Object Manipulation Based on Robust and Adaptive Control by Hemispherical Soft Fingertips

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Abstract: This paper presents an object manipulation for a multi-fingered robot hand with hemispherical soft fingertips. The proposed controller is based on robust control and adaptive control. In the adaptive controller, the dynamic parameters of the object and multi-fingered robot hand are estimated adaptively in order to track a desired trajectory of the object. Feedback compensation of the robust controller is generated in order to decrease the error between the desired trajectory and real trajectory. The system with the proposed control law proved to be uniformly ultimately bounded for the tracking error. The importance of the damping factor of the soft fingertip is shown in the stability analysis. A simulation of object manipulation by the human-type robot hand using five fingers is shown.

Keywords: Adaptive Control; Robust Control; Multi-fingered Robot Hand.

1. INTRODUCTION

Many researchers have studied the control of grasping and manipulation of an object by a multi-fingered robot hand (Yoshikawa (2000); Bicchi (2000)). In many of these studies, fingertips were designed as rigid bodies because the mathematical analysis can be easily performed. However, it is difficult to manipulate an object because the contact friction is small. Various research has been conducted on the manipulations of objects by a multi-fingered robot hand using soft fingertips. The advantages of using soft fingertips are: a higher friction coefficient, stable grasping by plane contact, and ease-up of impulse. The disadvantages are that it is difficult to perform the mathematical analyses, such as kinematics, dynamics, and stability.

Yokokohji et al. (1999) proposed the dynamic control of a multi-fingered hand manipulating an object with frictional spin moment compensation. They implemented this dynamic control with vision-aided pose estimation of an object, and the experimental results confirmed its performance (Yokokohji et al. (2000)). However, in these papers, such effects of the deformation of soft fingertips as kinematics and dynamics were not studied. Inoue et al. (2009) proposed a two-stage control law via virtual desired angles. Using their control law, a desired joint angle is computed by integrating an error of object orientation. However, their control method has been considered in 2D space. Arimoto et al. (2000, 2001) analyzed the dynamics and control of pinch motions generated by a pair of two multi-degree-of-freedom robot fingers with soft and deformable tips pinching a rigid object. They found that there are separate feedback signals with different Jacobian coefficient vectors for realizing stable grasping, regulating the posture, and regulating the position of the object.

However, these control methods are considered in 2D space. Their recent studies analyzed the dynamics and control of pinching by a pair of two multi-degree-of-freedom robot fingers with soft and deformable tips in a 3D object (Arimoto et al. (2007)). However, this research focuses on grasping, not on manipulation.

For object manipulation, both adaptability and robustness is important. Because the dynamic parameter of the grasped object cannot be identified accurately. In order to solve this problem, we have proposed adaptive control of a multi-fingered robot hand (Ueki et al. (2006, 2008, 2009)). However, a robust controller has not been included in our controller, and our controllers have thus lacked robustness.

In this paper, an object manipulation by a multi-fingered robot hand with hemispherical soft fingertips based on robust control and adaptive control is proposed. The adaptive control is based on model-based adaptive control (Slotine et al. (1987)). In the adaptive controller, the dynamic parameter of the object and multi-fingered robot hand are estimated adaptively in order to track a desired trajectory of the object. The robust control is based on model-based robust control(Spong (1992)). Feedback compensation of the robust controller is generated in order to decrease errors between the desired trajectory and real trajectory. The basic idea of combining both controllers is based on a reference (Dawson et al. (1992); Nakada et al. (2006)). To our knowledge, these controllers were studied for a robot manipulator, and were not applied to a multi-fingered robot hand with soft fingertips. The proposed controller is extended to a multi-fingered robot hand with soft fingertips, which includes stability analysis. The system with the proposed control law is proved to be uniformly ultimately bounded for the tracking error.
Moreover, the importance of the damping factor of the soft fingertip is shown in the stability analysis. A simulation of object manipulation by the human-type robot hand using five fingers is shown.

