^2 + \omega_a^2}
\]  

(12)

Two curves of eqs.(10) and (11) have the intersection at

\[
\omega_0 = \sqrt{\frac{1 + R + R_0}{2}} \omega_a
\]  

(12)

and the amplitude there is given by

\[
\frac{\omega_L}{\omega_a} = \frac{1 + R}{R - R_0}
\]  

(13)

Interestingly, all curves having \( T_q \) pass this point. Hence, if \( T_q \) is selected so that this point is the local maximum, vibration suppression can be realized most effectively. Such \( T_q \) is given by the explicit formulation of

\[
T_q = \sqrt{\frac{1 + 3R + R_0}{4} \left(1 + \frac{R + R_0}{2}\right) \frac{1}{\omega_a}}
\]  

(14)

This gives the optimal estimation speed of the disturbance observer.

Finally, we should design \( K \), the ratio of \( R \) (the new inertia ratio that the resonance ratio control aims to realize) to \( R_0 \) (the original inertia ratio), i.e., \( R = K R_0 \). When \( K \) increases, the amplitude peak given by eq.(13) becomes smaller but \( \omega_0 (=1/T_q) \) becomes bigger, which means that faster estimation is required. It causes implementation problem. Hence we need a compromise. If we select \( K = 5 \), the peak is relatively small while keeping \( \omega_q \) not so big for a wide range of \( R_0 \).

Next, we should design the speed controller. In eq.(9), \( \omega_0/T_q \) converges to \( 1/\omega \) when \( s \to 0 \), because we designed \( Q \) so as to keep their DC gains same regardless of \( R_0 \). It means that, in any \( R_0 \) cases, the 2-inertia system can be seen 1-inertia system having \( J_{M0} + J_L = 1 \) as the total inertia moment. It is very convenient if we can use the unique P&I speed controller designed for the 1-inertia system. Here, we use the following simple controller.

\[
K_p = \frac{1}{T_q} \quad K_i = \frac{K_p}{2.5 T_q}
\]  

(15), (16)

\( T_q \) is the specified speed response time. Here, we put \( T_q = 1/\omega_a \). Eq.(16) means that we selected the integral time constant to be 2.5 times of the speed control response. In simulation, Two-Degree-Of-Freedom P&I controller is used to reduce the overshoot in command response.

5. Simulation

Fig. 5 shows the time response simulation of the "slow resonance ratio control". At \( t = 5 \), the speed command \( \omega_0 = 1 \) is given to see the command response characteristics. At \( t = 25 \), step disturbance of \( T_2 = 0.5 \) is given to see the disturbance response. The model constants include 10~20% errors, and backlash (+/-0.01) and torque limiter (+/-1.2) are introduced.

We can know that the performances of the proposed method are same or even superior to other methods, e.g., the resonance ratio control, the optimal PID control, and even the state feedback control.
The speed controller here is designed for 1-inertia system without considering vibration suppression. Such an independent design has a great advantage in actual industrial application systems.

In the fast resonance ratio control shown in Fig.8, low frequency vibration can be suppressed effectively. However, the transfer function from \( T_q \) to \( \omega_n \) has a harmful frequency peak around 200 [rad/s]. Due to this peak, relatively big high frequency vibration remains in the motor torque.

The slow resonance ratio control shown in Fig.9 gives sufficiently stable vibration-less responses in frequency characteristics and motor torque waveforms. This is because there are no fast parts in the controller.

### 6. Experimental Results

Fig.6 illustrates the "Torsional Vibration System Experimental Setup" specially made by Mitsubishi Heavy Industry. It consists of two brushless DC motors, changeable backlash and friction mechanism, the load equipment and so on. Sensor information from shaft encoders and tacho-generators are read into the microcomputer via counter boards and A/D converters. Control algorithm is written by C language.

Table 1 gives the experimented control methods and their parameters. The inertia moments are converted to the load-side (i.e. the torsional shaft side) quantities. In all experiments, IP speed controllers are used by putting \( b=0 \) in Fig.4.

In the experiments of the command response shown in Fig.7(a)~Fig.9(a), the speed reference \( \omega_{ref}=10 \) [rad/s] is given at \( t=0 \). In experiments of the disturbance response shown in (b)'s, disturbance torque 2 [Nm] is added from the loadside motor at \( t=0 \).

\( \omega_{Ls}, \omega_{M}, \) and \( \omega_{m} \) are load, gear, and motor speeds. \( T_m \) is the motor torque. We tried to set the gear backlash to be 0, but there still remains little backlash. The motor torque is actually limited by +/-3.84[Nm]. Please note that the torque commands are drawn in the figures.

Fig.7 shows the responses of the original disturbance observer designed for 1-inertia system. It just suppresses the disturbance injected into the motor axis. As the result, big vibration was induced in the load speed and it considerably affected to the motor speed, too.

### 7. Conclusion

In this paper, we proposed the "slow resonance ratio control" as an effective torsional system control method. The optimal estimation speed of disturbance observer is given by the explicit formula. We can use the speed controller independently designed for 1-inertia system. We confirmed its superior performances by simulation and real experiment.
Fig. 7 Original disturbance observer.
Fig. 8 Fast resonance ratio control.
Fig. 9 Slow resonance ratio control.

References