2. TARGET SYSTEM

Consider the robot hand with \( k(\geq 3) \) fingers with 3 DOF (degrees of freedom) manipulating a rigid object in three-dimensional space, as shown in Fig. 1, in which the \( i \)-th robot finger contacts the object at point \( C_i \). The coordinate systems are defined as follows: \( \Sigma_p \) is the task coordinate system, \( \Sigma_o \) is the object coordinate system fixed on the object, and \( \Sigma_{f,i} \) is the \( i \)-th fingertip coordinate system fixed on the \( i \)-th fingertip. We also use notations defined as follows: \( p_{i} \in R^{3} \) is a position vector of the origin of the object coordinate system \( \Sigma_o \) with respect to \( \Sigma_p \), \( q_{f,i} \in R^{3} \) is the unit quaternion for orientation of the origin of the object coordinate system \( \Sigma_o \) with respect to \( \Sigma_p \), \( \omega_{i} \in R^{3} \) is the angular velocity vector of the origin of the object coordinate system \( \Sigma_o \) with respect to \( \Sigma_p \), \( q_{f} \in R^{4k} \) is the joint angle vector of robot fingers, \( \mathbf{p}_{\infty} \in R^{3} \) is the position vector from \( \Sigma_o \) to the contact point \( C_i \) with respect to \( \Sigma_o \), and \( \mathbf{p}_{f,i} \in R^{3} \) is the position vector from \( \Sigma_{f,i} \) to the contact point \( C_i \) with respect to \( \Sigma_{f,i} \).

To facilitate the dynamic formulation, the following assumptions are made.

(A1) All the finger-tips contact the common object at one point with frictional point contact (the point contact means that the contact force has three elements), and the frictional force at each contact point follows Coulomb’s law.

(A2) The deformation of the soft fingertip is composed only of compressive displacement \( d_{s,i} \).

(A3) The object and finger-tip surfaces are described by twice continuously differentiable hypersurfaces.

(A4) The relative motion at the contact point is the rolling contact with the fingertip’s radius \( r_i - d_{s,i} \), where \( r_i \) is the radius of the \( i \)-th fingertips when there is non-compression.

(A5) The constraint at each contact point is described by the rolling contact. The force generated by the constraint does not work on the system (d’Alembert’s principle).

2.1 Constraint condition

To perform system modeling, it is necessary to describe a contact condition that takes into account the deformation effect of soft-fingertips. When the \( i \)-th finger manipulates the object at a condition of rolling contact without slip, the tangent elements of relative translational velocities must be equal to zero, and the normal element of relative translational velocities is the speed of the soft-fingertip’s deformation. That is, the constraint condition is given by

\[
\mathbf{p}_{f,i} + \left[ \mathbf{w}_{f,i} \times \mathbf{R}_{f,i} \right] \mathbf{p}_{f,i} - \left( \mathbf{p}_{o} + \left[ \mathbf{w}_{o} \times \mathbf{R}_{o} \mathbf{n}_{\infty} \right] \mathbf{p}_{\infty} \right) = n_{c,i} d_{s,i},
\]

\( i = 1,2,...,k \),

where \( d_{s,i} \) is the velocity of deformation displacement, \( n_{c,i} \in R^{3} \) is the unit normal vector at \( C_i \), \( \mathbf{p}_{f,i} \) is the velocity vector of the origin of \( \mathbf{f}_{i} \) with respect to \( \Sigma_p \), \( \omega_{f,i} \in R^{3} \) is the angular velocity vector of the origin of \( \mathbf{f}_{i} \) with respect to \( \Sigma_p \), \( \mathbf{R}_{f,i} \in R^{3 \times 3} \) is a rotation matrix from \( \Sigma_p \) to \( \Sigma_{f,i} \), \( \mathbf{R}_{o} \in R^{3 \times 3} \) is a rotation matrix from \( \Sigma_p \) to \( \Sigma_{o} \), and \( \mathbf{x} = \left[ \mathbf{x} \right] \in R^{3 \times 3} \) is a skew-symmetric matrix expressing the cross product form of the \( \mathbf{x} \). Therefore, the relation between the object velocity and angular velocity of the finger joint is given by

\[
\mathbf{W}_{i}^{T}(\mathbf{R}_{\omega_{i},\mathbf{n}_{\infty}}) \mathbf{v}_{o} = \mathbf{J}_{c,i}(\mathbf{q}_{f,i}) \dot{\mathbf{q}}_{f,i} - n_{c,i} \dot{d}_{s,i},
\]

where \( \mathbf{v}_{o} = \left[ \mathbf{p}_{o}^{T} \mathbf{\omega}_{o}^{T} \right]^{T} \in R^{6} \) is the velocity vector of the object with respect to the task coordinate system \( \Sigma_p \), \( \mathbf{W}_{i} \in R^{6 \times 3} \) is the grasp form matrix, \( \mathbf{J}_{c,i} \in R^{3 \times 3} \) is the Jacobian matrix at the contact point, and \( \mathbf{q}_{f,i} \) is the joint angle vector of the \( i \)-th finger. For all fingers, (2) is represented as follows:

\[
\mathbf{W}^{T} \mathbf{v}_{o} = \mathbf{J}_{c} \dot{\mathbf{q}}_{f} - \mathbf{N}_{c} \dot{d}_{s},
\]

where \( \mathbf{W} = [\mathbf{W}_{1} \cdots \mathbf{W}_{k}] \), \( \mathbf{J}_{c} = \text{block diag} \left[ \mathbf{J}_{c_{1}} \cdots \mathbf{J}_{c_{k}} \right] \), \( \mathbf{N}_{c} = \text{block diag} \left[ \mathbf{n}_{c_{1}} \cdots \mathbf{n}_{c_{k}} \right] \), and \( \mathbf{d}_{s} = \left[ d_{s_{1}} \cdots d_{s_{k}} \right]^{T} \).

Furthermore, by defining \( \dot{\mathbf{y}} = \left[ \mathbf{v}_{o}^{T} \mathbf{q}_{f}^{T} \mathbf{d}_{s}^{T} \right]^{T} \) and \( \mathbf{G} = \left[ \mathbf{W}^{T} - \mathbf{J}_{c} \mathbf{N}_{c} \right] \), (3) is represented in a more compact form as follows:

\[
\mathbf{G} \dot{\mathbf{y}} = \mathbf{0}.
\]

2.2 Dynamic equation

The total kinetic energy for the target system is given by

\[
K = \frac{1}{2} \mathbf{q}_{f}^{T} \mathbf{M}_{f}(\mathbf{q}_{f}) \dot{\mathbf{q}}_{f} + \frac{1}{2} \mathbf{v}_{o}^{T} \mathbf{M}_{o}(\mathbf{R}_{o}) \mathbf{v}_{o},
\]

where \( \mathbf{M}_{f} \in R^{3k \times 3k} \) is the inertia matrix of the robot fingers, and \( \mathbf{M}_{o} \in R^{6 \times 6} \) is the inertia matrix of the object. Actually, a kinetic energy of deformation of the soft fingertip is thought to exist. Because the kinetic energy of deformation of the soft fingertip is small, it is possible to disregard. On the other hand, the total potential energy for the target system is given by

Fig. 1. Grasped object and multi-fingered hand coordinate system

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\[ P = P_f(q_f) + P_o(R_o) + \sum_{i=1}^k \int_{0}^{d_i} k_s(x)dx, \quad (6) \]

where \( P_f \) is the potential energy for the robot fingers caused by the effect of gravity, \( P_o \) is the potential energy for the object caused by the effect of gravity, the 3rd term is the elastic potential energy for deformation of soft fingertips, and \( k_s(x) \) is a function of the non-linear elastic model for soft fingertips. Hence, the Lagrangian of the target system is given by \( L = K - P \). Using the Lagrange multipliers, Lagrange’s equation of motion is derived by applying the variational principle in such a way

\[ \int_{t_1}^{t_2} \left( \delta L + u^T \delta y - f_c^T G \delta y \right) dt = 0, \quad (7) \]

where \( f_c \in \mathbb{R}^{3k} \) is Lagrange multipliers corresponding to the contact force, and \( u \) is a nonconservative and external force. In this paper, let us define \( u \) as

\[ [0^T \tau^T - (D_s x_s)^T]^T, \]

where \( \tau \in \mathbb{R}^{3k} \) is the input joint torque, and \( D_s \in \mathbb{R}^{k \times 3k} \) is a diagonal damping coefficient matrix. The dynamic equations for the target system are given by

\[ M_o(R_o)\ddot{v}_o + C_o(R_o, \omega_o) v_o + g_o(R_o) = W f_c, \quad (8) \]

\[ M_f(q_f)\ddot{q}_f + C_f(q_f, \dot{q}_f) \dot{q}_f + g_f(q_f) = \tau - J_c^T f_c, \quad (9) \]

\[ D_s x_s + k_s x_s = N_s^T f_c, \quad (10) \]

where \( C_o v_o \in \mathbb{R}^6 \) is the Coriolis and Centripetal forces of the object, \( g_o \in \mathbb{R}^6 \) is the gravity force term of the object, \( C_f \dot{q}_f \in \mathbb{R}^{3k} \) is the Coriolis and Centripetal forces of the robot fingers, \( g_f \in \mathbb{R}^{3k} \) is the gravity force term of the robot fingers, and \( k_s = [k_s \cdots k_s] \in \mathbb{R}^6 \) is an elastic force vector. In this paper, \( k_s \) refers to a reference (Arimoto et al. (2000)).

The dynamic equations (8) and (9) are characterized by the following structural properties, which are utilized in our controller design.

(P1) \( M_o \) and \( M_f \) are symmetric positive definite matrices.
(P2) A suitable definition of \( C_o \) and \( C_f \) makes the matrices \( M_o = 2C_o \) and \( M_f = 2C_f \) skew-symmetric.
(P3) The dynamic model is linear in its dynamic parameter.

3. PROPOSED CONTROLLER

To propose the control law, the following assumptions are made:

(A6) \( p_o, q_o, v_o, q_f, \dot{q}_f, f_c, \) and \( f_{fc} \) are measurable.
(A7) The desired trajectory \( p_{od}, q_{od}, v_{od}, \) and \( \dot{v}_{od} \) of the object are bounded and uniformly continuous at time \( t = \infty \).
(A8) The desired internal force \( f_{intd} \) is constant at \( t = \infty \).
(A9) The properties of force-closure and manipulable are satisfied, and there exists an internal force.
(A10) There exists known bound \( \rho_{o,i} \) and \( \rho_{f,i} \) on parametric uncertainty such that

\[ |\Delta \bar{a}_{o,i}| = |\bar{a}_{o,i} - a_{o,i}| \leq \rho_{o,i} \quad (11) \]

\[ |\Delta \bar{a}_{f,i}| = |\bar{a}_{f,i} - a_{f,i}| \leq \rho_{f,i} \quad (12) \]

where the subscript \( i \) of \( x \) means the \( i \)-th element of \( x \), \( \sigma_o \in \mathbb{R}^{\alpha_o} \) is a dynamic parameter vector of the object, \( \alpha_o \) is the number of the object dynamic parameter, \( \sigma_f \in \mathbb{R}^{\alpha_f} \) is a dynamic parameter vector of the robot finger, \( \alpha_f \) is the number of the robot finger dynamic parameter, and \( \bar{a}_o \) and \( \bar{a}_f \) represents the fixed parameters in dynamic equations.

3.1 Desired contact force

Let us define a position and orientation error of the object by

\[ e_o = \begin{bmatrix} s(\Delta p_o) \\ R_o(q_{od}, v_{od}) \end{bmatrix}, \quad (13) \]

where \( \Delta p_o = p_{od} - p_o \in \mathbb{R}^3 \) is the position error of the object, \( s(\Delta p_o) = [s(\Delta p_{o,2}) s(\Delta p_{o,3})]^T \in \mathbb{R}^3 \) is the position error using saturation function \( s(x) \), \( q_{od,v} \) is the vector part of quaternion \( q_{od} = q_{od}^0 \otimes q_{od}^v \) and \( \otimes \) is an operator of quaternion multiplication. The saturation function \( s(x) \) is a first derivative of \( S(x) \) with respect to \( x \) (i.e. \( s(x) = \frac{\partial S(x)}{\partial x} \)), the function \( S(x) \) and \( s(x) \) have the following properties:

1) \( S(x) > 0 \) for \( x \neq 0 \), and \( S(x) = 0 \) for \( x = 0 \);
2) \( S(x) \) is twice continuously differentiable, and \( s(x) \) is strictly increasing in \( x \) for \( \|x\| < \beta \) and saturated for \( \|x\| \geq \beta \);
3) There exist constants \( c_1 > 0, c_2 > 0, \) and \( c_3 > c_2 > 0 \) such that

\[ c_3 s^2(x) \geq s(x) \geq c_2 s^2(x) > 0 \]

\[ (S(x) \geq c_1 s^2(x) \text{ for } x \neq 0) \quad (14) \]

In the experiment, the functions \( S(x) \) and \( s(x) \) refer to a reference (Dong et al. (2006)).

Using (P3), a reference model of the object for adaptive control input is defined as

\[ \dot{M}_o \dot{v}_o + \dot{C}_o v_o + \dot{\dot{g}}_o = Y_o(R_o, \omega_o, v_{od}, \dot{v}_{od}) \dot{\sigma}_o \quad (15) \]

where \( Y_o \in \mathbb{R}^{6 \times \alpha_o} \) is a regressor with respect to the dynamic parameter, \( \dot{\sigma}_o \) is a parameter estimate of \( \sigma_o \), and \( \dot{M}_o, \dot{C}_o, \) and \( \dot{\dot{g}}_o \) are estimates of \( M_o, C_o, \) and \( g_o \), respectively, which are computed using \( \dot{\sigma}_o \). Moreover, the reference model for robust control input is defined as

\[ \dot{M}_o \dot{\varepsilon}_o + \dot{C}_o \varepsilon_o = E_o(R_o, \omega_o, \varepsilon_o, \dot{\varepsilon}_o)(\dot{\sigma}_o + \mu_o) \quad (16) \]

where \( E_o \in \mathbb{R}^{6 \times \alpha_o} \) is a regressor matrix, \( \dot{M}_o \) and \( \dot{C}_o \) are compensations of \( M_o \) and \( C_o \), respectively, which are computed using \( \dot{\sigma}_o + \mu_o \), \( \Lambda = \text{block diag} \{w_p I_3 w_c I_3 \} \in \mathbb{R}^{6 \times 6} \), \( w_p > 0 \) and \( w_c > 0 \) are scalar constants, and \( I_n \in \mathbb{R}^{n \times n} \) is an identity matrix.

Then the following desired external force \( F_{od} \in \mathbb{R}^6 \) is generated by:

\[ F_{od} = Y_o \dot{\sigma}_o + E_o(\dot{\sigma}_o + \mu_o) - K_s \dot{s}_o \quad (17) \]

where \( \mu_o = [\mu_{o,1} \cdots \mu_{o,\alpha_o}]^T \) is given by
\( \mu_{o,i} = \begin{cases} -\rho_{o,i}, & \text{if } |\xi_{o,i}| > \epsilon_{o,i} \\ -\rho_{o,i} |\xi_{o,i}|, & \text{if } |\xi_{o,i}| \leq \epsilon_{o,i} \end{cases} \) \hspace{1cm} (18)

\( \xi_{o} = E_{o}^{T}(R_{o}, o_{w}, \Lambda_{o}, \dot{\Lambda}_{o}) s_{o}. \) \hspace{1cm} (19)

where \( \epsilon_{o,i} > 0 \) is a design constant, \( K_{o} = \text{block diag} \{k_{p}I_{3}, k_{d}I_{3}\} \in \mathbb{R}^{6 \times 6} \) is a feedback gain matrix, \( k_{p} > 0 \) and \( k_{d} > 0 \) are scalar constants, and \( s_{o} \in \mathbb{R}^{6} \) is a residual error given by

\( s_{o} = -\Delta v_{o} - \Lambda e_{o} = v_{o} - v_{od} - \Lambda e_{o} \) \hspace{1cm} (20)

An adaptive law of the dynamic parameter of the object is given by

\[ \dot{\sigma}_{o} = -\Gamma_{o} Y_{o}^{T} s_{o} - s(|s||s|) \Gamma_{o}(\sigma_{o} - \tilde{\sigma}_{o}) \] \hspace{1cm} (21)

where \( \Gamma_{o} > 0 \in \mathbb{R}^{o_{x} \times o_{x}} \) is a symmetric adaptive gain matrix.

A desired contact force \( f_{cd} \in \mathbb{R}^{3k} \) is generated using \( F_{od}. \) The desired contact force should satisfy

\[ F_{od} = W f_{cd} \] \hspace{1cm} (22)

Moreover, the force at the contact points generates the internal force in the object. Hence, the general solution of the desired contact force is given by

\[ f_{cd} = W^{T} F_{od} + f_{intd} \] \hspace{1cm} (23)

where \( f_{intd} \) (in this paper, \( f_{intd} = (I_{3k} - W^{T}W)x, x \) is suitable inverse vector) is a desired internal force, and \( W^{T} \) is a pseudo inverse of \( W \) given by

\[ W^{T} = W^{T}(WW^{T})^{-1}. \] \hspace{1cm} (24)

3.2 Control input

Let us define the desired velocity of the joint angle \( \dot{q}_{fd} \) as follows:

\[ \dot{q}_{fd} = J_{c}^{-1} W^{T} v_{od}. \] \hspace{1cm} (25)

Similarly, let us define an error for the joint angle as follows:

\[ e_{f} = J_{c}^{-1} W^{T} \Lambda e_{o} + \Omega_{f}, \] \hspace{1cm} (26)

where \( \Omega > 0 \in \mathbb{R}^{ological} \times 3k \) is a symmetric gain matrix, and \( \nu \) is calculated to satisfy the following relation:

\[ \dot{\nu} + \kappa \nu = \kappa J_{c}^{T} \Delta f_{c}. \] \hspace{1cm} (27)

where \( \kappa > 0 \) is a design constant.

Using (P3), a reference model of the finger for adaptive control input is defined as

\[ \dot{M}_{f} \dot{q}_{fd} + \ddot{C}_{f} \dot{q}_{fd} + g_{f} = Y_{f}(q_{f}, \dot{q}_{f}, \ddot{q}_{fd}, \dot{q}_{fd}) \sigma_{f} \] \hspace{1cm} (28)

where \( Y_{f} \in \mathbb{R}^{3k \times 3k} \) is a regressor with respect to the dynamic parameter, \( \sigma_{f} \) is a parameter estimate of \( \sigma_{f} \), and \( \dot{M}_{f}, \ddot{C}_{f} \) and \( g_{f} \) are estimates of \( M_{f}, C_{f} \) and \( g_{f} \) respectively, which are computed using \( \tilde{\sigma}_{f} \). Moreover, the reference model for robust control input is defined as

\[ \dot{M}_{f} \dot{q}_{fd} + \ddot{C}_{f} \dot{q}_{fd} + g_{f} = E_{f}(q_{f}, \dot{q}_{f}, \ddot{q}_{f}, \dot{e}_{f})(\tilde{\sigma}_{f} + \mu_{f}) \] \hspace{1cm} (29)

where \( E_{f} \in \mathbb{R}^{3k \times 3k} \) is a regressor matrix, \( \dot{M}_{f} \) and \( \ddot{C}_{f} \) are compensations of \( M_{f} \) and \( C_{f} \) respectively, which are computed using \( \tilde{\sigma}_{f} \) and \( \mu_{f}. \)

Then a control law of the robot finger is given by

\[ \tau = Y_{f} \tilde{\sigma}_{f} + E_{f}(\tilde{\sigma}_{f} + \mu_{f}) - K_{f} s_{f} + J_{c}^{T} f_{cd} + \beta J_{c}^{T} \Delta f_{c}. \] \hspace{1cm} (30)

where \( \mu_{f} = \begin{pmatrix} \mu_{f,1}, \ldots, \mu_{f,3k} \end{pmatrix}^{T} \) is given by

\[ \mu_{f,i} = \begin{cases} -\rho_{f,i}, & \text{if } |\xi_{f,i}| > \epsilon_{f,i} \\ -\rho_{f,i} |\xi_{f,i}|, & \text{if } |\xi_{f,i}| \leq \epsilon_{f,i} \end{cases} \] \hspace{1cm} (31)

\( \epsilon_{f,i} > 0 \) is the design constant, \( K_{f} > 0 \) is the feedback gain matrix, \( \beta > 0 \) is the force feedback gain scalar constant, and \( s_{f}(\dot{\tilde{q}}_{f} - \dot{q}_{fd} - e_{f}) \) is a residual error. An adaptive law of the parameter estimate of the robot finger is given by

\[ \dot{\tilde{\sigma}}_{f} = -\Gamma_{f} Y_{f}^{T} s_{f} - s(|s||s|) \Gamma_{f}(\tilde{\sigma}_{f} - \tilde{\sigma}_{f}) \] \hspace{1cm} (33)

where \( \Gamma_{f} > 0 \in \mathbb{R}^{\sigma_{f} \times \sigma_{f}} \) is a symmetric adaptive gain matrix.

3.3 Stability analysis

It is easy to show the following equations:

\[ M_{o} \dot{s}_{o} + C_{o} s_{o} = Y_{o} \Delta \tilde{\sigma}_{o} + E_{o}(\Delta \sigma_{o} + \mu_{o}) - K_{o} s_{o} - W \Delta f_{c}. \] \hspace{1cm} (34)

\[ M_{f} \dot{s}_{f} + C_{f} s_{f} = Y_{f} \Delta \tilde{\sigma}_{f} + E_{f}(\Delta \sigma_{f} + \mu_{f}) - K_{f} s_{f} + \beta J_{c}^{T} \Delta f_{c}. \] \hspace{1cm} (35)

where \( k_{f} = \beta + 1 \), \( \Delta \tilde{\sigma}_{o} = \tilde{\sigma}_{o} - \sigma_{o} \) is an estimate error vector of the object dynamic parameter, \( \Delta \sigma_{o} = \tilde{\sigma}_{o} - \sigma_{o} \) is an uncertainty error vector of the object dynamic parameter, \( \Delta \sigma_{f} = \tilde{\sigma}_{f} - \sigma_{f} \) is an estimate error vector of the robot finger dynamic parameter, and \( \Delta \tilde{\sigma}_{f} = \tilde{\sigma}_{f} - \sigma_{f} \) is an uncertainty error vector of the robot finger dynamic parameter. Consider as a candidate for a Lyapunov function the following equation:

\[ V = \frac{k_{f}}{2}(s_{o}^{T} M_{o} s_{o} + \Delta \tilde{\sigma}_{o}^{T} \Gamma_{o}^{-1} \Delta \tilde{\sigma}_{o}) + \frac{1}{2} s_{f}^{T} M_{f} s_{f} + \frac{1}{2} \Delta \sigma_{f}^{T} \Gamma_{f}^{-1} \Delta \sigma_{f} + k_{f}^{2} \nu^{T} \Omega \nu + k_{f} k_{p} \nu_{p} (S(\Delta p_{o} \alpha) + S(\Delta p_{o} \gamma) + S(\Delta p_{o} \beta)) + k_{f} k_{p} \nu_{p} (q_{\text{oes}} - (q_{\text{oes}} - 1)^{2}) + k_{f} \int_{0}^{t} (d_{i} - d_{i-1}) k_{s}(y + d_{i}) - k_{s}(d_{i}) dy, \hspace{1cm} (36)\]

where \( q_{\text{oes}} \) is a scalar part of \( q_{\text{oes}} \), and \( d_{i} = k_{p}^{-1} n_{i} f_{\text{intd}} \) at time \( t = \infty \). A time derivative along the solution of the error equation gives the following equation using (P2), (34), (35), and (3):
\[ 
\dot{V} = -k_f \Delta v_o^T K_o \Delta v_o - k_f (\Delta e_o)^T K_o \Delta e_o - s_f^T K_f \dot{s}_f \\
- k_f \mu^T \Omega \nu - k_f \dot{d}_s^T (D_s \dot{d}_s - N^T (W' F_{od} + \Delta f_{intd})) \\
- k_f s(||s_o||)(|\Delta \sigma_o| + k_f s_o E_o (\Delta \sigma_o + \mu_o)) \\
- s(||s_f||)(|\Delta \sigma_f| + s_f^T E_f (\Delta \sigma_f + \mu_f)) \tag{37} 
\]

From (A10), the following inequality is satisfied
\[ 
\xi_{s,i}(\rho_{s,i} - \xi_{s,i}) \geq |\xi_{s,i}||\rho_{s,i} - |\Delta \sigma_{s,i}|| \geq 0, \tag{38} 
\]
where the subscript * is a wildcard character (i.e. o or f).

Therefore,
\[ 
s_f^T E_s (\Delta \sigma_s + \mu_s) \leq \sum_{i=1}^{\alpha_s} \xi_{s,i}(\rho_{s,i} - \xi_{s,i}) + \mu_{s,i}. \tag{39} \]
\[ 
\xi_{s,i}(\rho_{s,i} - \xi_{s,i}) + \mu_{s,i} = \\
0 \quad \text{if } |\xi_{s,i}| > \epsilon_{s,i} \\
\frac{\rho_{s,i} - \xi_{s,i}}{2} + \frac{\mu_{s,i}}{4} \quad \text{if } |\xi_{s,i}| \leq \epsilon_{s,i} \tag{40} \]

Furthermore, the following inequalities are satisfied
\[ 
-(\Delta \sigma - \hat{\Delta} \sigma_s)^T \Delta \dot{\sigma}_s \leq -\frac{1}{2} \|\Delta \sigma_s\|^2 + \frac{1}{4} \|\Delta \dot{\sigma}_s\|^2 \tag{41} \]
\[ 
-\frac{d_s^T (D_s \dot{d}_s - N^T (W' F_{od} + \Delta f_{intd})) \leq -\frac{\delta_D}{\lambda D_s} (\|\dot{d}_s\|^2 + \frac{\delta_f}{\lambda D_s}) \tag{42} \]

where \( \lambda D_s \) is the minimum of the eigen value of \( D_s \), and \( \delta_D \) is the maximum of \( N^T (W' F_{od} + \Delta f_{intd}) \). As these results, \( \dot{V} \) represents as
\[ 
\dot{V} \leq -k_f \Delta v_o^T K_o \Delta v_o - k_f (\Delta e_o)^T K_o \Delta e_o - s_f^T K_f \dot{s}_f \\
- k_f \mu^T \Omega \nu - k_f \lambda D_s (\|\dot{d}_s\|^2 + \frac{\delta_D}{\lambda D_s} + \frac{\delta_f}{\lambda D_s}) \\
- k_f s(||s_o||)(|\Delta \sigma_o| + k_f s_o E_o (\Delta \sigma_o + \mu_o)) \\
- s(||s_f||)(|\Delta \sigma_f| + s_f^T E_f (\Delta \sigma_f + \mu_f)) \tag{44} 
\]

On the other hand, if \( \|\dot{x}_f\| \leq (\frac{\delta_f}{\lambda Q_f})^2 \), then \( V \) increases.

This result means that \( \|\dot{x}_f\| \) and/or \( |\Delta \dot{\sigma}_s| \) increase. If \( \|\dot{x}_f\| \) increases, then \( \|\dot{x}_f\| > (\frac{\delta_f}{\lambda Q_f})^2 \) and Eq. (44) are satisfied in sequence. If \( \|\dot{x}_f\| \) does not increase, \( |\Delta \dot{\sigma}_s| \) is bounded because of Eqs. (21) and (33). Therefore, the system has uniformly ultimately bounded.

4. SIMULATION

Simulation of box handling by the human-type robot hand named Gifu-Hand III(Mouri et al. (2007)) was performed to show the performance of the proposed control method. In the simulation, as shown in Fig. 2, five fingers grasp a box (size : 0.07 × 0.07 × 0.18 (m), mass : 0.2 (kg)). The simulation conditions are as follows: The initial values of the unknown dynamic parameter are set to zero. The gain matrices are given by trial and error. The desired trajectory of the box is given repeatedly by a 5 order polynomial in time with the initial \( p_{od}(0) = p_{o}(0), q_{od}(0) = q_{o}(0) \) and terminal \( p_{od}(0.5) = p_{o}(0) + [0.0, 0.0, -0.02]^T, q_{od}(0.5) = q_{o}(0) \). \( \gamma_{ld} \) is given as follows
\[ 
\gamma_{ld} = 2.0(I_{15} - W' W) \begin{bmatrix} p_{f_{c_1}}^T \\
p_{f_{c_2}}^T \\
\vdots \\
p_{f_{c_{15}}}^T \end{bmatrix}^T, 
\]
where \( I_{15} \in R^{15 \times 15} \) is an identity matrix.

The simulation result is shown in Fig. 3. Fig. 3 (a) shows \( z \) of the desired object position \( p_{ad,z} \) and the actual object position \( p_{a,z} \). Fig. 3 (b) shows the norm of the object position error \( \Delta p_{od} \). Fig. 3 (c) shows the norm of the object orientation error \( q_{od,z} \). Fig. 3 (d) shows the norm of the contact force error \( \Delta f_{c} \). Fig. 3 (a) shows that the actual \( z \) element of the object position tracks the desired object position well. The error norms of object position, object orientation, and contact force decreased by the adaptive controller within 6 seconds from the beginning of the simulation.

5. CONCLUSION

An object manipulation by a multi-fingered robot hand with hemispherical soft fingertips based on robust control and adaptive control has been proposed. In the adaptive controller, the dynamic parameter of the object and multi-fingered robot hand are estimated adaptively in order
Fig. 3. Experimental result of proposed controller to track a desired trajectory of the object. Feedback compensation of the robust controller is generated in order to decrease the error between the desired trajectory and real trajectory. The system with the proposed control law proved to have uniformly ultimately bounded tracking error. The simulation result shows that the proposed controller had a bounded tracking error, and the objective of the controller design was achieved.

REFERENCES


